Chapter Goal: To understand and apply the wave model of light.
Important Concepts

The wave model of light considers light to be a wave propagating through space. Diffraction and interference are important. The ray model of light considers light to travel in straight lines like little particles. Diffraction and interference are not important. Diffraction is important when the width of the diffraction pattern of an aperture equals or exceeds the size of the aperture.

For a circular aperture, the crossover between the ray and wave models occurs for an opening of diameter \( D_c \approx \sqrt{2\lambda L} \).

In practice, \( D_c \approx 1 \text{ mm} \) for visible light. Thus

- Use the wave model when light passes through openings < 1 mm in size. Diffraction effects are usually important.
- Use the ray model when light passes through openings > 1 mm in size. Diffraction is usually not important.

General Principles

**Huygens’ principle** says that each point on a wave front is the source of a spherical wavelet. The wave front at a later time is tangent to all the wavelets.

**Diffraction** is the spreading of a wave after it passes through an opening.

Constructive and destructive interference are due to the overlap of two or more waves as they spread behind openings.

Applications

**Single slit** of width \( a \).

A bright central maximum of width

\[
W = \frac{2\lambda L}{a}
\]

is flanked by weaker secondary maxima. Dark fringes are located at angles such that

\[
\alpha \sin \theta_p = p\lambda, \quad p = 1, 2, 3, \ldots
\]

If \( \lambda a \ll 1 \), then from the small-angle approximation

\[
\theta_p = \frac{p\lambda}{a}
\]

**Circular aperture** of diameter \( D \).

A bright central maximum of diameter

\[
W = \frac{2.44\lambda L}{D}
\]

is surrounded by circular secondary maxima. The first dark fringe is located at

\[
\theta_1 = \frac{1.22\lambda}{D}, \quad y_1 = \frac{1.22\lambda L}{D}
\]

For an aperture of any shape, a smaller opening causes a more rapid spreading of the wave behind the opening.

**Interference due to wave-front division**

Waves overlap as they spread out behind slits. Constructive interference occurs along antinodal lines. Bright fringes are seen where the antinodal lines intersect the viewing screen.

**Double slit** with separation \( d \).

Equally spaced bright fringes are located at

\[
\theta_m = \frac{m\lambda}{d}, \quad y_m = \frac{m\lambda L}{d}, \quad m = 0, 1, 2, \ldots
\]

The fringe spacing is \( \Delta y = \frac{\Delta L}{d} \).

**Diffraction grating** with slit spacing \( d \).

Very bright and narrow fringes are located at angles and positions

\[
d \sin \theta_m = m\lambda, \quad y_m = L \tan \theta_m
\]

**Interference due to amplitude division**

An interferometer divides a wave, lets the two waves travel different paths, then recombines them. Interference is constructive if one wave travels an integer number of wavelengths more or less than the other wave. The difference can be due to an actual path-length difference or to a different index of refraction.

**Michelson interferometer**

The number of bright-dark-bright fringe shifts as mirror \( M_2 \) moves distance \( \Delta L_2 \) is

\[
\Delta m = \frac{\Delta L_2}{\lambda/2}
\]
Interference of Light
Wave Optics
Diffraction depends on SLIT WIDTH: the smaller the width, relative to wavelength, the more bending and diffraction.
Ray Optics: Ignores Diffraction and Interference of waves!

Diffraction depends on SLIT WIDTH: the smaller the width, relative to wavelength, the more bending and diffraction.

Ray Optics assumes that $\lambda \ll d$, where $d$ is the diameter of the opening.
This approximation is good for the study of mirrors, lenses, prisms, etc.

Wave Optics assumes that $\lambda \sim d$, where $d$ is the diameter of the opening.
This approximation is good for the study of interference.
Geometric RAY Optics (Ch 23)

\[ \theta_i = \theta_r \]

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
James Clerk Maxwell
1860s

Light is wave.

