E: The Electric Field

Lines Shows how the Force acts on a Positive Test Charge.
B: The Magnetic Field

Lines point from North (+) to South (-) or form closed loops
It is important to review the gravitational force and field because it has the same mathematical form as the electric force and field.
Man of the Millennium
Sir Issac Newton

(1642 -1727)
Universal Law of Gravity

\[ \mathbf{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{\mathbf{r}}_{12} \]

(Minus because of the direction of the unit vector. Attractive Central Forces are negative!)

\[ G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2} \]
Gravity: Inverse Square Law

\[ F = \frac{G m M}{d^2} \]
Inside the Earth the Gravitational Force is Linear.

Acceleration decreases as you fall to the center (where your speed is the greatest) and then the acceleration increases but in the opposite direction, slowing you down to a stop at the other end...but then you would fall back in again, bouncing back and forth forever!
Superposition of Gravitational Force Vectors

\[ \vec{F} = \vec{F}_{12} + \vec{F}_{13} \]

What is the net force on mass 2?

Problem Solving Strategy
1. Draw FBD
2. Calculate Force Magnitudes
3. Resolve into components
4. Add components
5. Express as ijk vector
6. Find magnitude and direction of resultant force.
Gravitational Force & Field

\[ \vec{F}_G = \frac{Gm_1m_2}{r^2} \hat{r} \]

\[ \vec{F}_G = m_2 \vec{g} \]

\[ \mathbf{g} = \frac{\vec{F}_g}{m} = -\frac{GM}{r^2} \hat{r} \]

Gravity field is the force that +1kg would feel if it were placed at this location. The gravitational Field of the Earth at the surface of the Earth is simply, \( g \):

\[ g = \frac{GM_E}{r^2} = 9.81 \text{N/kg} = 9.81 \text{m/s}^2 \]
In general, $g$ for any Planet:

The gravitational field describes the “effect” that any object has on the empty space around itself in terms of the force that would be present if a second object were somewhere in that space

$$g = \frac{F_g}{m} = -\frac{GM}{r^2} \hat{r}$$
Electric Field
Two flavors of charge (+/-)

More on this later.....
Electric FORCE
What is Charge?
Electrons are Fundamental and have a unit of charge that cannot be divided.

In bound systems, the electron is a standing wave of Probability...

\[ q = -1.6 \times 10^{-19} \text{ Coulombs (C)} \]
Quarks are Fundamental and can not be divided. Quarks have fractional charge.

<table>
<thead>
<tr>
<th>Flavor</th>
<th>Mass</th>
<th>Elect. Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>u up</td>
<td>.005</td>
<td>+2/3</td>
</tr>
<tr>
<td>d down</td>
<td>.01</td>
<td>-1/3</td>
</tr>
<tr>
<td>c charm</td>
<td>1.5</td>
<td>+2/3</td>
</tr>
<tr>
<td>s strange</td>
<td>0.2</td>
<td>-1/3</td>
</tr>
<tr>
<td>t top</td>
<td>180</td>
<td>+2/3</td>
</tr>
<tr>
<td>b bottom</td>
<td>4.7</td>
<td>-1/3</td>
</tr>
</tbody>
</table>
Quarks combine to make particles with unit charge.

\[
\frac{2}{3} + \frac{2}{3} - \frac{1}{3} = +1
\]

An up, up, down makes a proton for a total charge of plus one.
How many protons in a Coulomb?

\[ 1.00 \text{C} \times (1 \text{ proton}) / (1.60 \times 10^{-19} \text{C}) \]
\[ = 6.25 \times 10^{18} \text{ protons!} \]
elementary charge

• the charge on one proton is
  \[ q = +1.6 \times 10^{-19} \text{Coulombs (C)} \]
• the charge on the electron is \(-q\)
• like charges repel, opposites attract
• the force between charges decreases with distance, obeying an inverse square Law
• charge is conserved
• charge is not observed to be divided
Charge is Conserved

Before
$q_{\text{tot}} = 0$

(b)

After
$q_{\text{tot}} = 0$
Charge is Conserved

\[ \Delta m = 2m_e = \frac{E}{c^2} \]

Before \( q_{\text{tot}} = 0 \)  

After \( q_{\text{tot}} = 0 \)  

(a)
This looks like the Big Bang! So if charge is conserved where is all the antimatter?

