This box contains

1. a net positive charge.
2. no net charge.
3. a net negative charge.
4. a positive charge.
5. a negative charge.
Quiz Fun!

This box contains

1. a net positive charge.
2. no net charge.
3. a net negative charge. **(Correct Answer)**
4. a positive charge.
5. a negative charge.
Maxwell’s Equations
Gauss is #1!

### General case

The Equations are given in SI units. See below for CGS units.

<table>
<thead>
<tr>
<th>Name</th>
<th>Differential form</th>
<th>Integral form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauss’s law:</td>
<td>( \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} )</td>
<td>( \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{Q_S}{\varepsilon_0} )</td>
</tr>
<tr>
<td>Gauss’ law for magnetism (absence of magnetic monopoles):</td>
<td>( \nabla \cdot \mathbf{B} = 0 )</td>
<td>( \oint_S \mathbf{B} \cdot d\mathbf{A} = 0 )</td>
</tr>
<tr>
<td>Faraday’s law of induction:</td>
<td>( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} )</td>
<td>( \oint_{\partial S} \mathbf{E} \cdot dl = -\frac{d\Phi_{E,S}}{dt} )</td>
</tr>
<tr>
<td>Ampère’s Circuital Law (with Maxwell’s correction):</td>
<td>( \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} )</td>
<td>( \oint_{\partial S} \mathbf{B} \cdot dl = \mu_0 \mathbf{I}<em>S + \mu_0 \varepsilon_0 \frac{d\Phi</em>{E,S}}{dt} )</td>
</tr>
</tbody>
</table>
Gauss’s Law

The net electric flux through ANY closed surface is due to the net charge contained inside that surface (divided by $\varepsilon_0$)!

$$\Phi = \oint_S \mathbf{E} \cdot d\mathbf{A} = \frac{1}{\varepsilon_0} \int_V \rho \, dV = \frac{Q_A}{\varepsilon_0}$$

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \frac{q_{\text{in}}}{\varepsilon_0}$$
The amount of FLOW perpendicular to a surface!

**Electric Flux**

\[ d\Phi = \vec{E} \cdot dA\hat{n} \]

\[ d\Phi = E\cos\theta dA \]

\[ \Phi = \int \vec{E} \cdot d\vec{A} \]

Remember that dot products ‘select’ mutually parallel components of vectors. For flux, a vector normal to the surface area determines the direction. On a closed surface, positive flux always points OUTWARDS.
1. The electric field in the region of space shown is given by \( E = (8i + 2yj) \) N/C where \( y \) is in m. What is the magnitude of the electric flux through the top face of the cube shown?

\[
\Phi = \int \vec{E} \cdot d\vec{A}
\]

a. 90 N \cdot m^2/C
b. 6.0 N \cdot m^2/C
c. 54 N \cdot m^2/C
d. 12 N \cdot m^2/C
e. 126 N \cdot m^2/C
Gauss’s Law

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E\hat{r} \cdot dA\hat{r} = E \oint dA \]

\[ = EA = \frac{kq}{r^2} \frac{4\pi r^2}{4\pi \varepsilon_0} = \frac{1}{4\pi \varepsilon_0} q 4\pi = \frac{q}{\varepsilon_0} \]

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0} \]
Gauss’s Law

The net electric flux through ANY closed surface is due to the net charge contained inside that surface (divided by $\varepsilon_0$)!

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0} \]

\[ \Phi = \int_S \vec{E} \cdot d\vec{A} = \frac{1}{\varepsilon_0} \int_V \rho \, dV = \frac{Q_A}{\varepsilon_0} \]
Why/How Use Gauss’s Law?

• Electric fields of some continuous distributions of charge can be found much more easily than using Coulomb’s Law.
• Gauss’s Law is valid for moving charges, but Coulomb’s law is not. Thus, Gauss’s Law is more fundamental.
• Use Gauss’s Law to find the Electric field when you have symmetry and already know what the field should look like.
• The Gaussian surface must have the same symmetry and every part of it must either be tangent to or perpendicular to the electric field.
• You need not enclose all the charge within the Gaussian surface.
Solid Sphere: Insulator

\[ \Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0} \]
Recall: Gravitational Force INSIDE the Earth

Inside the Earth the Gravitational Force is Linear.

Acceleration decreases as you fall to the center (where your speed is the greatest) and then the acceleration increases but in the opposite direction, slowing you down to a stop at the other end…but then you would fall back in again, bouncing back and forth forever!
4. A solid nonconducting sphere (radius = 12 cm) has a charge of uniform density (30 nC/m³) distributed throughout its volume. Determine the magnitude of the electric field 15 cm from the center of the sphere.

a. 22 N/C
b. 49 N/C
c. 31 N/C
d. 87 N/C
e. 26 N/C
These are two-dimensional cross sections through three-dimensional closed spheres and a cube. Rank order, from largest to smallest, the electric fluxes $\Phi_a$ to $\Phi_e$ through surfaces a to e.

A. $\Phi_a > \Phi_c > \Phi_b > \Phi_d > \Phi_e$
B. $\Phi_b = \Phi_e > \Phi_a = \Phi_c = \Phi_d$
C. $\Phi_e > \Phi_d > \Phi_b > \Phi_c > \Phi_a$
D. $\Phi_b > \Phi_a > \Phi_c > \Phi_e > \Phi_d$
E. $\Phi_d = \Phi_e > \Phi_c > \Phi_a = \Phi_b$
These are two-dimensional cross sections through three-dimensional closed spheres and a cube. Rank order, from largest to smallest, the electric fluxes $\Phi_a$ to $\Phi_e$ through surfaces a to e.