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.0 \times 10^8 \text{ m/s} \]

Speed of Light in a vacuum:
186,000 miles per second
300,000 kilometers per second
3 x 10^8 m/s
The Electromagnetic Spectrum
Incandescent Light Bulb
Full Spectrum of Light
All frequencies excited!
Visible Light

- Different wavelengths correspond to different colors.
- The range is from red ($\lambda \approx 7 \times 10^{-7} \text{ m}$) to violet ($\lambda \approx 4 \times 10^{-7} \text{ m}$).
Hydrogen Spectra
Transition probabilities of electrons correspond to the intensity of light emission.

\[ P(x) \propto |A(x)|^2 \]
Where does light actually come from?

Light comes from the acceleration of charges.
Light is emitted when an electron in an atom jumps between energy levels either by excitation or collisions.
Atoms are EM Tuning Forks

They are ‘tuned’ to particular frequencies of light energy.
Atomic Emission of Light

Each chemical element produces its own unique set of spectral lines when it burns.
Light Emission

- Ultraviolet
- Visible
- Infrared

- Paschen series
- Lyman series
- Balmer series

Absorption
Emission
Ionization
Ground state

Energy levels: 13.6 eV, 12.73 eV, 12.07 eV, 10.19 eV, 0 eV
Hydrogen Spectra
If you pass white light through a prism, it separates into its component colors.
If you pass white light through a prism, it separates into its component colors. 

R.O.Y.G.B.I.V spectrum

long wavelengths  R.O.Y. G. B.I.V  short wavelengths

spectrum
Thin Film Interference
Diffraction & Interference
Iridescence
Dispersion via Diffraction

constructive: \( d \sin \theta = m \lambda \), \( m = 0, 1, 2, 3 \)
Radiation of Visible Sunlight
Additive Primary Colors
Red, Green, Blue
RGB Color Theory

Visible light

- Infrared
- Red
- Orange
- Yellow
- Green
- Blue
- Violet
- Ultraviolet

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Additive Complementary Colors
Yellow, Cyan, Magenta
The color you have to add to get white light.

Red + Green = Yellow
Blue + Green = Cyan
Red + Blue = Magenta

Red + Blue + Green = White

White light – red light = ??
White light – yellow light = ??
FYI: Mixing Colored Pigments

Subtractive Colors

Pigments subtract colors from white light.

Yellow + Cyan = Green

Cyan + Magenta = Purple

Yellow + Magenta = Red

Yellow + Cyan + Magenta = Black
Why are some materials colored?
Why is a Rose Red?

Colored materials absorb certain colors that resonate with their electron energy levels and reject & reflect those that do not.
Why is the Ocean Cyan?

White light minus cyan is red. Ocean water absorbs red.
Shine cyan light on a red rose and what color do you see?
Shine cyan light on a red rose and what color do you see?
I am Watching YOU!!
Human Retina

Sharp Spot: Fovea

Blind Spot: Optic Nerve
Human Vision
An optical Tuning Fork

Anatomy of the Human Eye

- Retina
- Choroid
- Cornea
- Iris
- Pupil
- Aqueous Humor
- Lens
- Fovea
- Optic Nerve
- Sclera
- Vitreous Humor

Figure 1
Optical Antennae: Rods & Cones

Rods: Intensity
Cones: Color
Double Slit is VERY IMPORTANT because it is evidence of waves. Only waves interfere like this.
If light were made of hard bullets, there would be no interference pattern.
In reality, light does show an interference pattern.
Light acts like a wave going through the slits but arrive at the detector like a particle.
Particle Wave Duality
1924: De Broglie Waves

If photons can be particles, then why can’t electrons be waves?

\[ p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \]

Electrons are *STANDING WAVES* in atomic orbitals.

\[ \lambda_e = \frac{h}{p} \]

\[ \lambda_e = 2.4 \times 10^{-11} \, m \]

\[ h = 6.626 \times 10^{-34} \, J \cdot s \]
Double Slit for Electrons shows Wave Interference! Key to Quantum Theory!
Interference pattern builds one electron at a time.

