### 17.3 Intensity of Periodic Sound Waves

In Chapter 16, we showed that a wave traveling on a taut string transports energy, consistent with the notion of energy transfer by mechanical waves in Equation 8.2. Naturally, we would expect sound waves to also represent a transfer of energy. Consider the element of gas acted on by the piston in Figure 17.5. Imagine that the piston is moving back and forth in simple harmonic motion at angular frequency \( \omega \). Imagine also that the length of the element becomes very small so that the entire element moves with the same velocity as the piston. Then we can model the element as a particle on which the piston is doing work. The rate at which the piston is doing work on the element at any instant of time is given by Equation 8.19:

\[
P_{\text{power}} = \mathbf{F} \cdot \mathbf{\dot{x}}
\]

where we have used \( P_{\text{power}} \) rather than \( P \) so that we don’t confuse power \( P \) with pressure \( P \). The force \( \mathbf{F} \) on the element of gas is related to the pressure and the velocity \( \dot{x} \) of the element is the derivative of the displacement function, so we find

\[
P_{\text{power}} = [\Delta P(x, t) A] \mathbf{i} \cdot \frac{\partial}{\partial t} [s(x, t) \mathbf{i}]
\]

\[
= [\rho \omega A s_{\text{max}} \sin (kx - \omega t)] \left\{ \frac{\partial}{\partial t} [s_{\text{max}} \cos (kx - \omega t)] \right\}
\]

\[
= \rho \omega A s_{\text{max}} \sin (kx - \omega t) \left[ \omega s_{\text{max}} \sin (kx - \omega t) \right]
\]

\[
= \rho \nu \omega^2 A s_{\text{max}}^2 \sin^2 (kx - \omega t)
\]

We now find the time average power over one period of the oscillation. For any given value of \( x \), which we can choose to be \( x = 0 \), the average value of \( \sin^2 (kx - \omega t) \) over one period \( T \) is

\[
\frac{1}{T} \int_0^T \sin^2 (0 - \omega t) \, dt = \frac{1}{T} \int_0^T \sin^2 \omega t \, dt = \frac{1}{T} \left[ \left( \frac{t}{2} + \sin 2\omega t \right) \right]_0^T = \frac{1}{2}
\]

Therefore,

\[
(P_{\text{power}})_{\text{avg}} = \frac{1}{2} \rho \nu \omega^2 A s_{\text{max}}^2
\]

We define the intensity \( I \) of a wave, or the power per unit area, as the rate at which the energy transported by the wave transfers through a unit area \( A \) perpendicular to the direction of travel of the wave:

\[
I = \frac{(P_{\text{power}})_{\text{avg}}}{A}
\]

(17.11) \hspace{1cm} \blacktriangleleft \text{Intensity of a sound wave}

In this case, the intensity is therefore

\[
I = \frac{1}{2} \rho \nu (\omega s_{\text{max}})^2
\]

Hence, the intensity of a periodic sound wave is proportional to the square of the displacement amplitude and to the square of the angular frequency. This expression can also be written in terms of the pressure amplitude \( \Delta P_{\text{max}} \); in this case, we use Equation 17.10 to obtain

\[
I = \frac{(\Delta P_{\text{max}})^2}{2\rho \nu}
\]

(17.12)

The string waves we studied in Chapter 16 are constrained to move along the one-dimensional string, as discussed in the introduction to this chapter. The sound waves we have studied with regard to Figures 17.1 through 17.3 and 17.5 are constrained to move in one dimension along the length of the tube. As we mentioned
in the introduction, however, sound waves can move through three-dimensional bulk media, so let’s place a sound source in the open air and study the results.

Consider the special case of a point source emitting sound waves equally in all directions. If the air around the source is perfectly uniform, the sound power radiated in all directions is the same, and the speed of sound in all directions is the same. The result in this situation is called a spherical wave. Figure 17.6 shows these spherical waves as a series of circular arcs concentric with the source. Each arc represents a surface over which the phase of the wave is constant. We call such a surface of constant phase a wave front. The radial distance between adjacent wave fronts that have the same phase is the wavelength \( \lambda \) of the wave. The radial lines pointing outward from the source, representing the direction of propagation of the waves, are called rays.

The average power emitted by the source must be distributed uniformly over each spherical wave front of area \( 4\pi r^2 \). Hence, the wave intensity at a distance \( r \) from the source is

\[
I = \frac{(\text{Power})_{\text{avg}}}{4\pi r^2}
\]

The intensity decreases as the square of the distance from the source. This inverse-square law is reminiscent of the behavior of gravity in Chapter 13.

Quick Quiz 17.2 A vibrating guitar string makes very little sound if it is not mounted on the guitar body. Why does the sound have greater intensity if the string is attached to the guitar body? (a) The string vibrates with more energy. (b) The energy leaves the guitar at a greater rate. (c) The sound power is spread over a larger area at the listener’s position. (d) The sound power is concentrated over a smaller area at the listener’s position. (e) The speed of sound is higher in the material of the guitar body. (f) None of these answers is correct.

Example 17.1 Hearing Limits

The faintest sounds the human ear can detect at a frequency of 1 000 Hz correspond to an intensity of about \( 1.00 \times 10^{-12} \text{ W/m}^2 \), which is called threshold of hearing. The loudest sounds the ear can tolerate at this frequency correspond to an intensity of about \( 1.00 \text{ W/m}^2 \), the threshold of pain. Determine the pressure amplitude and displacement amplitude associated with these two limits.

**SOLUTION**

Conceptualize Think about the quietest environment you have ever experienced. It is likely that the intensity of sound in even this quietest environment is higher than the threshold of hearing.

Categorize Because we are given intensities and asked to calculate pressure and displacement amplitudes, this problem requires the concepts discussed in this section.