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D. $\Phi_b > \Phi_a > \Phi_c > \Phi_e > \Phi_d$

E. $\Phi_d = \Phi_e > \Phi_c > \Phi_a = \Phi_b$
Chapter 23: The Electric Field of a Uniform Rod along the Bisector

\[ E_y = \frac{kQ}{y\sqrt{y^2 + (l/2)^2}} \]

If \( y \gg l \)

\[ E_y \rightarrow \frac{kQ}{y^2} \]

Far away from the rod, the field looks like that of a point charge.
Far Away from the Finite Line:
The rod looks like a point particle

\[ E_y \rightarrow \frac{kQ}{y^2} \]
Infinite Line of Charge

\[
\lim_{L \to \infty} E_x = \lim_{L \to \infty} \frac{kQ}{y\sqrt{y^2 + (L/2)^2}} = \frac{k2Q}{yL} = 2k \frac{\lambda}{y}
\]

\[E_r = 2k \frac{\lambda}{r} = \frac{2}{4\pi \varepsilon_0} \frac{\lambda}{r} = \frac{\lambda}{2\pi \varepsilon_0 r}
\]

Notice: The electric field of an infinitely long charged rod decreases as \(1/r\)! This is a good approximation for a finite line of charge points far from the ends.
An infinite line of charge: We can get the same result using Gauss’s Law with much greater ease using symmetry arguments. The rod is VERY Important because it models a wire!!

\[ E_r = \frac{\lambda}{2\pi r\varepsilon_0} \]
Consider a long cylindrical charge distribution of radius $R$ with a uniform charge density $\rho$. Find the electric field at distance $r$ from the axis where $r < R$, $r > R$. What if the Cylinder is a conductor?
Cylinder: Insulator

2. A long nonconducting cylinder (radius = 12 cm) has a charge of uniform density (5.0 nC/m³) distributed throughout its column. Determine the magnitude of the electric field 5.0 cm from the axis of the cylinder.

a. 25 N/C
b. 20 N/C
c. 14 N/C
d. 31 N/C
e. 34 N/C
Conductors

Free electrons move to the surface until the interior is neutral. They move until the parallel component of the field is zero and the conductor is at **Electrostatic Equilibrium**.

**At Electrostatic Equilibrium:**
- The field inside is zero.
- External field lines are perpendicular to the surface.
- External field lines end on charges on the surface.
- Any excess charges lies entirely on the surface.
- At **Electrostatic Equilibrium**, the conducting surface **shields** the interior of external fields.
- The magnitude of the Electric field near a conducting surface is: $\sigma/\varepsilon_0$.
- On asymmetric objects, charges collect at pointy edges, such as on lightening rods.
3. A long straight metal rod has a radius of 2.0 mm and a surface charge of density 0.40 nC/m². Determine the magnitude of the electric field 3.0 mm from the axis.

a. 18 N/C
b. 23 N/C
c. 30 N/C
d. 15 N/C
e. 60 N/C
5. Charge of uniform density (80 nC/m³) is distributed throughout a hollow cylindrical region formed by two coaxial cylindrical surfaces of radii 1.0 mm and 3.0 mm. Determine the magnitude of the electric field at a point which is 4.0 mm from the symmetry axis.

a. 7.9 N/C  
b. 10 N/C  
c. 9.0 N/C  
d. 8.9 N/C  
e. 17 N/C
Spherical Conducting Shells

\[ E_{in} = 0 \]
Concentric Spherical Shells

6. A charge of 5.0 pC is distributed uniformly on a spherical surface (radius = 2.0 cm), and a second charge of -2.0 pC is distributed uniformly on a concentric spherical surface (radius = 4.0 cm). Determine the magnitude of the electric field 3.0 cm from the center of the two surfaces.

a. 30 N/C
b. 50 N/C
c. 40 N/C
d. 20 N/C
e. 70 N/C
Faraday Cage

A Faraday cage or Faraday shield is an enclosure formed by conducting material, or by a mesh of such material. Such an enclosure blocks out external static electrical fields. Faraday cages are named after physicist Michael Faraday, who built one in 1836. Externally applied electric fields produce forces that rearranges the charges in the conductor so as to cancel the applied external field inside. Coaxial cable is a faraday cage made from concentric cylinders.
From 23: Infinite Charged Plane

$$\lim_{R \to \infty} E_{Disk} = \lim_{R \to \infty} 2\pi k\sigma \left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right) = 2\pi k\sigma$$

$$E = 2\pi k\sigma = \frac{2\pi}{4\pi\varepsilon_0} \sigma = \frac{\sigma}{2\varepsilon_0}$$

Infinite Charged Plane has a constant Uniform Field EVERYWHERE!!!!
Infinite Plane using Gauss

\[ E = \frac{\sigma}{2\varepsilon_0} \]
Parallel Plates: In the center we assume they are infinite and that the field is constant and Uniform!

\[ E = \frac{\sigma}{\varepsilon_0} \]
Ch25: ENERGY & VOLTAGE

E is the FORCE FIELD
V is the ENERGY FIELD

The force field lines are everywhere perpendicular to the potential energy field lines.
Next Week’s Lab

The field lines are perpendicular to the equipotential surfaces.

The equipotential surfaces gradually change from the shape of one electrode to that of the other.