Physics 41 Chapter 22 HW

1. A heat engine performs 200 J of work in each cycle and has an efficiency of 30.0%. For each cycle, how much energy is (a) taken in and (b) expelled as heat?

\[ W_{\text{eng}} = |Q_h| - |Q_c| = 200 \text{ J} \quad (1) \]

\[ e = \frac{W_{\text{eng}}}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} = 0.300 \quad (2) \]

From (2),

\[ |Q_c| = 0.700|Q_h| \quad (3) \]

Solving (3) and (1) simultaneously, we have

(a) \[ |Q_h| = 667 \text{ J} \]

and (b) \[ |Q_c| = 467 \text{ J} \].

2. A refrigerator has a coefficient of performance of 3.00. The ice tray compartment is at \(-20.0^\circ\text{C}\), and the room temperature is \(22.0^\circ\text{C}\). The refrigerator can convert \(30.0 \text{ g}\) of water at \(22.0^\circ\text{C}\) to \(30.0 \text{ g}\) of ice at \(-20.0^\circ\text{C}\) each minute. What input power is required? Give your answer in watts.

\[ \text{COP} = 3.00 = \frac{Q_c}{W} \]. Therefore, \[ W = \frac{Q_c}{3.00} \].

The heat removed each minute is

\[ \frac{Q_c}{t} = (0.030 \text{ kg})(4186 \text{ J/kg}^\circ\text{C})(22.0^\circ\text{C}) + (0.030 \text{ kg})(3.33 \times 10^5 \text{ J/kg}^\circ\text{C}) \]

\[ + (0.030 \text{ kg})(2090 \text{ J/kg}^\circ\text{C})(20.0^\circ\text{C}) = 1.40 \times 10^4 \text{ J/min} \]

or,

\[ \frac{Q_c}{t} = 233 \text{ J/s} \]

Thus, the work done per second is

\[ P = \frac{233 \text{ J/s}}{3.00} = [77.8 \text{ W}] \].

3. An electric power plant that would make use of the temperature gradient in the ocean has been proposed. The system is to operate between \(20.0^\circ\text{C}\) (surface water temperature) and \(5.00^\circ\text{C}\) (water temperature at a depth of about 1 km). (a) What is the maximum efficiency of such a system? (b) If the useful power output of the plant is \(75.0 \text{ MW}\), how much energy is taken in from the warm reservoir per hour? (c) In view of your answer to part (a), do you think such a system is worthwhile? Note that the “fuel” is free.

a) \[ \eta_{\text{max}} = 1 - \frac{T_c}{T_h} = 1 - \frac{278}{293} = 5.12 \times 10^{-2} = [5.12\%] \]

(b) \[ P = \frac{W_{\text{eng}}}{\Delta t} = 75.0 \times 10^6 \text{ J/s} \]

Therefore,

\[ W_{\text{eng}} = (75.0 \times 10^6 \text{ J/s})(3600 \text{ s/h}) = 2.70 \times 10^{11} \text{ J/h} \]

From \[ e = \frac{W_{\text{eng}}}{|Q_h|} \] we find

\[ |Q_h| = \frac{W_{\text{eng}}}{e} = \frac{2.70 \times 10^{11} \text{ J/h}}{5.12 \times 10^{-2}} = 5.27 \times 10^{12} \text{ J/h} = [5.27 \text{ TJ/h}] \]

(c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.
4. The temperature at the surface of the Sun is approximately 5700 K, and the temperature at the surface of the Earth is approximately 290 K. What entropy change occurs when 1000 J of energy is transferred by radiation from the Sun to the Earth?

\[ \Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = \left( \frac{1000}{290} - \frac{1000}{5700} \right) \text{ J/K} = 3.27 \text{ J/K} \]

5. A 2.00-L container has a center partition that divides it into two equal parts, as shown. The left side contains H\textsubscript{2} gas, and the right side contains O\textsubscript{2} gas. Both gases are at room temperature and at atmospheric pressure. The partition is removed and the gases are allowed to mix. What is the entropy increase of the system?