Analyze To find the amplitude of the pressure variation at the threshold of hearing, use Equation 17.12, taking the speed of sound waves in air to be \( v = 343 \text{ m/s} \) and the density of air to be \( \rho = 1.20 \text{ kg/m}^3 \):

\[
\Delta P_{\text{max}} = \sqrt{2\rho v I}
\]

\[
= \sqrt{2(1.20 \text{ kg/m}^3)(343 \text{ m/s})(1.00 \times 10^{-12} \text{ W/m}^2)}
\]

\[
= 2.87 \times 10^{-5} \text{ N/m}^2
\]

Calculate the corresponding displacement amplitude using Equation 17.10, recalling that \( \omega = 2\pi f \) (Eq. 16.9):

\[
\delta_{\text{max}} = \frac{\Delta P_{\text{max}}}{\rho \omega v}
\]

\[
= \frac{2.87 \times 10^{-5} \text{ N/m}^2}{(1.20 \text{ kg/m}^3)(343 \text{ m/s})(2\pi \times 1000 \text{ Hz})}
\]

\[
= 1.11 \times 10^{-11} \text{ m}
\]

In a similar manner, one finds that the loudest sounds the human ear can tolerate (the threshold of pain) correspond to a pressure amplitude of \( 28.7 \text{ N/m}^2 \) and a displacement amplitude equal to \( 1.11 \times 10^{-5} \text{ m} \).
Finalize Because atmospheric pressure is about $10^5$ N/m$^2$, the result for the pressure amplitude tells us that the ear is sensitive to pressure fluctuations as small as $3$ parts in $10^{10}$! The displacement amplitude is also a remarkably small number! If we compare this result for $s_{\text{max}}$ to the size of an atom (about $10^{-10}$ m), we see that the ear is an extremely sensitive detector of sound waves.

Example 17.2 Intensity Variations of a Point Source

A point source emits sound waves with an average power output of 80.0 W.

(A) Find the intensity 3.00 m from the source.

**SOLUTION**

**Conceptualize** Imagine a small loudspeaker sending sound out at an average rate of 80.0 W uniformly in all directions. You are standing 3.00 m away from the speakers. As the sound propagates, the energy of the sound waves is spread out over an ever-expanding sphere.

**Categorize** We evaluate the intensity from an equation generated in this section, so we categorize this example as a substitution problem.

Because a point source emits energy in the form of spherical waves, use Equation 17.13 to find the intensity:

$$I = \frac{(\text{Power})_{\text{avg}}}{4\pi r^2} = \frac{80.0 \text{ W}}{4\pi (3.00 \text{ m})^2} = 0.707 \text{ W/m}^2$$

This intensity is close to the threshold of pain.

(B) Find the distance at which the intensity of the sound is $1.00 \times 10^{-8}$ W/m$^2$.

**SOLUTION**

Solve for $r$ in Equation 17.13 and use the given value for $I$:

$$r = \sqrt{\frac{(\text{Power})_{\text{avg}}}{4\pi I}} = \sqrt{\frac{80.0 \text{ W}}{4\pi (1.00 \times 10^{-8} \text{ W/m}^2)}} = 2.52 \times 10^4 \text{ m}$$

Sound Level in Decibels

Example 17.1 illustrates the wide range of intensities the human ear can detect. Because this range is so wide, it is convenient to use a logarithmic scale, where the sound level $\beta$ (Greek letter beta) is defined by the equation

$$\beta = 10 \log \left( \frac{I}{I_0} \right)$$

(17.14)

The constant $I_0$ is the reference intensity, taken to be at the threshold of hearing ($I_0 = 1.00 \times 10^{-12}$ W/m$^2$), and $I$ is the intensity in watts per square meter to which the sound level $\beta$ corresponds, where $\beta$ is measured in decibels (dB). On this scale, the threshold of pain ($I = 1.00 \text{ W/m}^2$) corresponds to a sound level of $\beta = 10 \log \left[ (1 \text{ W/m}^2)/(10^{-12} \text{ W/m}^2) \right] = 10 \log (10^{12}) = 120 \text{ dB}$, and the threshold of hearing corresponds to $\beta = 10 \log \left[ (10^{-12} \text{ W/m}^2)/(10^{-12} \text{ W/m}^2) \right] = 0 \text{ dB}$.

Prolonged exposure to high sound levels may seriously damage the human ear. Ear plugs are recommended whenever sound levels exceed 90 dB. Recent evidence suggests that “noise pollution” may be a contributing factor to high blood pressure, anxiety, and nervousness. Table 17.2 gives some typical sound levels.

<table>
<thead>
<tr>
<th>Source of Sound</th>
<th>$\beta$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearby jet airplane</td>
<td>150</td>
</tr>
<tr>
<td>Jackhammer; machine gun</td>
<td>130</td>
</tr>
<tr>
<td>Siren; rock concert</td>
<td>120</td>
</tr>
<tr>
<td>Subway; power lawn mower</td>
<td>100</td>
</tr>
<tr>
<td>Busy traffic</td>
<td>80</td>
</tr>
<tr>
<td>Vacuum cleaner</td>
<td>70</td>
</tr>
<tr>
<td>Normal conversation</td>
<td>60</td>
</tr>
<tr>
<td>Mosquito buzzing</td>
<td>40</td>
</tr>
<tr>
<td>Whisper</td>
<td>30</td>
</tr>
<tr>
<td>Rustling leaves</td>
<td>10</td>
</tr>
<tr>
<td>Threshold of hearing</td>
<td>0</td>
</tr>
</tbody>
</table>

The unit bel is named after the inventor of the telephone, Alexander Graham Bell (1847–1922). The prefix deci- is the SI prefix that stands for $10^{-1}$.