Matter-Antimatter Anisotropy

\[ \Delta m = 2m_e = E/c^2 \]

Before
\[ q_{\text{tot}} = 0 \]

(a) After
\[ q_{\text{tot}} = 0 \]
Conductivity

- **Conductors:** loosely bound valence electrons; charge flows freely  
  Ex: metals, salt water

- **Insulators:** tightly bound electrons; charge hard to move  
  Ex: organic materials, wood, glass, water

- Electrons move, protons are bound in a lattice
- Positively charged ions flow in fluid (liquid or gas)
- Positive & Negative charges rearrange and become Polarized in Insulators
- Most materials are Electrically NEUTRAL (lowest potential energy – electron shells want to be filled!)
Electric forces for charges that are not moving. Though static charge DO move! But not continuously as with current.

Static Cling: Adhesion
Most materials are Electrically NEUTRAL.
TOTAL charge is conserved for any process.
TOTAL charge is conserved for any process. Net Charge is still zero after rubbing.
Charging by Induction
Next Lab: Charging by Induction

Sample: Charging a metal sphere by *induction* using a negatively charged rod

(a) A neutral metallic sphere, with equal numbers of positive and negative charges.

(b) The electrons on the neutral sphere are redistributed when a charged **negatively** charge rubber rod is placed near the sphere.

(c) When the sphere is grounded, some of its electrons leave through the ground wire.

(d) When the ground connection is removed while leaving the rod close to the sphere, the sphere has excess positive charge that is nonuniformly distributed.

(e) When the rod is removed, the remaining electrons redistribute uniformly and there is a net uniform distribution of **positive charge** on the sphere.
Coulomb’s Law

\[ F_{12} = k \frac{|q_1| |q_2|}{r^2} = F_{21} \]

\[ k = 9 \times 10^9 \text{ N m}^2/\text{C}^2 \]

Likes Repel (+):

Opposites Attract (-):
Electric Force Vector

Like Charges: Repulsive: Positive

\[ F_{12} = +k \frac{q_1 q_2}{r^2} \hat{r} \]

Opposite Charges: Attractive: Negative

\[ F_{12} = -k \frac{q_1 q_2}{r^2} \hat{r} \]

Minus because of the definition of the direction of reference frame and the unit vector. Sign Convention: Attractive Central Forces are negative!
Electric Interaction

\[ F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1|q_2|}{r^2} = F_{21} \]

\[ \varepsilon_0 = 8.8542 \cdot 10^{-12} \]

\[ \mathcal{E}_0 : \text{Permittivity of free space} \]

Permittivity is a physical quantity that describes how an electric field affects and is affected by a dielectric medium, and is determined by the ability of a material to polarize in response to the field, and thereby reduce the total electric field inside the material. Thus, permittivity relates to a material's ability to transmit (or "permit") an electric field.
What is the Electric Force between two electrons separated by 1 μm?

\[ k = 9 \times 10^9 \text{ N m}^2/\text{C}^2 \]
\[ q_e = -1.6 \times 10^{-19} \text{ (C)} \]

\[ F_{12} = k \frac{|q_1||q_2|}{r^2} = F_{21} \]
\[ = (9.00 \times 10^9 \text{ Nm}^2/\text{C}^2) \times (-1.60 \times 10^{-19} \text{ C})^2 \]
\[ = 2.30 \times 10^{-16} \text{ N} \]
How strong is the Coulomb Force relative to Gravity?