\[ \Delta S = nR \ln \left( \frac{V_f}{V_i} \right) = (0.0440)(2)R \ln 2 \]

\[ \Delta S = 0.0880(8.314) \ln 2 = 0.507 \text{ J/K} \]

6. At point A in a Carnot cycle, 2.34 mol of a monatomic ideal gas has a pressure of 1400 kPa, a volume of 10.0 L, and a temperature of 720 K. It expands isothermally to point B, and then expands adiabatically to point C where its volume is 24.0 L. An isothermal compression brings it to point D, where its volume is 15.0 L. An adiabatic process returns the gas to point A. (a) Draw the PV diagram. (b) Determine all the unknown pressures, volumes and temperatures as you fill in the following table. (c) Find the energy added by heat, the work done by the engine, and the change in internal energy for each of the steps A→B, B→C, C→D, and D→A. But values in the table. (d) Calculate the efficiency \( W_{net}/Q_h \). Show that it is equal to 1 – TC/TA, the Carnot efficiency.

Results:

<table>
<thead>
<tr>
<th>State</th>
<th>( P ) (kPa)</th>
<th>( V ) (L)</th>
<th>( T ) (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>1400</td>
<td>10.0</td>
<td>720</td>
</tr>
<tr>
<td>( B )</td>
<td>875</td>
<td>16.0</td>
<td>720</td>
</tr>
<tr>
<td>( C )</td>
<td>445</td>
<td>24.0</td>
<td>549</td>
</tr>
<tr>
<td>( D )</td>
<td>712</td>
<td>15.0</td>
<td>549</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process</th>
<th>( Q ) (kJ)</th>
<th>( W ) (kJ)</th>
<th>( \Delta E_{in} ) (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \rightarrow B )</td>
<td>+6.58</td>
<td>−6.58</td>
<td>0</td>
</tr>
<tr>
<td>( B \rightarrow C )</td>
<td>0</td>
<td>−4.98</td>
<td>−4.98</td>
</tr>
<tr>
<td>( C \rightarrow D )</td>
<td>−5.02</td>
<td>+5.02</td>
<td>0</td>
</tr>
<tr>
<td>( D \rightarrow A )</td>
<td>0</td>
<td>+4.98</td>
<td>+4.98</td>
</tr>
<tr>
<td>( ABCDA )</td>
<td>+1.56</td>
<td>−1.56</td>
<td>0</td>
</tr>
</tbody>
</table>
#6. Calculations:

(a) First, consider the adiabatic process \( D \rightarrow A \):

\[
P_D V_D^\gamma = P_A V_A^\gamma \quad \text{so} \quad P_D = P_A \left( \frac{V_A}{V_D} \right)^\gamma = 1400 \text{ kPa} \left( \frac{10.0 \text{ L}}{15.0 \text{ L}} \right)^{\frac{2}{3}} = 712 \text{ kPa}
\]

Also \( \left( \frac{nRT_D}{V_D} \right) V_D = \left( \frac{nRT_A}{V_A} \right) V_A \) or \( T_D = T_A \left( \frac{V_A}{V_D} \right)^{-1} = 720 \text{ K} \left( \frac{10.0 \text{ L}}{15.0 \text{ L}} \right)^2 = 549 \text{ K} \)

Now, consider the isothermal process \( C \rightarrow D \) : \( T_C = T_D = 549 \text{ K} \)

\[
P_C = P_D \left( \frac{V_D}{V_C} \right) = \left[ P_A \left( \frac{V_A}{V_D} \right)^\gamma \right] \left( \frac{V_D}{V_C} \right) = P_A \left( \frac{V_A}{V_C} \right)^\gamma \left( \frac{V_D}{V_C} \right)
\]

\[
P_C = 1400 \text{ kPa} \left( \frac{10.0 \text{ L}}{24.0 \text{ L}} \right)^{\frac{2}{3}} = 445 \text{ kPa}
\]

Next, consider the adiabatic process \( B \rightarrow C \) : \( P_B V_B^\gamma = P_C V_C^\gamma \)

But, \( P_C = P_A \left( \frac{V_A}{V_B} \right)^\gamma \) from above. Also considering the isothermal process, \( P_B = P_A \left( \frac{V_A}{V_B} \right)^\gamma \)