Electrons act like waves going through the slits but arrive at the detector like a particle.

\[ \lambda_e = 2.4 \times 10^{-11} m \]
Limits of Vision

Electron Waves

\[ \lambda_e = 2.4 \times 10^{-11} m \]
Electron Diffraction with Crystals

[Diagram showing the setup of an electron diffraction experiment with a metal target and scattered electron beam.]
Electron Microscope

Electron microscope picture of a fly. The resolving power of an optical lens depends on the wavelength of the light used. An electron-microscope exploits the wave-like properties of particles to reveal details that would be impossible to see with visible light.
Intereference of 2-D Coherent Sound Waves

Phase Difference at P: $\Delta \phi = \frac{2\pi}{\lambda} \Delta r$, $\Delta \phi_0 = 0$
Interference of 2-D Coherent Light Waves

1. A plane wave is incident on the double slit.
2. Waves spread out behind each slit.
3. The waves interfere in the region where they overlap.
4. Bright fringes occur where the antinodal lines intersect the viewing screen.
To observe interference in light waves, the following two conditions must be met:

1) The sources must be **coherent**
   - They must maintain a constant phase with respect to each other

2) The sources should be **monochromatic**
   - Monochromatic means they have a single wavelength
Interference of 2-D Coherent Light Waves

Double Slit Interference of Light Waves
(light goes thru both slits, hits screen at right)
Double Slit Interference Dependence on Slit Separation

KEY:
20 pix.

Slit Width = 48
Wavelength = 16

Crests
Troughs
Derive Fringe Equations

“m” is the fringe order.

• Maxima: bright fringes

\[ d \sin \theta = m \lambda \]

\[ y_{\text{bright}} = \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2 \ldots) \]

• Minima: dark fringes

\[ d \sin \theta = \left( m + \frac{1}{2} \right) \lambda \]

\[ y_{\text{dark}} = \frac{\lambda L}{d} \left( m + \frac{1}{2} \right) \quad (m = 0, \pm 1, \pm 2 \ldots) \]
Phase Difference at P: $\Delta \phi = \frac{2\pi}{\lambda} \Delta r$

Constructive interference occurs at point $O$ when the waves combine.

Constructive interference also occurs at point $P$.

Destructive interference occurs at point $R$ when the two waves combine because the lower wave falls one-half a wavelength behind the upper wave.

Constructive: $\Delta \phi = 2m\pi$, $\Delta r = m\lambda$, $m = 0,1,2,3...$

Destructive: $\Delta \phi = (2m + 1)\pi$, $\Delta r = (m + \frac{1}{2})$, $m = 0,1,2,3...$
Phase Difference at P: \( \Delta \phi = \frac{2\pi}{\lambda} \Delta r \)

Constructive: \( \Delta \phi = 2m\pi, \quad \Delta r = m\lambda, \quad m = 0, 1, 2, 3... \)

Destructive: \( \Delta \phi = (2m + 1)\pi, \quad \Delta r = (m + \frac{1}{2}), \quad m = 0, 1, 2, 3... \)
Derive Fringe Equations

“m” is the fringe order.

- Maxima: bright fringes

\[ d \sin \theta = m \lambda \]

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- Minima: dark fringes

\[ d \sin \theta = \left( m + \frac{1}{2} \right) \lambda \]

\[ y_{\text{dark}} = \frac{\lambda L}{d} \left( m + \frac{1}{2} \right) \quad (m = 0, \pm 1, \pm 2 \ldots) \]
Problem

Red light ($\lambda=664\text{nm}$) is used in Young’s double slit as shown. Find the distance $y$ on the screen between the central bright fringe and the third order bright fringe.

$$y_{\text{bright}} = \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2 \ldots)$$
Measuring the wavelength of light

\[ \Delta y = \frac{\lambda L}{d} \] Fringe Spacing.

A double-slit interference pattern is observed on a screen 1.0 m behind two slits spaced 0.30 mm apart. 9 bright fringes span a distance of 1.7 cm. What is the wavelength of light?
**EXAMPLE 22.2** Measuring the wavelength of light

**SOLVE** The fringe spacing is

\[ \Delta y = \frac{1.7 \text{ cm}}{9} = 1.89 \times 10^{-3} \text{ m} \]

Using this fringe spacing in Equation 22.7, we find that the wavelength is

\[ \lambda = \frac{d}{L} \Delta y = 5.7 \times 10^{-7} \text{ m} = 570 \text{ nm} \]

It is customary to express the wavelengths of visible light in nanometers. Be sure to do this as you solve problems.
Double Slit PreLab

