**P26.37**

(a) Because the capacitors are connected in parallel, their voltage remains the same:

\[ U = \frac{1}{2} C (\Delta V)^2 + \frac{1}{2} C (\Delta V)^2 = C (\Delta V)^2 = \left(10.0 \times 10^{-6} \, \mu F\right) (50.0 \, V)^2 = 2.50 \times 10^{-2} \, J \]

(b) Because \( C = \frac{\varepsilon_0 A}{d} \) and \( d \rightarrow 2d \), the altered capacitor has new capacitance to \( C' = \frac{C}{2} \).

The total charge is the same as before:

\[ Q \text{\textsubscript{initial}} = Q \text{\textsubscript{final}} \]

\[ C (\Delta V) + C (\Delta V) = C (\Delta V') + \frac{C}{2} (\Delta V') \]

\[ 2C (\Delta V) = \frac{3}{2} C (\Delta V') \]

\[ \Delta V' = \frac{4}{3} \Delta V = \frac{4}{3} (50.0 \, V) = 66.7 \, V \]

(c) New \( U' = \frac{1}{2} C (\Delta V')^2 + \frac{1}{2} C \left(\frac{1}{2} C\right) (\Delta V')^2 = \frac{3}{4} C (\Delta V')^2 = \frac{3}{4} C \left(\frac{4\Delta V}{3}\right)^2 \)

\[ U' = \frac{4}{3} C (\Delta V)^2 = \frac{4}{3} U = \frac{4}{3} \left(2.50 \times 10^{-2} \, J\right) = 3.30 \times 10^{-2} \, J \]

(d) Positive work is done by the agent pulling the plates apart.

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**P26.46**

The area \( A \) has dimensions of 0.0700 m by \( \ell \).

\[ C = \frac{\varepsilon_0 A}{d} \]

or \[ 9.50 \times 10^{-8} = \frac{3.70 \left(8.85 \times 10^{-12}\right) (0.0700 \, \ell)}{0.0250 \times 10^{-3}} \]

\[ \ell = 1.04 \, m \]

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**P26.48**

The given combination of capacitors is equivalent to the circuit diagram shown to the right.

\[ \begin{array}{c|c|c|c|c}
   A & B & C & D \\
   \hline
   40 \mu F & 10 \mu F & 40 \mu F & \\
   \end{array} \]

ANS FIG. P26.48

Put charge \( Q \) on point A. Then,

\[ Q = (40.0 \, \mu F) \Delta V_{AB} = (10.0 \, \mu F) \Delta V_{BC} = (40.0 \, \mu F) \Delta V_{CD} \]

So, \( \Delta V_{BC} = 4 \Delta V_{AB} = 4 \Delta V_{CD} \), and the center capacitor will break down first, at \( \Delta V_{BC} = 15.0 \, V \).

When this occurs,

\[ \Delta V_{AB} = \Delta V_{CD} = \frac{1}{4} (\Delta V_{BC}) = 3.75 \, V \]

and \( V_{AD} = V_{AB} + V_{BC} + V_{CD} = 3.75 \, V + 15.0 \, V + 3.75 \, V = 22.5 \, V \).
The electric field produced by the line of charge has radial symmetry about the y axis. According to equation 24.7 in Example 24.4, the electric field to the right of the y axis is

$$\vec{E} = E(r) \hat{i} = 2k_e \frac{\lambda}{r} \hat{i}$$

Let $x = 25.0 \text{ cm}$ represent the coordinate of the center of the dipole charge, and let $2a = 2.00 \text{ cm}$ represent the distance between the charges. Then $r_+ = x - a \cos \theta$ is the coordinate of the negative charge and $r_- = x + a \cos \theta$ is the coordinate of the positive charge.