\[ F_{12} = k \frac{|q_1||q_2|}{r^2} \quad F_G = \frac{Gm_1m_2}{r^2} \]

\[ k = 9 \times 10^9 \text{ N m}^2/\text{C}^2 \]
\[ G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \]
\[ q_e = 1.6 \times 10^{-19} \text{ C} \]
\[ m_p = 1.67 \times 10^{-27} \text{ kg}, \quad m_e = 9.11 \times 10^{-31} \text{ kg} \]
How strong is the Coulomb Force relative to Gravity?

\[ F_{12} = k \frac{|q_1||q_2|}{r^2} \]

\[ F_G = \frac{Gm_1m_2}{r^2} \]

\[ F_C = 8.20 \times 10^{-8} \, N \]

\[ F_G = 3.61 \times 10^{-47} \, N \]

\[ \frac{F_C}{F_G} = 2.27 \times 10^{39} \]
Electric Force

Two point charges are lying on the y-axis. Ignore gravity. They are equidistant from the point P, which lies on the x-axis. An electron is placed at P and released. What direction will the electron move? What is the net force acting on the electron? Draw the force diagram showing the net force.

\[ q_1 = +4.00 \mu C, \quad q_2 = +4.00 \mu C \]

\[ k = 9 \times 10^9 \text{ N m}^2/\text{C}^2 \]
Electric Field

Shows how the field acts on a POSTIVE test charge!

\[ E = \text{Force/unit Charge} \]
Electric Field

• Definition:

\[ \vec{F} = q\vec{E} \quad E = \frac{F}{q} \]

• Electric field is the force per unit coulomb: it is the force that +1C would feel if it were placed at this location.

• E field is in N/C.

• E field is a VECTOR.

• E field shows how the force acts on a POSTIIVE test charge!

Electric Field E due to \( q_1 \):

\[ E = \frac{kq_1}{r^2} \]
Electric Field Vector

The electric field at $P$ due to $q$. To find the sign and directions ask, “how would the charge act on a POSITIVE ‘test’ charge?

\[ \vec{E}_p = \pm k \frac{q}{r^2} \hat{r} \]

Positive Charge: Repulsive: Positive

Negative Charge: Attractive: Negative
**Electric Field Lines:**

- Direction is for a POSITIVE Test charge
- Direction of E-field at any point is tangent to the field line
- Magnitude of E-field at any point is proportional to line density
- Number of lines leaving or entering a charged object is proportional to the charge on the object
- Electric Force is in the direction of the E field for positive charges and in the opposite direction for negative charges.
Equal Distance – Equal Force
Number of Field Lines ~ Electric Field Strength
A conductor in an external Electric Field will Polarize
Electric Field Visualization

http://www.falstad.com/vector3de/
MIT Shockwave Visual EM

http://web.mit.edu/8.02t/www/802TEAL3D/visualizations/guidedtour/Tour.htm
Superposition of Electric Field Vectors

$$E_P = E_{1P} + E_{2P}$$

What is the net field at P due to all charges?

Problem Solving Strategy
1. Draw FBD
2. Calculate Field Magnitudes
3. Resolve into components
4. Add components
5. Express as ijk vector
6. Find magnitude and direction of resultant field.
Two Positive Charges

Two positive point charges of equal magnitude are lying on the y-axis. Ignore gravity. They are equidistant from the point P, which lies on the x-axis. What is the net electric field at P? Draw the field vectors showing the net field.

\[ q_1 = +4.00 \mu C, \quad q_2 = +4.00 \mu C \]

Where is the E field zero?
Rank the Field Strength

Fig 23-23a, p.725
Electric Dipole: Two point charges of equal but opposite magnitude are lying on the y-axis. Ignore gravity. They are equidistant from the point P, which lies on the x-axis. What is the net electric field at P? Draw the field vectors showing the net field. Derive a general solution too. What is the electric field for a point much further away than the charge separation distance?

\[ q_1 = +4.00 \mu C, \quad q_2 = -4.00 \mu C \]
Dipole Electric Field Bisector

In practice, we almost always observe the electric field of a dipole only for distance much greater than the charge separation and the $E \sim 1/r^3$ holds everywhere.