Hence, \( P_A \left( \frac{V_A}{V_B} \right)^\gamma = \left( \frac{P_A V_A}{V_C V_D} \right) V_D \) which reduces to \( V_B = V_A V_C \left( \frac{V_D}{V_C} \right) = 10.0 \text{ L} \left( \frac{24.0 \text{ L}}{15.0 \text{ L}} \right) = 16.0 \text{ L} \)

Finally, \( P_B = P_A \left( \frac{V_A}{V_B} \right) = 1400 \text{ kPa} \left( \frac{10.0 \text{ L}}{16.0 \text{ L}} \right) = 875 \text{ kPa} \)

(b) For the isothermal process \( A \rightarrow B \) : \( \Delta E_{\text{int}} = nC_v \Delta T = 0 \)

\[Q = -W = nRT \ln \left( \frac{V_B}{V_A} \right) = 2.34 \text{ mol} \left( 8.314 \text{ J/mol} \cdot \text{K} \right) \left( 720 \text{ K} \right) \ln \left( \frac{16.0}{10.0} \right) = +6.58 \text{ kJ}\]

For the adiabatic process \( B \rightarrow C \) :

\[Q = 0\]

\[\Delta E_{\text{int}} = nC_v (T_C - T_B) = 2.34 \text{ mol} \left[ \frac{3}{2} (8.314 \text{ J/mol} \cdot \text{K}) \right] (549 - 720) \text{ K} = -4.98 \text{ kJ}\]

and \( W = -Q + \Delta E_{\text{int}} = 0 + (-4.98 \text{ kJ}) = -4.98 \text{ kJ}\)

For the isothermal process \( C \rightarrow D \) : \( \Delta E_{\text{int}} = nC_v \Delta T = 0 \)

and \( Q = -W = nRT \ln \left( \frac{V_B}{V_C} \right) = 2.34 \text{ mol} \left( 8.314 \text{ J/mol} \cdot \text{K} \right) \left( 549 \text{ K} \right) \ln \left( \frac{15.0}{24.0} \right) = -5.02 \text{ kJ}\)

Finally, for the adiabatic process \( D \rightarrow A \):

\[Q = 0\]

\[\Delta E_{\text{int}} = nC_v (T_A - T_D) = 2.34 \text{ mol} \left[ \frac{3}{2} (8.314 \text{ J/mol} \cdot \text{K}) \right] (720 - 549) \text{ K} = +4.98 \text{ kJ}\]

and \( W = -Q + \Delta E_{\text{int}} = 0 + 4.98 \text{ kJ} = +4.98 \text{ kJ}\)

The work done by the engine is the negative of the work input. The output work \( W_{\text{eng}} \) is given by the work column in the table with all signs reversed.

(c) \[e = \frac{W_{\text{eng}}}{Q_i} = \frac{-W_{ABC}}{Q_{A \rightarrow B}} = \frac{1.56 \text{ kJ}}{6.58 \text{ kJ}} = 0.237 \quad \text{or} \quad 23.7\% \]

\[e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{549}{720} = 0.237 \quad \text{or} \quad 23.7\%\]
7. A heat engine with a diatomic gas as the working substance uses the closed cycle shown. How much work does this engine do per cycle and what is its thermal efficiency? But all your values in the tables as usual.

<table>
<thead>
<tr>
<th></th>
<th>( P ) (atm)</th>
<th>( V ) (m(^3))</th>
<th>( T ) (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
<td>900</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2700</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>900</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>300</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process</th>
<th>( W ) (kJ)</th>
<th>( Q ) (kJ)</th>
<th>( \Delta E_{int} ) (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \rightarrow ) 2</td>
<td>-608</td>
<td>2127</td>
<td>1519</td>
</tr>
<tr>
<td>2 ( \rightarrow ) 3</td>
<td>0</td>
<td>-1519</td>
<td>-1519</td>
</tr>
<tr>
<td>3 ( \rightarrow ) 4</td>
<td>+203</td>
<td>-709</td>
<td>-506</td>
</tr>
<tr>
<td>4 ( \rightarrow ) 1</td>
<td>0</td>
<td>5.06</td>
<td>506</td>
</tr>
<tr>
<td>Net</td>
<td>-405</td>
<td>405</td>
<td>0</td>
</tr>
</tbody>
</table>

**Example 19.3 Analyzing a heat engine**

A heat engine with a diatomic gas at the working substance uses the closed cycle shown in FIGURE 19.16. How much work does this engine do per cycle, and what is its thermal efficiency?