A Young’s interference experiment is performed with monochromatic light. The separation between the slits is 0.500 mm, and the interference pattern on a screen 3.30 m away shows the first side maximum 3.40 mm from the center of the pattern. What is the wavelength?
Double Slit

The image shows the light intensity on a screen behind a double slit. The slit spacing is 0.20 mm and the wavelength of light is 600 nm. What is the distance from the slits to the screen?

$$y_{\text{bright}} = \frac{\lambda L}{d} m \quad (m = 0, \pm 1, \pm 2 \ldots)$$
A laboratory experiment produces a double-slit interference pattern on a screen. The point on the screen marked with a dot is how much farther from the left slit than from the right slit?

A. 1.0 $\lambda$.
B. 1.5 $\lambda$.
C. 2.0 $\lambda$.
D. 2.5 $\lambda$.
E. 3.0 $\lambda$. 
A laboratory experiment produces a double-slit interference pattern on a screen. The point on the screen marked with a dot is how much farther from the left slit than from the right slit?

A. 1.0 $\lambda$.
B. 1.5 $\lambda$.
C. 2.0 $\lambda$.
D. 2.5 $\lambda$.
E. 3.0 $\lambda$. 
A laboratory experiment produces a double-slit interference pattern on a screen. If the screen is moved farther away from the slits, the fringes will be

A. Closer together.
B. In the same positions.
C. Farther apart.
D. Fuzzy and out of focus.
A laboratory experiment produces a double-slit interference pattern on a screen. If the screen is moved farther away from the slits, the fringes will be

A. Closer together.
B. In the same positions.
C. **Farther apart.**
D. Fuzzy and out of focus.
A laboratory experiment produces a double-slit interference pattern on a screen. If green light is used, with everything else the same, the bright fringes will be

A. Closer together
B. In the same positions.
C. Farther apart.
D. There will be no fringes because the conditions for interference won’t be satisfied.
A laboratory experiment produces a double-slit interference pattern on a screen. If green light is used, with everything else the same, the bright fringes will be

\[ \Delta y = \frac{\lambda L}{d} \]

and green light has a shorter wavelength.

A. Closer together.
B. In the same positions.
C. Farther apart.
D. There will be no fringes because the conditions for interference won’t be satisfied.
Type in the values of the parameters and then click CALCULATE. The applet will calculate the new pattern. When the Cancel button at bottom left changes to Forward, Click Forward to see the animation.

This Java program is a Physlet. Copyright Wolfgang Christian, Davidson College. Used here with his permission.

Click here for more information about Physlets.

Intensity of Light Waves

\[ E = E_{\text{max}} \cos (kx - \omega t) \]
\[ B = B_{\text{max}} \cos (kx - \omega t) \]

\[
\frac{E_{\text{max}}}{B_{\text{max}}} = \frac{\omega}{k} = \frac{E}{B} = \frac{c}{c}
\]

\[ I = S_{\text{av}} = \frac{E_{\text{max}} B_{\text{max}}}{2\mu_0} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{c B_{\text{max}}^2}{2\mu_0} \]

\[ I \propto E_{\text{max}}^2 \]
Intensity Distribution Resultant Field

• The magnitude of the resultant electric field comes from the superposition principle
  \[ E_P = E_1 + E_2 = E_0 [\sin \omega t + \sin (\omega t + \varphi)] \]

• This can also be expressed as
  \[ E_P = 2E_0 \cos \left( \frac{\varphi}{2} \right) \sin \left( \omega t + \frac{\varphi}{2} \right) \]
  – \( E_P \) has the same frequency as the light at the slits
  – The amplitude at \( P \) is given by \( 2E_0 \cos (\varphi / 2) \)

• Intensity is proportional to the square of the amplitude:

\[ I_P \propto E_P^2 = 4E_0^2 \cos^2 \left( \frac{\varphi}{2} \right) \]

• The intensity at \( P \) is 4 times one source.
Amplitude is twice as big
But intensity is proportional to the amplitude SQUARED so the Intensity is four times as big as the source. This is energy being conserved! The light energy is redistributed on the screen.