Near: $E_y = \frac{2kq}{(x^2 + a^2)^{3/2}}$

Far: $E_y = \frac{2kq}{x^3}$
Electric Dipole: H2O

Although an electric dipole has zero net charge, it does have a net electric field. Some molecules, such as the water molecule, have permanent electric dipoles. The polar nature of water molecules allows them to bond to each other in groups and is associated with the high surface tension of water. The dipole moment of water provides a "handle" for interaction with microwave electric fields in a microwave oven.

More on electric dipoles in CH 26
Book Solves the Dipole with this orientation: SAME RESULT!
Recall from Physics 40

**Center of Mass**

The geometric ‘center’ or average location of the mass.

Extended Body:

\[ \mathbf{r}_{\text{CM}} = \frac{1}{M} \int \mathbf{r} \, dm \]

System of Particles:

\[ x_{\text{CM}} = \frac{\sum_{i} m_i x_i}{M} \]
Rotational Inertia

The resistance of an object to rotate.

Extended Body:

$$I = \int r^2 \, dm$$

System of Particles:

$$I = \sum m_i r_i^2$$

Solid cylinder or disk

$$I_{CM} = \frac{1}{2} MR^2$$
Continuous Charge Distribution

Note: For spheres, cylinders, infinite planes, rods, etc, the electric field is found using Gauss’s Law which we’ll do next chapter!

\[ E = \int dE = k \int \frac{dq}{r^2} \hat{r} \]

\[ \lambda = \frac{Q}{L} \quad \text{linear charge density} \]
\[ \sigma = \frac{Q}{A} \quad \text{surface charge density} \]
\[ \rho = \frac{Q}{V} \quad \text{volume charge density} \]

\[ dq = \lambda dx \]
\[ dq = \sigma dA \]
\[ dq = \rho dV \]
Finite Uniform Rod: Bisector

A thin rod of length $\ell$ and uniform charge per unit length $\lambda$ lies along the $x$ axis, as shown. Find the electric field at $P$, a distance $y$ from the rod along its perpendicular bisector, and the electric field at $P$ of the rod if it is of infinite length.
Finite Uniform Rod: Bisector

\[ E_y = \frac{kQ}{y \sqrt{y^2 + (l/2)^2}} \]

If \( y \gg l \)

\[ E_y \rightarrow \frac{kQ}{y^2} \]

Far away from the rod, the field looks like that of a point charge.
Far Away from the Finite Line: \[ E_y \rightarrow \frac{kQ}{y^2} \]
Infinite Line of Charge
Bisecting the Rod

\[ \lim_{L \to \infty} E_x = \lim_{L \to \infty} \frac{kQ}{y\sqrt{y^2 + (L/2)^2}} = \frac{k2Q}{yL} = 2k \frac{\lambda}{y} \]

**Notice:** The electric field of an infinitely long charged rod decreases as \(1/r!\)

This is a good approximation for a finite line of charge points far from the ends.
NonUniform Charge Distribution

28. A line of charge starts at \( x = +x_0 \) and extends to positive infinity. The linear charge density is \( \lambda = \lambda_0 x_0 / x \). Determine the electric field at the origin.

