Interference: Two Spherical Sources
Superposition
Interference

Waves ADD: Constructive Interference.

Waves SUBTRACT: Destructive Interference.

In Phase

Out of Phase
Superposition

Traveling waves move through each other, interfere, and keep on moving!

(a) Overlap begins

(b) Total overlap; the slinky has twice the height of either pulse

(c) The receding pulses
Pulsed Interference
Superposition

Waves ADD in space.

Any complex wave can be built from simple sine waves.

Simply add them point by point.
Fourier Synthesis of a Square Wave

Any periodic function can be represented as a series of sine and cosine terms in a **Fourier series**:

\[ y(t) = \sum_{n} (A_n \sin 2\pi f_n t + B_n \cos 2\pi f_n t) \]
Superposition of Sinusoidal Waves

- **Case 1**: Identical, same direction, with phase difference (Interference) Both 1-D and 2-D waves.

- **Case 2**: Identical, opposite direction (standing waves)

- **Case 3**: Slightly different frequencies (Beats)
Superposition of Sinusoidal Waves

- Assume two waves are traveling in the same direction, with the same frequency, wavelength and amplitude.
- The waves differ in phase.
- \[ y_1 = A \sin (kx - \omega t) \]
- \[ y_2 = A \sin (kx - \omega t + \phi) \]
- \[ y = y_1 + y_2 = 2A \cos (\phi/2) \sin (kx - \omega t + \phi/2) \]

Resultant Amplitude Depends on phase:

Spatial Interference Term
Sinusoidal Waves with Constructive Interference

\[ y = y_1 + y_2 = 2A \cos \left( \frac{\phi}{2} \right) \sin \left( kx - wt + \frac{\phi}{2} \right) \]

- When \( \phi = 0 \), then \( \cos \left( \frac{\phi}{2} \right) = 1 \)
- The amplitude of the resultant wave is \( 2A \)
  - The crests of one wave coincide with the crests of the other wave
- The waves are everywhere in phase
- The waves interfere constructively
Sinusoidal Waves with Destructive Interference

\[ y = y_1 + y_2 = 2A \cos \left( \frac{\phi}{2} \right) \sin \left( kx - wt + \frac{\phi}{2} \right) \]

- When \( \phi = \pi \), then
  \[ \cos \left( \frac{\phi}{2} \right) = 0 \]
  - Also any even multiple of \( \pi \)
- The amplitude of the resultant wave is 0
  - Crests of one wave coincide with troughs of the other wave
- The waves interfere destructively
**Sinusoidal Waves Interference**

\[ y = y_1 + y_2 = 2A \cos(\phi/2) \sin(kx - wt + \phi/2) \]

- When \( \phi \) is other than 0 or an even multiple of \( \pi \), the amplitude of the resultant is between 0 and 2A
- The wave functions still add
Superposition of Sinusoidal Waves

\[ y = y_1 + y_2 = 2A \cos \left( \frac{\phi}{2} \right) \sin \left( kx - \omega t + \frac{\phi}{2} \right) \]

- The resultant wave function, \( y \), is also sinusoidal
- The resultant wave has the same frequency and wavelength as the original waves
- The amplitude of the resultant wave is \( 2A \cos \left( \frac{\phi}{2} \right) \)
- The phase of the resultant wave is \( \frac{\phi}{2} \)

**Constructive**

**Destructive**

**Interference**
Wave Interference

\[ y = y_1 + y_2 = 2A \cos \left( \frac{\phi}{2} \right) \sin \left( kx - \omega t + \frac{\phi}{2} \right) \]

Resultant Amplitude: \( 2A \cos \left( \frac{\Delta \phi}{2} \right) \)

Constructive Interference: \( \Delta \phi = 2n\pi, \quad n = 0, 1, 2, 3... \)

Destructive Interference: \( \Delta \phi = (2n + 1)\pi, \quad n = 0, 1, 2, 3... \)
1. Two sinusoidal waves are described by the wave functions

\[ y_1 = (5.00 \text{ m}) \sin[\pi(4.00x - 1200t)] \quad \text{and} \quad y_2 = (5.00 \text{ m}) \sin[\pi(4.00x - 1200t - 0.250)] \]

where \( x, y_1, \) and \( y_2 \) are in meters and \( t \) is in seconds. (a) What is the amplitude of the resultant wave? (b) What is the frequency of the resultant wave?

\[ y = y_1 + y_2 = 2A \cos (\phi/2) \sin (kx - \omega t + \phi/2) \]

1-D Sound Wave Interference

(a) Constructive interference
These two waves are in phase. Their crests are aligned.