\[ E_P = 2E_o \cos \left( \frac{\varphi}{2} \right) \sin \left( \omega t + \frac{\varphi}{2} \right) \]

\[ I_P \propto E_P^2 = 4E_o^2 \cos^2 \left( \frac{\varphi}{2} \right) \]
WARNING! We are going to ignore the intensity drop due to distance in inverse square law. We assume that the amplitude remains constant over the short distances considered. We will only considering the intensity change due to interference! This is not a bad approximation using lasers as sources.

\[ I = \frac{P}{4\pi r^2} \quad \text{[W/m}^2\text{]} \]
Light Intensity: Ignoring Diffraction

- The interference pattern consists of equally spaced fringes of equal intensity.
- This result is valid only if $L \gg d$ and for small values of $\theta$.

$$I = I_{\text{max}} \cos^2(\Delta \phi / 2)$$

Phase Difference at P: $\Delta \phi = \frac{2\pi}{\lambda} \Delta r$

$$I = I_{\text{max}} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \approx I_{\text{max}} \cos^2 \left( \frac{\pi d}{\lambda L} y \right)$$
Intensity

In a double-slit experiment, the distance between the slits is 0.2 mm, and the distance to the screen is 150 cm. What wavelength (in nm) is needed to have the intensity at a point 1 mm from the central maximum on the screen be 80% of the maximum intensity?

a. 900
b. 700
c. 500
d. 300
e. 600

\[ I = I_{\text{max}} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \approx I_{\text{max}} \cos^2 \left( \frac{\pi d}{\lambda L} y \right) \]
Two slits are illuminated with green light (\(\lambda = 540\) nm). The slits are 0.05 mm apart and the distance to the screen is 1.5 m. At what distance (in mm) from the central maximum on the screen is the average intensity 50\% of the intensity of the central maximum?

\(\begin{align*}
\text{a. } & 1 \\
\text{b. } & 3 \\
\text{c. } & 2 \\
\text{d. } & 4 \\
\text{e. } & 0.4
\end{align*}\)

\[I = I_{\text{max}} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \approx I_{\text{max}} \cos^2 \left( \frac{\pi d}{\lambda L} y \right)\]
Double Slit Interference Reality
Combination of Single and Double
Double Slit Interference Reality
Combination of Single and Double

1. A plane wave is incident on the double slit.
2. Waves spread out behind each slit.
3. The waves interfere in the region where they overlap.
4. Bright fringes occur where the antinodal lines intersect the viewing screen.

In reality, the fringe intensity decreases because the intensity of the light from a single slit is not uniform.
Diffraction depends on SLIT WIDTH: the smaller the width, relative to wavelength, the more bending and diffraction.
Single Slit Interference Is called Diffraction

(a) Without diffraction  (b) With diffraction
Single Slit
Light interferes with itself
**Fraunhofer Diffraction**: screen is far away, approximate plane waves and parallel rays.

**Fresnel Diffraction**: Screen is close and curvature of wave fronts complicates the analysis.
Single Slit Diffraction

(a) Greatly magnified view of slit

The wavelets from each point on the initial wave front overlap and interfere, creating a diffraction pattern on the screen.
Single Slit Diffraction

When the path length differs by half a wavelength then the rays will interfere destructively. For Rays 1 & 3:

\[ \Delta r = \frac{a}{2} \sin \theta = \frac{\lambda}{2} \]

Dark fringe: \( \sin \theta = m \frac{\lambda}{a} \) \( m = \pm 1, \pm 2, \pm 3, \ldots \)
Dark fringe: \( \sin \theta = m \frac{\lambda}{a} \) \( m = \pm 1, \pm 2, \pm 3, \ldots \)
Dark fringe: $\sin \theta \equiv \theta = m \frac{\lambda}{a}$

Single Slit Diffraction for Different Slit Widths

These single slit diffraction patterns were photographed with a helium-neon laser as the light source and a micrometer-controlled single slit. The sketches of the slit widths at right were scaled to the difference between the first minima of the diffraction patterns. If the geometry is such that the small angle approximation is valid, the width of the pattern is inversely proportional to the slit width.
Single Slit Problem