The force on the positive charge is

$$\vec{F}_+ = qE(r_+) \hat{i} = q \left( 2k_e \frac{\lambda}{r_+} \hat{i} \right) = 2k_e \frac{q \lambda}{x + a \cos \theta} \hat{i},$$

and the force on the negative charge is

$$\vec{F}_- = -qE(r_-) \hat{i} = -q \left( 2k_e \frac{\lambda}{r_-} \hat{i} \right) = -2k_e \frac{q \lambda}{x - a \cos \theta} \hat{i}.$$

The force on the dipole is

$$\vec{F} = \vec{F}_+ + \vec{F}_- = \left( 2k_e \frac{q \lambda}{x + a \cos \theta} - 2k_e \frac{q \lambda}{x - a \cos \theta} \right) \hat{i} = 2k_e q \lambda \left( \frac{1}{x + a \cos \theta} - \frac{1}{x - a \cos \theta} \right) \hat{i}$$

$$= 2k_e q \lambda \left[ \frac{(x - a \cos \theta) - (x + a \cos \theta)}{x^2 + (a \cos \theta)^2} \right] \hat{i} = 4k_e q \lambda \frac{a \cos \theta}{x^2 + (a \cos \theta)^2} \hat{i}$$

$$= -4 \left( 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \left( 0.0100 \text{ m} \right) \left( 10.0 \times 10^{-6} \text{ C} \right) \left( 2.00 \times 10^{-6} \text{ C/m} \right) \cos 35.0^\circ \hat{i}$$

$$= -9.42 \times 10^{-2} \hat{i} \text{ N}$$

Let $x$ represent the coordinate of the negative charge. Then $x + 2a \cos \theta$ is the coordinate of the positive charge. The force on the negative charge is

$$\vec{F}_- = -qE(x) \hat{i} \text{. The force on the positive charge is}$$

$$\vec{F}_+ = qE(x + 2a \cos \theta) \hat{i} = q \left[ E(x) + \frac{dE}{dx} (2a \cos \theta) \right] \hat{i}$$

The force on the dipole is altogether

$$\vec{F} = \vec{F}_+ + \vec{F}_- = q \frac{dE}{dx} (2a \cos \theta) \hat{i} = \rho \frac{dE}{dx} \cos \theta \hat{i} \text{.}$$
The general form of Gauss’s law describes how a charge creates an electric field in a material, as well as in vacuum. It is
\[ \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon} \]
where \( \varepsilon = \kappa \varepsilon_0 \) is the permittivity of the material. (a) A sheet with charge \( Q \) uniformly distributed over its area \( A \) is surrounded by a dielectric. Show that the sheet creates a uniform electric field at nearby points, with magnitude \( E = \frac{Q}{2A\varepsilon} \). (b) Two large sheets of area \( A \), carrying opposite charges of equal magnitude \( Q \), are a small distance \( d \) apart. Show that they create uniform electric field in the space between them, with magnitude \( E = \frac{Q}{dA\varepsilon} \). (c) Assume that the negative plate is at zero potential. Show that the positive plate is at potential \( QdA\varepsilon \). (d) Show that the capacitance of the pair of plates is \( A\varepsilon/d = \kappa \varepsilon_0 A/d \).

Consider a gaussian surface in the form of a cylindrical pillbox with ends of area \( A' \ll A \) parallel to the sheet. The side wall of the cylinder passes no flux of electric field since this surface is everywhere parallel to the field. Gauss’s law becomes

\[ EA' + EA' = \frac{Q}{\varepsilon} A' \] directed away from the positive sheet.

(b) In the space between the sheets, each creates field \( \frac{Q}{2\varepsilon A} \) away from the positive and toward the negative sheet. Together, they create a field of

\[ E = \frac{Q}{\varepsilon A} \]

(c) Assume that the field is in the positive \( x \)-direction. Then, the potential of the positive plate relative to the negative plate is

\[ \Delta V = -\int_{\text{+ plate}} \mathbf{E} \cdot d\mathbf{s} - \int_{\text{+ plate}} \frac{Q}{\varepsilon A} i \cdot (\hat{i} dx) = \frac{Qd}{\varepsilon A} \]

(d) Capacitance is defined by:

\[ C = \frac{Q}{\Delta V} = \frac{Q}{Qd/A} = \frac{\varepsilon A}{\kappa \varepsilon_0 A/d} = \frac{\kappa \varepsilon_0 A}{d} \]

To repair a power supply for a stereo amplifier, an electronics technician needs a 100-\( \mu \)F capacitor capable of withstanding a potential difference of 90 V between the plates. The only available supply is a box of five 100-\( \mu \)F capacitors, each having a maximum voltage capability of 50 V. Can the technician substitute a combination of these capacitors that has the proper electrical characteristics? If so, what will be the maximum voltage across any of the capacitors used? (*Suggestion:* The technician may not have to use all the capacitors in the box.)