(b) Destructive interference
These two waves are out of phase. The crests of one wave are aligned with the troughs of the other.

Their superposition produces a wave with amplitude $2a$. This is constructive interference.

Their superposition produces a wave with zero amplitude. This is destructive interference.
Superposition

Sound Waves

The superposition of two identical transverse waves in phase produces a wave of increased amplitude.

The superposition of two identical longitudinal waves in phase produces a wave of increased intensity.

Two identical transverse waves that are out of phase destroy each other when they are superimposed.

Two identical longitudinal waves that are out of phase destroy each other when they are superimposed.
2-D Wave Interference?
These two loudspeakers are in phase. They emit equal-amplitude sound waves with a wavelength of 1.0 m. At the point indicated, is the interference maximum constructive, perfect destructive or something in between?

A. perfect destructive  
B. maximum constructive  
C. something in between
These two loudspeakers are in phase. They emit equal-amplitude sound waves with a wavelength of 1.0 m. At the point indicated, is the interference maximum constructive, perfect destructive or something in between?

A. perfect destructive

✓ B. maximum constructive

C. something in between
2-D Phase Difference Different than 1-D
You have to consider the Path Difference!

\[ \Delta \phi = \omega \Delta t = 2\pi f \Delta t = 2\pi \frac{v}{\lambda} \Delta t = \frac{2\pi}{\lambda} (v\Delta t) = \frac{2\pi}{\lambda} \Delta r \]

2-D Phase Difference at P: \( \Delta \phi \) is different from the phase difference \( \phi \) between the two source waves!

\[ \Delta \phi = \phi_2 - \phi_1 \]

Phase Difference at P: \( \Delta \phi = \frac{2\pi}{\lambda} \Delta r \)

Path Difference at P: \( \Delta r = \frac{\lambda}{2\pi} \Delta \phi \)
Spherically Symmetric Waves

The wave fronts are crests, separated by $\lambda$.

Troughs are halfway between wave fronts.

This graph shows the displacement of the medium.
Two in-phase sources emit circular or spherical waves.

- Points of constructive interference. A crest is aligned with a crest, or a trough with a trough.
- Points of destructive interference. A crest is aligned with a trough of another wave.
• At A, $\Delta r_A = \lambda$, so this is a point of constructive interference.

• At B, $\Delta r_B = \frac{1}{2}\lambda$, so this is a point of destructive interference.
Antinodal lines, constructive interference. $A = 2a$

Nodal lines, destructive interference. $A = 0$
\[ \Delta r = \frac{5}{2} \lambda \]

\[ \Delta r = 2\lambda \]

\[ \Delta r = \lambda \]

\[ \Delta r = 0 \]

\[ \Delta r = \frac{3}{2} \lambda \]

\[ \Delta r = \frac{1}{2} \lambda \]

**Antinodal lines, constructive interference. Intensity is at its maximum value.**

**Nodal lines, destructive interference. Intensity is zero.**

Intensity
Antinodal lines, constructive interference. Intensity is at its maximum value.

Nodal lines, destructive interference. Intensity is zero.
Contour Map of Interference Pattern of Two Sources

(a) Two identical sources

(b) Two out-of-phase sources
Constructive or Destructive?

(Identical in phase sources)

Phase Difference at P: \[ \Delta \phi = \frac{2\pi}{\lambda} \Delta r + \phi_0 \]

\[ \Delta \phi = \frac{2\pi}{\lambda} (1\lambda) = 2\pi \]

*Constructive!*

Resultant Amplitude: \[ 2A \cos \left( \frac{\Delta \phi}{2} \right) \]

Constructive Interference: \[ \Delta r = n\lambda, \quad \Delta \phi = 2n\pi, \quad n = 0,1,2,3... \]

Destructive Interference: \[ \Delta r = (2n+1) \frac{\lambda}{2}, \quad \Delta \phi = (2n+1)\pi, \quad n = 0,1,2,3... \]
Constructive or Destructive?
(Source out of Phase by 180 degrees)

Phase Difference at P: \( \Delta \phi = \frac{2\pi}{\lambda} \Delta r + \phi_0 \)

\[ \Delta \phi = \frac{2\pi}{\lambda} (1\lambda) + \pi = 3\pi \]

Destructive!