The work done during one cycle is \( W_{net} = 4.05 \times 10^4 \) J. Heat enters the system from the hot reservoir during processes 1 \( \rightarrow \) 2 and 4 \( \rightarrow \) 1, where \( Q \) is positive. Summing these gives \( Q_{in} = 26.33 \times 10^4 \) J. Thus the thermal efficiency of this engine is

\[
\eta = \frac{W_{net}}{Q_{in}} = 4.05 \times 10^4 = 0.15 = 15\%
\]

**Assess** The verification that \( W_{net} = Q_{in} \) and \( (\Delta E)_{int} = 0 \) gives us great confidence that we didn’t make any calculational error. This engine may not seem very efficient, but \( \eta \) is quite typical of many real engines.
Otto Cycle: Approximates the process occurring in an internal combustion engine. The Otto Cycle represents a four-stroke cycle consisting of two upstrokes and two downstrokes. The compression ratio \( \frac{V_1}{V_2} = 8 \) and gas is diatomic:

\[
\begin{align*}
\text{Otto Cycle:} & \quad e = 1 - \frac{1}{(V_1 / V_2)^{\gamma-1}} = 1 - \frac{T_A}{T_B} = 1 - \frac{T_D}{T_C}, \\
\end{align*}
\]

The tables look like:

<table>
<thead>
<tr>
<th>State</th>
<th>( T(K) )</th>
<th>( P \text{ (kPa)} )</th>
<th>( V \text{ (m}^3)</th>
<th>( E_{\text{int}} \text{ (J)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>293</td>
<td>100</td>
<td>500</td>
<td>125</td>
</tr>
<tr>
<td>( B )</td>
<td>673</td>
<td>1 840</td>
<td>62.5</td>
<td>287</td>
</tr>
<tr>
<td>( C )</td>
<td>1 023</td>
<td>2 790</td>
<td>62.5</td>
<td>436</td>
</tr>
<tr>
<td>( D )</td>
<td>445</td>
<td>152</td>
<td>500</td>
<td>190</td>
</tr>
<tr>
<td>( A )</td>
<td>293</td>
<td>100</td>
<td>500</td>
<td>125</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process</th>
<th>( Q(J) )</th>
<th>output ( W(J) )</th>
<th>( \Delta E_{\text{int}} \text{ (J)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( AB )</td>
<td>0</td>
<td>-162</td>
<td>162</td>
</tr>
<tr>
<td>( BC )</td>
<td>149</td>
<td>0</td>
<td>149</td>
</tr>
<tr>
<td>( CD )</td>
<td>0</td>
<td>246</td>
<td>-246</td>
</tr>
<tr>
<td>( DA )</td>
<td>-65.0</td>
<td>0</td>
<td>-65.0</td>
</tr>
<tr>
<td>( ABCDA )</td>
<td>84.3</td>
<td>84.3</td>
<td>0</td>
</tr>
</tbody>
</table>

(a), (b) The quantity of gas is

\[
\begin{align*}
n & = \frac{P_A V_A}{R T_A} = \frac{(100 \times 10^3 \text{ Pa})(500 \times 10^{-6} \text{ m}^3)}{(8.314 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 0.0205 \text{ mol}  \\
E_{\text{int},A} & = \frac{5}{2} n R T_A = \frac{5}{2} P_A V_A = \frac{5}{2} (100 \times 10^3 \text{ Pa})(500 \times 10^{-6} \text{ m}^3) = 125 \text{ J} \\
\end{align*}
\]