Placing two identical capacitor in series will split the voltage evenly between them, giving each a voltage of 45 V, but the total capacitance will be half of what is needed. To double the capacitance, another pair of series capacitors must be placed in parallel with the first pair, as shown in the upper figure. The equivalent capacitance is

\[ \left( \frac{1}{100 \ \mu F} + \frac{1}{100 \ \mu F} \right)^{-1} + \left( \frac{1}{100 \ \mu F} + \frac{1}{100 \ \mu F} \right)^{-1} = 100 \ \mu F \]

Another possibility shown in the lower figure: two capacitors in parallel, connected in series to another pair of capacitors in parallel; the voltage across each parallel section is then 45 V. The equivalent capacitance is

\[ \frac{1}{(100 \ \mu F + 100 \ \mu F)^{-1} + (100 \ \mu F + 100 \ \mu F)^{-1}} = 100 \ \mu F \]
One capacitor cannot be used by itself—it would burn out. She can use two capacitors in series, connected in parallel to another two capacitors in series. Another possibility is two capacitors in parallel, connected in series to another two capacitors in parallel. In either case, one capacitor will be left over.

Each of the four capacitors will be exposed to a maximum voltage of 45 V.

\[ E_{\text{max}} = \frac{2k_e \lambda}{a} \text{ from an equation derived about this situation in Chapter 24.} \]

\[ \Delta V = 2k_e \lambda \ln \left( \frac{b}{a} \right) \text{ from Example 26.1.} \]

\[ E_{\text{max}} = \frac{\Delta V}{a \ln (b/a)} \]

\[ \Delta V_{\text{max}} = E_{\text{max}} a \ln \left( \frac{b}{a} \right) = \left( 18.0 \times 10^6 \text{ V/m} \right) \left( 0.800 \times 10^{-3} \text{ m} \right) \ln \left( \frac{3.00}{0.800} \right) = 19.0 \text{ kV} \]

Chapter 27: 26, 63, 67

(a) A thin cylindrical shell of radius \( r \), thickness \( dr \), and length \( L \) contributes resistance

\[ dR = \frac{\rho dl}{A} = \frac{\rho dr}{(2\pi r)L} = \left( \frac{\rho}{2\pi L} \right) \frac{dr}{r} \]

The resistance of the whole annulus is the series summation of the contributions of the thin shells:

\[ R = \frac{\rho}{2\pi L} \int_{a}^{b} \frac{dr}{r} = \frac{\rho}{2\pi L} \ln \left( \frac{b}{a} \right) \]

(b) In this equation

\[ \frac{\Delta V}{L} = \frac{\rho}{2\pi L} \ln \left( \frac{b}{a} \right) \]

we solve for

\[ \rho = \frac{2\pi L \Delta V}{I \ln \left( \frac{b}{a} \right)} \]
(a) We require two conditions:

\[ R = \frac{\rho_1 \ell_1}{\pi r^2} + \frac{\rho_2 \ell_2}{\pi r^2} \]  

(1)

where carbon = 1 and Nichrome = 2, and for any \( \Delta T \)

\[ R = \frac{\rho_1 \ell_1}{\pi r^2} (1 + \alpha_1 \Delta T) + \frac{\rho_2 \ell_2}{\pi r^2} (1 + \alpha_2 \Delta T) \]  

(2)