Resultant Amplitude: \( 2A \cos \left( \frac{\Delta \phi}{2} \right) \)

Constructive Interference: \( \Delta r = n\lambda, \ \Delta \phi = 2n\pi, \ n = 0,1,2,3... \)

Destructive Interference: \( \Delta r = (2n+1)\frac{\lambda}{2}, \ \Delta \phi = (2n+1)\pi, \ n = 0,1,2,3... \)
In Phase or Out of Phase?

A

B

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Constructive or Destructive?
The interference at point C in the figure at the right is

A. maximum constructive.
B. destructive, but not perfect.
C. constructive, but less than maximum.
D. perfect destructive.
E. there is no interference at point C.
The interference at point C in the figure at the right is

A. maximum constructive.
B. destructive, but not perfect.
C. constructive, but less than maximum.

D. **perfect destructive.**
E. there is no interference at point C.
2. Two loudspeakers are placed on a wall 2.00 m apart. A listener stands 3.00 m from the wall directly in front of one of the speakers. A single oscillator is driving the speakers at a frequency of 300 Hz. (a) What is the phase difference between the two waves when they reach the observer? (b) What If? What is the frequency closest to 300 Hz to which the oscillator may be adjusted such that the observer hears minimal sound?

Phase Difference at P: \[ \Delta \phi = \frac{2\pi}{\lambda} \Delta r \]

Path Difference at P: \[ \Delta r = \frac{\lambda}{2\pi} \Delta \phi \]
3. An observer stands 3 m from speaker A and 4 m from speaker B. Both speakers, oscillating in phase, produce 170 Hz waves. The speed of sound in air is 340 m/s. What is the phase difference (in radians) between the waves from A and B at the observer’s location, point P?

a. 0
b. \( \frac{\pi}{2} \)
c. \( \pi \)
d. \( 2\pi \)
e. \( 4\pi \)
Reflected PULSE:

Free End

Bound End
Reflected PULSE:
Standing Waves
Created by Boundary Conditions
Standing Waves on Strings
Standing Wave
Standing Wave:

Wave 1

Wave 2

Resultant

$t = 0$

$t = \frac{1}{4} T$

$t = \frac{1}{2} T$

$t = \frac{3}{4} T$

$t = T$

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Transverse Standing Wave

Produced by the superposition of two identical waves moving in opposite directions.
Standing Waves on a String

Harmonics
Standing Waves

Superposition of two identical waves moving in opposite directions.

\[ y_1 = A \sin (kx - \omega t) \quad y_2 = A \sin (kx + \omega t) \]

\[ y = (2A \sin kx) \cos \omega t \]

• There is no \( kx - \omega t \) term, and therefore it is not a traveling wave!
• Every element in the medium oscillates in simple harmonic motion with the same frequency, \( \omega \): \( \cos \omega t \)
• The amplitude of the simple harmonic motion depends on the location of the element within the medium: \( 2A \sin kx \)
Note on Amplitudes

\[ y = (2A \sin kx) \cos \omega t \]

There are three types of amplitudes used in describing waves

- The amplitude of the individual waves, \( A \)
- The amplitude of the simple harmonic motion of the elements in the medium, \( 2A \sin kx \)
- The amplitude of the standing wave, \( 2A \)
  - A given element in a standing wave vibrates within the constraints of the envelope function \( 2A \sin kx \), where \( x \) is the position of the element in the medium
Node & Antinodes

- A **node** occurs at a point of zero amplitude

  \[ x = \frac{n\lambda}{2} \quad n = 0, 1, \ldots \]

- An **antinode** occurs at a point of maximum displacement, \(2A\)
Two harmonic waves traveling in opposite directions interfere to produce a standing wave described by $y = 2 \sin (\pi x) \cos (3 \pi t)$ where $x$ is in m and $t$ is in s. What is the distance (in m) between the first two antinodes?

a. 8
b. 2
c. 4
d. 1
e. 0.5
- **Intensity** of a wave is proportional to the square of the amplitude: \( I \propto A^2 \).
- Intensity is maximum at points of constructive interference and zero at points of destructive interference.


