Physics 41 Chapter 21 Lecture Problems

\[ PV = nRT = NkT, \quad Q = mc\Delta T, \quad R = 8.31 \text{ J/mol} \cdot \text{K}, \quad k = 1.38 \times 10^{-23} \text{ J/K}, \quad N_A = 6.022 \times 10^{23} \text{ / mol} \]

\[ 1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}, \quad u = 1.66 \times 10^{-27} \text{ kg}, \quad v_{rms} = \sqrt{\frac{3k_B T}{m}}, \quad v_{avg} = \sqrt{\frac{8k_B T}{\pi m}}, \quad v_{mp} = \sqrt{\frac{2k_B T}{m}} \]

<table>
<thead>
<tr>
<th>System</th>
<th>Degrees of freedom</th>
<th>( E_{th} )</th>
<th>( C_V )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monatomic gas</td>
<td>3</td>
<td>( \frac{3}{2}Nk_B T = \frac{3}{2}nRT )</td>
<td>( \frac{3}{2}R = 12.5 \text{ J/mol K} )</td>
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<tr>
<td>Diatomic gas</td>
<td>5</td>
<td>( \frac{5}{2}Nk_B T = \frac{5}{2}nRT )</td>
<td>( \frac{5}{2}R = 20.8 \text{ J/mol K} )</td>
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<tr>
<td>Elemental solid</td>
<td>6</td>
<td>( 3Nk_B T = 3nRT )</td>
<td>( 3R = 25.0 \text{ J/mol K} )</td>
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1. A 5.00-L vessel contains nitrogen gas at 27.0°C and 3.00 atm. Find (a) the total translational kinetic energy of the gas molecules and (b) the average kinetic energy per molecule.

(a) \[ PV = nRT = \frac{N m^2}{3} \quad \text{The total translational kinetic energy is} \quad \frac{N m^2}{2} = E_{\text{trans}}: \]

\[ E_{\text{trans}} = \frac{3}{2} PV = \frac{3}{2} (3.00 \times 1.013 \times 10^5) (5.00 \times 10^{-3}) = 2.28 \text{ kJ} \]

(b) \[ \frac{m v^2}{2} = \frac{3k_B T}{2} = \frac{3RT}{2N_A} = \frac{3(8.314)(300)}{2(6.02 \times 10^{23})} = 6.21 \times 10^{-21} \text{ J} \]

2. Calculate the RMS speed of an oxygen molecule in the air if the temperature is 5.00 °C. The mass of an oxygen molecule is 32.00 u

\[ v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(278 \text{ K})}{(32u)(1.66 \times 10^{-27} \text{ kg/u})}} = 466 \text{ m/s} \]

3. A cylinder contains a mixture of helium and argon gas in equilibrium at 150°C. (a) What is the average kinetic energy for each type of gas molecule? (b) What is the root-mean-square speed of each type of molecule?

(a) \[ K = \frac{3}{2} k_B T = \frac{3}{2} (1.38 \times 10^{-23} \text{ J/K})(423 \text{ K}) = 8.76 \times 10^{-21} \text{ J} \]
(b) \( \bar{R} = \frac{1}{2} m v_{rms}^2 = 8.76 \times 10^{-21} \text{ J} \)

so
\[
v_{rms} = \sqrt{\frac{1.75 \times 10^{-20} \text{ J}}{m}}
\]

For helium,
\[
m = \frac{4.00 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.64 \times 10^{-24} \text{ g/molecule}
\]
\[
m = 6.64 \times 10^{-27} \text{ kg/molecule}
\]

Similarly for argon,
\[
m = \frac{39.9 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.63 \times 10^{-23} \text{ g/molecule}
\]
\[
m = 6.63 \times 10^{-26} \text{ kg/molecule}
\]

Substituting in (1) above,
we find for helium,
\[
v_{rms} = 1.62 \text{ km/s}
\]
and for argon,
\[
v_{rms} = 514 \text{ m/s}
\]

4. In a constant-volume process, 209 J of energy is transferred by heat to 1.00 mol of an ideal monatomic gas initially at 300 K. Find (a) the increase in internal energy of the gas, (b) the work done on it, and (c) its final temperature

\[n = 1.00 \text{ mol}, \quad T_i = 300 \text{ K}\]

(b) Since \( V = \text{constant} \), \( W = 0 \)

(a) \( \Delta E_{int} = Q + W = 209 \text{ J} + 0 = 209 \text{ J} \)

(c) \( \Delta E_{int} = nC_v \Delta T = n\left(\frac{3}{2}R\right) \Delta T \)

so
\[
\Delta T = \frac{2\Delta E_{int}}{3nR} = \frac{2(209 \text{ J})}{3(1.00 \text{ mol})(8.314 \text{ J/mol·K})} = 16.8 \text{ K}
\]
\[
T = T_i + \Delta T = 300 \text{ K} + 16.8 \text{ K} = 317 \text{ K}
\]
5. A 2.00-mol sample of a diatomic ideal gas expands slowly and adiabatically from a pressure of 5.00 atm and a volume of 12.0 L to a final volume of 30.0 L. (a) What is the final pressure of the gas? (b) What are the initial and final temperatures? (c) Find $Q$, $W$, and $\Delta E_{\text{int}}$.

(a) $P_f V_f^\gamma = P_i V_i^\gamma$

$$P_f = P_i \left( \frac{V_i}{V_f} \right)^\gamma = 5.00 \text{ atm} \left( \frac{12.0}{30.0} \right)^{1.40} = 1.39 \text{ atm}$$

(b) $T_i = \frac{P_i V_i}{nR} = \frac{5.00 \left(1.013 \times 10^5 \text{ Pa} \right) \left(12.0 \times 10^{-3} \text{ m}^3 \right)}{2.00 \text{ mol} \left(8.314 \text{ J/mol \cdot K} \right)} = 365 \text{ K}$

$$T_f = \frac{P_f V_f}{nR} = \frac{1.39 \left(1.013 \times 10^5 \text{ Pa} \right) \left(30.0 \times 10^{-3} \text{ m}^3 \right)}{2.00 \text{ mol} \left(8.314 \text{ J/mol \cdot K} \right)} = 253 \text{ K}$$

(c) The process is adiabatic: $Q = 0$

$$\gamma = 1.40 = \frac{C_P}{C_V} = \frac{R + C_V}{C_V}, \quad C_V = \frac{5}{2} R$$

$$\Delta E_{\text{int}} = nC_V \Delta T = 2.00 \text{ mol} \left(\frac{5}{2} \left(8.314 \text{ J/mol \cdot K} \right) \right) (253 \text{ K} - 365 \text{ K}) = -4.66 \text{ kJ}$$

$$W = \Delta E_{\text{int}} - Q = -4.66 \text{ kJ} - 0 = -4.66 \text{ kJ}$$