A narrow slit is illuminated with sodium yellow light of wavelength 589 nm. If the central maximum extends to ±30°, how wide is the slit?

a. 0.50 mm
b. $2.2 \times 10^{-6}$ m
c. $3.3 \times 10^{-5}$ m
d. 1.18 μm
e. 5.89 μm

Dark fringe: $\sin \theta = m \frac{\lambda}{a}$
Width of the Central Maximum

Dark fringe: \( \sin \theta = m \frac{\lambda}{a} \) \( m = 1, 2, 3, ... \)

\( y \): the distance from the center of the central maximum to the fringe \( m \)

\( y_m = L \tan \theta_m \)

\( w = 2y \)
Single Slit Problem

If the incident light has $\lambda = 690\text{nm}$ (red) and $a = 4 \times 10^{-6}\text{m}$, find the width of the central bright fringe when the screen is 0.4m away.

Dark fringe: $\sin \theta = m \frac{\lambda}{a}$ \hspace{1cm} m = 1, 2, 3, ...

\[ y_m = L \tan \theta_m \]

\[ w = 2y \]
Single Slit Intensity

- The intensity can be expressed as

\[
I = I_{\text{max}} \left[ \frac{\sin(\pi a \sin \theta / \lambda)}{\pi a \sin \theta / \lambda} \right]^2
\]

- \( I_{\text{max}} \) is the intensity at \( \theta = 0 \)
  - This is the central maximum

- Minima occur at

\[
\frac{\pi a \sin \theta_{\text{dark}}}{\lambda} = m\pi \quad \text{or} \quad \sin \theta_{\text{dark}} = m\frac{\lambda}{a}
\]
Compare Fringe Equations for Single and Double Slits

maxima: \( d \sin \theta = m\lambda \quad (m = 0, \pm 1, \pm 2 \ldots) \)

minima: \( d \sin \theta = \left( m + \frac{1}{2} \right)\lambda \quad (m = 0, \pm 1, \pm 2 \ldots) \)

minima: \( a \sin \theta_{\text{dark}} = m\lambda, \quad m = \pm 1, \pm 2, \pm 3, \ldots \)
Combined Effects

Interference maximum coincides with the first diffraction minimum.
Two-Slit Diffraction Patterns, Maxima and Minima

• To find which interference maximum coincides with the first diffraction minimum

\[
\frac{d \sin \theta}{a \sin \theta} = \frac{m \lambda}{\lambda} \rightarrow \frac{d}{a} = m
\]

– The conditions for the first interference maximum
  • \(d \sin \theta = m \lambda\)
– The conditions for the first diffraction minimum
  • \(a \sin \theta = \lambda\)
Intensity of Two-Slit Diffraction
Chapter 38 in Serway

\[ I = I_{\text{max}} \cos^2 \left( \frac{\pi d \sin \theta}{\lambda} \right) \left[ \frac{\sin \left( \frac{\pi a \sin \theta}{\lambda} \right)}{\pi a \sin \theta / \lambda} \right]^2 \]
Single Slit Diffraction

Under the **Fraunhofer conditions**, the light curve of a multiple slit arrangement will be the interference pattern multiplied by the single slit diffraction envelope. This assumes that all the slits are identical.

- Single slit
- Diffraction and interference
- **Fraunhofer conditions**
Hyperphysics

Double Slit Diffraction

Under the **Fraunhofer conditions**, the light curve of a multiple slit arrangement will be the interference pattern multiplied by the single slit diffraction envelope. This assumes that all the slits are identical.
Three Slit Diffraction

Under the **Fraunhofer conditions**, the light curve of a multiple slit arrangement will be the interference pattern multiplied by the single slit diffraction envelope. This assumes that all the slits are identical.
Five Slit Diffraction

Under the Fraunhofer conditions, the light curve of a multiple slit arrangement will be the interference pattern multiplied by the single slit.
Multiple Slits: Diffraction Gratings

For $N$ slits, the intensity of the primary maxima is $N^2$ times greater than that due to a single slit.
Dispersion via Diffraction

constructive: \( d \sin \theta = m \lambda, \quad m = 0,1,2,3 \)
Hydrogen Spectra
Diffraction Gratings

constructive: \( d \sin \theta = m \lambda, \quad m = 0, 1, 2, 3 \)