Mode Number of Standing Waves

- $m$ is the number of antinodes on the standing wave.
- The *fundamental mode*, with $m = 1$, has $\lambda_1 = 2L$.
- The frequencies of the normal modes form a series: $f_1, 2f_1, 3f_1, \ldots$
- The fundamental frequency $f_1$ can be found as the *difference* between the frequencies of any two adjacent modes: $f_1 = \Delta f = f_{m+1} - f_m$.
- Below is a time-exposure photograph of the $m = 3$ standing wave on a string.
What is the mode number of this standing wave?

A. 4.
B. 5.
C. 6.
D. Can’t say without knowing what kind of wave it is.
What is the mode number of this standing wave?

A. 4.
B. 5. ☑
C. 6.
D. Can’t say without knowing what kind of wave it is.

Mode # = number of antinodes
Standing Waves on a String

Harmonics
Which harmonics (modes) are present on the string?

The Fundamental and third harmonic.
Standing Waves on a String

**Harmonics**

- **1st harmonic (fundamental)**: Frequency $= f_1$
- **2nd harmonic (1st overtone)**: Frequency $= 2f_1$
- **3rd harmonic (2nd overtone)**: Frequency $= 3f_1$
Standing Waves on a String

\[ \lambda_1 = 2L \]

\[ \lambda_2 = L \]

\[ \lambda_3 = \frac{2L}{3} \]
Standing Waves on a String

\[ \lambda_n = \frac{2L}{n} \]

\[ f_n = \frac{\nu}{\lambda_n} \]

\[ f_n = n \frac{\nu}{2L} \]
Standing Wave on a String

\[ f_n = n \frac{v}{2L} \]

\[ v = \sqrt{\frac{T}{\mu}} \]

\[ v = \lambda f \]

6. A string with a mass of 8.00 g and a length of 5.00 m has one end attached to a wall; the other end is draped over a pulley and attached to a hanging object with a mass of 4.00 kg. If the string is plucked, what is the fundamental frequency of vibration?
Longitudinal Standing Wave

Frequency = f

Frequency = 2f
- Shown are the displacement $\Delta x$ and pressure graphs for the $m = 2$ mode of standing sound waves in a closed-closed tube.

- The nodes and antinodes of the pressure wave are interchanged with those of the displacement wave.
Shown are displacement and pressure graphs for the first three standing-wave modes of a tube closed at both ends:

\[ \lambda_m = \frac{2L}{m} \]
\[ f_m = m \frac{v}{2L} \]

\[ m = 1, 2, 3, 4, \ldots \]
Shown are displacement and pressure graphs for the first three standing-wave modes of a tube open at both ends:

\[ \lambda_m = \frac{2L}{m} \]

\[ f_m = m \frac{v}{2L} \]

\[ m = 1, 2, 3, 4, \ldots \]
Standing Waves in an Open Tube

- Both ends are displacement antinodes
- The fundamental frequency is \( v/2L \)
  - This corresponds to the first diagram
- The higher harmonics are \( f_n = n f_1 = n (v/2L) \) where \( n = 1, 2, 3, \ldots \)

(a) Open at both ends
Standing Waves in a Tube Closed at One End

- The closed end is a displacement node
- The open end is a displacement antinode
- The fundamental corresponds to $\frac{1}{4} \lambda$
- The frequencies are $f_n = \frac{n}{\lambda} = \frac{n}{v/4L}$ where $n = 1, 3, 5, ...$

(b) Closed at one end, open at the other

First harmonic:
- $\lambda_1 = 4L$
- $f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$

Third harmonic:
- $\lambda_3 = \frac{4}{3} L$
- $f_3 = \frac{3v}{4L} = 3f_1$

Fifth harmonic:
- $\lambda_5 = \frac{4}{5} L$
- $f_5 = \frac{5v}{4L} = 5f_1$
An open-open tube of air has length $L$. Which is the displacement graph of the $m = 3$ standing wave in this tube?
An open-open tube of air has length $L$. Which is the displacement graph of the $m = 3$ standing wave in this tube?