In process \( AB \),

\[
\begin{align*}
P_B & = P_A \left( \frac{V_A}{V_B} \right)^{\gamma} = (100 \times 10^3 \text{ Pa})(8.00)^{\gamma} = 1.84 \times 10^6 \text{ Pa}  \\
T_B & = \frac{P_B V_B}{n R} = \frac{(1.84 \times 10^6 \text{ Pa})(500 \times 10^{-6} \text{ m}^3)(8.00)}{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})} = 673 \text{ K}  \\
E_{\text{int},B} & = \frac{5}{2} n R T_B = \frac{5}{2} (0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(673 \text{ K}) = 287 \text{ J} \\
\end{align*}
\]

so 

\[
\Delta E_{\text{int},AB} = 287 \text{ J} - 125 \text{ J} = 162 \text{ J} = Q - W_{\text{out}} = 0 - W_{\text{out}} \quad W_{AB} = -162 \text{ J} 
\]

Process \( BC \) takes us to:

\[
\begin{align*}
P_C & = \frac{n R T_C}{V_C} = \frac{(0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(1023 \text{ K})}{62.5 \times 10^{-6} \text{ m}^3} = 2.79 \times 10^6 \text{ Pa}  \\
E_{\text{int},C} & = \frac{5}{2} n R T_C = \frac{5}{2} (0.0205 \text{ mol})(8.314 \text{ J/mol} \cdot \text{K})(1023 \text{ K}) = 436 \text{ J}  \\
E_{\text{int},BC} & = 436 \text{ J} - 287 \text{ J} = 149 \text{ J} = Q - W_{\text{out}} = Q - 0  \\
Q_{BC} & = 149 \text{ J} 
\end{align*}
\]
In process CD:

\[
P_D = P_C \left( \frac{V_C}{V_D} \right) = \left( \frac{2.79 \times 10^6 \text{ Pa}}{8.00} \right)^{1.40} = 1.52 \times 10^5 \text{ Pa}
\]

\[
T_D = \frac{P_D V_D}{nR} = \frac{(1.52 \times 10^5 \text{ Pa})(500 \times 10^{-6} \text{ m}^3)}{(0.020 5 \text{ mol})(8.314 \text{ J/mol K})} = 445 \text{ K}
\]

\[
E_{\text{int,} D} = \frac{5}{2} nRT_D = \frac{5}{2}(0.020 5 \text{ mol})(8.314 \text{ J/mol K})(445 \text{ K}) = 190 \text{ J}
\]

\[
\Delta E_{\text{int,} CD} = 190 \text{ J} - 436 \text{ J} = -246 \text{ J} = Q - W_{\text{out}} = 0 - W_{\text{out}}
\]

\[
W_{CD} = 246 \text{ J}
\]

and \( \Delta E_{\text{int,} DA} = E_{\text{int,} A} - E_{\text{int,} D} = 125 \text{ J} - 190 \text{ J} = -65.0 \text{ J} = Q - W_{\text{out}} = Q - 0 \)

\[
Q_{DA} = -65.0 \text{ J}
\]

For the entire cycle, \( \Delta E_{\text{int, net}} = 162 \text{ J} + 149 - 246 = 0 \). The net work is

\[
W_{\text{eng}} = -162 \text{ J} + 0 + 246 \text{ J} + 0 = 84.3 \text{ J}
\]

\[
Q_{\text{net}} = 0 + 149 \text{ J} + 0 - 65.0 \text{ J} = 84.3 \text{ J}
\]

(c) The input energy is \( Q_h = 149 \text{ J} \), the waste is \( |Q_c| = 65.0 \text{ J} \), and \( W_{\text{eng}} = 84.3 \text{ J} \).

(d) The efficiency is: \( e = \frac{W_{\text{eng}}}{Q_h} = \frac{84.3 \text{ J}}{149 \text{ J}} = 0.565 \)

(e) Let \( f \) represent the angular speed of the crankshaft. Then \( \frac{f}{2} \) is the frequency at which we obtain work in the amount of 84.3 J/cycle:

\[
1000 \text{ J/s} = \left( \frac{f}{2} \right)(84.3 \text{ J/cycle})
\]

\[
f = \frac{2000 \text{ J/s}}{84.3 \text{ J/cycle}} = 23.7 \text{ rev/s} = 1.42 \times 10^4 \text{ rev/min}
\]