Solution With a continuous charge distribution we integrate over an electric field element, \( d\mathbf{E} \):

\[ \mathbf{E} = \int d\mathbf{E} = -\int \frac{dq}{r^2} \hat{i} \]

The minus sign is because the Electric field acts on a positive test charge at the origin and so point in the negative \( x \) direction. \( dq \) is the charge element found from the charge density, in this case, the linear charge density: 

\[ dq = \lambda dx = \frac{\lambda_0 x_0}{x} dx \]

Putting this all together and integrating from \( x_0 \) to infinity we have:

\[ \mathbf{E} = \int d\mathbf{E} = -\int_{-\infty}^{\infty} \left[ k_e \lambda_0 x_0 \frac{dx}{x^3} \right] \hat{i} \]

\[ = -k_e \lambda_0 x_0 \hat{i} \int_{x_0}^{\infty} x^{-3} dx = -k_e \lambda_0 x_0 \hat{i} \left( -\frac{1}{2x^2} \right)_{x_0}^{\infty} = -\left( \frac{k_e \lambda_0}{2x_0} \right) \hat{i} \]
Let’s Review these Examples!

\[ E_{\text{ring}} = \frac{kxQ}{\left(x^2 + a^2\right)^{3/2}} \]

\[ E_{\text{Disk}} = 2\pi k \sigma \left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right) \]
Infinite Charged Plane

$$\lim_{R \to \infty} E_{Disk} = \lim_{R \to \infty} 2\pi k \sigma \left( 1 - \frac{x}{\sqrt{x^2 + R^2}} \right) = 2\pi k \sigma$$

$$E = 2\pi k \sigma = \frac{2\pi}{4\pi \varepsilon_0} \sigma = \frac{\sigma}{2\varepsilon_0}$$

Infinite Charged Plane has a constant Uniform Field EVERYWHERE!!!!
Infinite Charged Plane

\[ E = \frac{\sigma}{2\varepsilon_0} \]
Rank in order, from largest to smallest, the electric field strengths $E_a$ to $E_e$ at these five points near an infinite plane of charge.

1. $E_a = E_b = E_c = E_d = E_e$
2. $E_a > E_c > E_b > E_e > E_d$
3. $E_b = E_c = E_d = E_e > E_a$
4. $E_a > E_b = E_c > E_d = E_e$
5. $E_e > E_d > E_c > E_b > E_a$
Rank in order, from largest to smallest, the electric field strengths $E_a$ to $E_e$ at these five points near an infinite plane of charge.

1. $E_a = E_b = E_c = E_d = E_e$
2. $E_a > E_c > E_b > E_e > E_d$
3. $E_b = E_c = E_d = E_e > E_a$
4. $E_a > E_b = E_c > E_d = E_e$
5. $E_e > E_d > E_c > E_b > E_a$
Parallel Plates: In the center we assume they are infinite and that the field is constant and Uniform!

$$E = \frac{\sigma}{\varepsilon_0}$$
Uniform Infinite Parallel Plates

\[ E = \frac{\sigma}{\varepsilon_0} \]

**IMPORTANT!!!!**

The electric field is uniform, therefore the force has equal magnitude at any distance from the plates. Therefore, the acceleration is constant between the plates.
Parallel Plate Capacitor

Two 6.0 cm diameter circular electrodes are spaced 5.0 mm apart. The capacitor is charged by transferring $10^{11}$ electrons from the right to the left electrode. An electron is released from rest at the surface of the negative electrode. How long does it take the electron to cross to the positive electrode? What speed does it collide? Assume the space between the plates is a vacuum.
Cathode Ray Tube
Which electric field is responsible for the trajectory of the proton?

(1)  
(2)  
(3)  
(4)  
(5)
Which electric field is responsible for the trajectory of the proton?

(1)  
(2)  
(3)  
(4)  
(5)
An electron enters the lower left side of a parallel plate capacitor and exits at the upper right side as shown. The initial speed of the electron is $v_i$. Assume that the electric field is uniform everywhere between the plates. Find the magnitude of the electric field.

$v_i = 7.00 \times 10^6 \text{ m/s}$
The TOTAL FORCE
Electric & Magnetic Fields

The Electric Force acts parallel to the Electric Field.

\[
F = qE + qv \times B
\]

The Magnetic Force acts perpendicular to the Magnetic Field and the velocity.