A. 

B. 3/2 wavelengths

C. Antinodes at open ends

D.
An open-closed tube of air of length $L$ has the closed end on the right. Which is the displacement graph of the $m = 3$ standing wave in this tube?
An open-closed tube of air of length $L$ has the closed end on the right. Which is the displacement graph of the $m = 3$ standing wave in this tube?

- A.
- B.
- C.
- D.

3/4 wavelengths
Node at closed end
At room temperature, the fundamental frequency of an open-open tube is 500 Hz. If taken outside on a cold winter day, the fundamental frequency will be

A. Less than 500 Hz.
B. 500 Hz.
C. More than 500 Hz.
At room temperature, the fundamental frequency of an open-open tube is 500 Hz. If taken outside on a cold winter day, the fundamental frequency will be

A. Less than 500 Hz.
B. 500 Hz.
C. More than 500 Hz.

\[ \lambda_m = \frac{2L}{m} \]
\[ f_m = m \frac{v}{2L} \]
\[ m = 1, 2, 3, 4, \ldots \]
9. A glass tube (open at both ends) of length $L$ is positioned near an audio speaker of frequency $f = 680$ Hz. For what values of $L$ will the tube resonate with the speaker?

10. A clarinet behaves like a tube closed at one end. If its length is 1.0 m, and the velocity of sound is 344 m/s, what is its fundamental frequency (in Hz)?

   a. 264
   b. 140
   c. 86
   d. 440
   e. 172
What is the difference between Noise and Music?

Regular Repeating Patterns
Multiple Harmonics can be present at the same time.
The amount that each harmonic is present determines the quality or *timbre* of the sound for each instrument.
Quality of Sound – Tuning Fork

• A tuning fork produces only the fundamental frequency
Quality of Sound – Flute

- The same note played on a flute sounds differently
- The second harmonic is very strong
- The fourth harmonic is close in strength to the first
Quality of Sound – Clarinet

• The fifth harmonic is very strong
• The first and fourth harmonics are very similar, with the third being close to them
Standing Waves in Membranes

- Two-dimensional oscillations may be set up in a flexible membrane stretched over a circular hoop
- The resulting sound is not harmonic because the standing waves have frequencies that are not related by integer multiples
- The fundamental frequency contains one nodal curve
Standing Waves

Standing waves form in certain *MODES* based on the length of the string or tube or the shape of drum or wire. Not all frequencies are permitted!
Standing Waves: Membranes
Standing Waves: Membranes
Strings & Atoms are Quantized

The possible frequency and energy states of an electron in an atomic orbit or of a wave on a string are quantized.

\[ f = n \frac{v}{2l} \]

\[ E_n = n hf, \quad n = 0, 1, 2, 3, \ldots \]

\[ h = 6.626 \times 10^{-34} \text{ Js} \]
Interference

\[ \text{Interference} = \text{Reinforcement} + \text{Cancellation} \]
Interference: Beats

beats frequency = difference in frequencies
Interference: Beats

\[ f_B = |f_2 - f_1| \]

\[ f_{ave} = \frac{f_2 + f_1}{2} \]
Interference: Beats
Beat Frequency

\[ A_{\text{resultant}} = 2A \cos 2\pi \left( \frac{f_1 - f_2}{2} \right) t \]

- The number of amplitude maxima one hears per second is the **beat frequency**: \( f_{\text{beat}} = |f_1 - f_2| \)
- The human ear can detect a beat frequency up to about 20 beats/sec
You hear three beats per second when two sound tones are generated. The frequency of one tone is known to be 610 Hz. The frequency of the other is

A. 604 Hz.
B. 607 Hz.
C. 613 Hz.
D. 616 Hz.
E. Either b or c.
You hear three beats per second when two sound tones are generated. The frequency of one tone is known to be 610 Hz. The frequency of the other is

A. 604 Hz.
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Beat Frequency #11

In certain ranges of a piano keyboard, more than one string is tuned to the same note to provide extra loudness. For example, the note at 110 Hz has two strings at this frequency. If one string slips from its normal tension of 600 N to 540 N, what beat frequency is heard when the hammer strikes the two strings simultaneously?