Note: The greater the wavelength, the greater the angle.
Example

constructive: \( d \sin \theta = m\lambda, \quad m = 0, 1, 2, 3 \)

A mixture of violet light (410 nm in vacuum) and red light (660 nm in vacuum) fall on a grating that contains 1.0 x 10^4 lines/cm. For each wavelength, find the angle and the distance from the central maximum to the first order maximum.
Diffraction Grating Problem

White light is spread out into spectral hues by a diffraction grating. If the grating has 1000 lines per cm, at what angle will red light \((l = 640 \text{ nm})\) appear in first order?

a. 14.68°
b. 7.35°
c. 17.73°
d. 3.67°
e. 1.84°
Resolving Power of a Diffraction Grating

• For two nearly equal wavelengths, \( \lambda_1 \) and \( \lambda_2 \), between which a diffraction grating can just barely distinguish, the **resolving power**, \( R \), of the grating is defined as

\[
R \equiv \frac{\lambda}{\lambda_2 - \lambda_1} = \frac{\lambda}{\Delta \lambda}
\]

• Therefore, a grating with a high resolution can distinguish between small differences in wavelength
Resolving Power of a Diffraction Grating

• The resolving power in the $m$th-order diffraction is $R = Nm$
  – $N$ is the number of slits
  – $m$ is the order number

• Resolving power increases with increasing order number and with increasing number of illuminated slits
Grating Resolution

Determine the number of grating lines necessary to resolve the 589.59 nm and 589.00 nm sodium lines in second order.

a. 999
b. 680
c. 500
d. 340
e. 380
Circular Aperature

If light travels in straight lines, the image on the screen is the same size as the hole. Diffraction will not be noticed unless the light spreads over a diameter larger than $D$.

Dark fringe: $\sin \theta = m \frac{\lambda}{D} \quad m = 0, 1, 2, 3, \ldots$
Circular Aperture Diffraction

Circular aperture

\[ \sin \theta = \frac{m \lambda}{d} \]
\[ d = \text{aperture diameter} \]

\[ y \approx D \frac{m \lambda}{d} \] for maxima and minima

\[ \frac{y}{D} = \tan \theta \approx \sin \theta \approx \theta \] for small angles \( \theta \)

- **Relative Intensity**
  - 0.0175
  - 0.0042
  - 0.00078

**m values for:**

<table>
<thead>
<tr>
<th>Minima</th>
<th>Maxima</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.220</td>
<td>1.635</td>
</tr>
<tr>
<td>2.233</td>
<td>2.679</td>
</tr>
<tr>
<td>3.238</td>
<td>3.69</td>
</tr>
</tbody>
</table>
Diffraction of a Penny

Central Bright Spot: Poisson Spot
Light from a small source passes by the edge of an opaque object and continues on to a screen. A diffraction pattern consisting of bright and dark fringes appears on the screen in the region above the edge of the object.
Resolution

- The ability of optical systems to distinguish between closely spaced objects is limited because of the wave nature of light.
- If two sources are far enough apart to keep their central maxima from overlapping, their images can be distinguished.
  - The images are said to be resolved.
- If the two sources are close together, the two central maxima overlap and the images are not resolved.
Diffraction Resolution

Fig 38-13, p.1215
Resolving Power

The diffraction limit: two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other.

\[ \theta_{\text{min}} = 1.22 \frac{\lambda}{D} \]

Rayleigh Criterion

D: Aperture Diameter
Standing back from a Georges Seurat painting, you cannot resolve the dots but a camera, at the same distance can. Assume that light enters your eyes through pupils that have diameters of 2.5 mm and enters the camera through an aperture with diameter of 25 mm. Assume the dots in the painting are separated by 1.5 mm and that the wavelength of the light is 550 nm in vacuum. Find the distance at which the dots can just be resolved by a) the camera b) the eye.
Michelson Interferometer