Setting equations (1) and (2) equal to each other, we have

\[ \frac{\rho_1 \ell_1}{\pi r^2} + \frac{\rho_2 \ell_2}{\pi r^2} = \frac{\rho_1 \ell_1}{\pi r^2} (1 + \alpha_1 \Delta T) + \frac{\rho_2 \ell_2}{\pi r^2} (1 + \alpha_2 \Delta T) \]

\[ \Rightarrow \frac{\rho_1 \ell_1}{\pi r^2} + \frac{\rho_2 \ell_2}{\pi r^2} = \frac{\rho_1 \ell_1}{\pi r^2} \alpha_1 \Delta T + \frac{\rho_2 \ell_2}{\pi r^2} \alpha_2 \Delta T \]

\[ \Rightarrow \frac{\rho_2 \ell_2}{\pi r^2} \alpha_2 \Delta T = -\frac{\rho_1 \ell_1}{\pi r^2} \alpha_1 \Delta T \]

\[ \rho_2 \ell_2 \alpha_2 = -\frac{\rho_1 \ell_1}{\rho_2 \alpha_2} \alpha_1 \]

(3)

The two equations (1) and (3) are just sufficient to determine \( \ell_1 \) and \( \ell_2 \).

The design goal can be met.

(b) From Table 27.2, \( \alpha_1 = -0.5 \times 10^{-3} \, (^\circ \text{C})^{-1} \) and \( \alpha_2 = 0.4 \times 10^{-3} \, (^\circ \text{C})^{-1} \).

Use equation (3) to solve for \( \ell_2 \) in terms of \( \ell_1 \):

\[ \ell_2 = -\frac{\rho_1}{\rho_2 \alpha_2} \alpha_1 \ell_1 \]

then substitute this into equation (1):

\[ R = \frac{\rho_1 \ell_1}{\pi r^2} + \frac{\rho_2}{\pi r^2} \left( -\frac{\rho_1}{\rho_2 \alpha_2} \alpha_1 \ell_1 \right) = \frac{\rho_1}{\pi r^2} \left( 1 - \frac{\alpha_1}{\alpha_2} \right) \ell_1 \]

\[ 10.0 \, \Omega = \frac{(3.5 \times 10^{-5} \, \Omega \cdot \text{m})}{\pi (1.50 \times 10^{-3} \, \text{m})^2} \left( 1 - \frac{-0.5 \times 10^{-3}}{0.4 \times 10^{-3}} \right) \ell_1 \]

\[ \Rightarrow \ell_1 = 0.898 \, \text{m} \]

and so \( \ell_2 = -\frac{\rho_1}{\rho_2 \alpha_2} \alpha_1 \ell_1 = -\frac{(3.5 \times 10^{-5} \, \Omega \cdot \text{m})}{(1.50 \times 10^{-6} \, \Omega \cdot \text{m})} \left( -\frac{0.5 \times 10^{-3}}{0.4 \times 10^{-3}} \right) \ell_1 = 26.2 \, \text{m} \)

Therefore, \( \ell_1 = 0.898 \, \text{m} \) and \( \ell_2 = 26.2 \, \text{m} \).
(a) Think of the device as two capacitors in parallel. The one on the left has \( \kappa_1 = 1 \),

\[
A_1 = \left( \frac{\ell}{2} + x \right) \ell .
\]
The equivalent capacitance is

\[
\frac{\kappa_1 \varepsilon_0 A_1}{d} + \frac{\kappa_2 \varepsilon_0 A_2}{d} = \frac{\varepsilon_0 \ell}{d} \left( \frac{\ell}{2} + x \right) + \frac{\kappa \varepsilon_0 \ell}{d} \left( \frac{\ell}{2} - x \right) = \frac{\varepsilon_0 \ell}{2d} \left( \ell + 2x + \kappa \ell - 2\kappa x \right)
\]

(b) The charge on the capacitor is \( Q = C \Delta V \)

\[
Q = \frac{\varepsilon_0 \ell \Delta V}{2d} \left( \ell + 2x + \kappa \ell - 2\kappa x \right)
\]

The current is

\[
I = \frac{dQ}{dt} = \frac{dQ}{dx} \frac{dx}{dt} = \frac{\varepsilon_0 \ell \Delta V}{2d} \left( 0 + 2 + 0 - 2\kappa \right)v = -\frac{\varepsilon_0 \ell \Delta V}{d} (\kappa - 1)
\]

The negative value indicates that the current drains charge from the capacitor. Positive current is clockwise \( \frac{\varepsilon_0 \ell \Delta V}{d} (\kappa - 1) \).