- A ray of light is split into two rays by the mirror $M_0$
  - The mirror is at 45° to the incident beam
  - The mirror is called a beam splitter
- It transmits half the light and reflects the rest
- After reflecting from $M_1$ and $M_2$, the rays eventually recombine at $M_0$ and form an interference pattern
- The fringe pattern shifts by one-half fringe each time $M_1$ is moved a distance $\lambda/4$
- The wavelength of the light is then measured by counting the number of fringe shifts for a given displacement of $M_1$
Michelson Interferometer

The interference changes from a max to a min and back to a max every time $L$ increases by half a wavelength. The number of maxima that appear as the Mirror shifts a length $\Delta L$:

$$\Delta m = \frac{\Delta L}{\lambda / 2}$$
10. Monochromatic light is beamed into a Michelson interferometer. The movable mirror is displaced 0.382 mm, causing the interferometer pattern to reproduce itself 1,700 times. Determine the wavelength of the light. What color is it?
James Clerk Maxwell 1860s

Light is wave. The medium is the Ether.

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3.0 \times 10^8 \text{ m/s} \]
Measure the Speed of the Ether Wind

The **Luminiferous Aether** was imagined by physicists since Isaac Newton as the invisible "vapor" or "gas aether" filling the universe and hence as the carrier of heat and light.
Rotate arms to produce interference fringes and find different speeds of light caused by the Ether Wind, due to Galilean Relativity: light should travel slower against the Ether Wind. From that you can find the speed of the wind.
http://www.youtube.com/watch?v=XavC4w_Y9b8&feature=related

http://www.youtube.com/watch?v=4KFMeKJySwA&feature=related

http://www.youtube.com/watch?v=ETLG5SLFMZo

http://www.youtube.com/watch?v=Z8K3gcHQiqk&feature=related
Michelson-Morely Experiment 1887

The speed of light is independent of the motion and is always $c$. The speed of the Ether wind is zero.

OR….

Lorentz Contraction

The apparatus shrinks by a factor:

$$\sqrt{1 - \frac{v^2}{c^2}}$$
Clocks slow down and rulers shrink in order to keep the speed of light the same for all observers!
Time is Relative!
Space is Relative!
Only the SPEED OF LIGHT is Absolute!
On the Electrodynamics of Moving Bodies
1905
LIGO in Richland, Washington

http://www.youtube.com/watch?v=RzZgFKolfQI&feature=related
LISA

http://www.youtube.com/watch?v=DrWwWcA_Hgw&feature=related
http://www.youtube.com/watch?v=tUpiohbBv6o
Space Shuttle Resolution

What, approximately, are the dimensions of the smallest object on Earth that the astronauts can resolve by eye at 200 km height from the space shuttle? Assume $\lambda = 500$ nm light and a pupil diameter $D = 0.50$ cm. Assume eye fluid has an average $n = 1.33$.

a. 150 m
b. 100 m
c. 250 m
d. 25 m
e. 18 m
Interference in Thin Films

When reflecting off a medium of greater refractive index, a light wave undergoes a phase shift of $\frac{1}{2}$ a wavelength. Wave 1 undergoes a phase shift of 180 degrees.

![Diagram showing interference in thin films](image)
From Low to High, a phase change of pi!
From High to Low, a phase change? NO!
Interference in Thin Films

- The wavelength of ray 1 in the film is $\lambda/n$
- For constructive interference
  
  $$2t = (m + \frac{1}{2}) \frac{\lambda}{n} \quad (m = 0, 1, 2 \ldots)$$

  This takes into account both the difference in optical path length for the two rays and the $180^\circ$ phase change

- For destructive interference
  
  $$2t = m\lambda/n \quad (m = 0, 1, 2 \ldots)$$
Problem: Thin Films

A thin film of gasoline floats on a puddle of water. Sunlight falls almost perpendicularly on the film and reflects into your eyes a yellow hue. Interference in the thin gasoline film has eliminated blue (469nm in vacuum) from the reflected light. The refractive indices of the blue light in gasoline and water are 1.40 and 1.33 respectively. Determine the minimum nonzero thickness of the film. What color do you see?
Thin Film Interference

The light reflected from a soap bubble (n = 1.40) appears red (\(\lambda = 640 \text{ nm}\)). What is the minimum thickness (in nm)?

a. 124
b. 104
c. 114
d. 134
e. 234
Interference of Light
Wave Optics