Conceptual Questions

15.1. (a) The density will not change. Although the mass and volume both increase by $2^3 = 8$ times, their ratio is unchanged.

(b) The volume $V$ has increased by $2^3 = 8$ times, but the mass has not changed, so the new density $\rho'$ is $\rho' = m/(8V) = \rho/8$. The new density has decreased by a factor of 8.

15.2. The pressure only depends on the depth with respect to the top surface of the liquid. Since points a, b, and c are all at the same depth (i.e., with respect to point e) the pressure is the same for each, so $p_a = p_b = p_c$.

15.3. The pressure only depends on the depth from the surface of the liquid. Since point d is the deepest and point e the highest, then $p_d > p_l > p_e$.

A point halfway between point e and point b would have a pressure about the same as the pressure at point d.

15.4. (a) The pressure at the bottom of each tank is given by $p = \rho gd$, so it will be the same because the depth of the water in each tank is the same. The area of the bottom of tank A is larger than the area of the bottom of tank B. From $p = F/A$, we have $F = \rho A$, so the $F_A > F_B$ because the bottom area of tank A is larger.

(b) The pressure at any given depth is the same in both tanks because the water depth is the same. Since the area of the sides indicated in Figure Q15.4 is the same for each tank, the force on these sides is also the same, so $F_A = F_B$. This makes sense. Since there’s more water in tank A, the total force of the water on the bottom of tank A is larger.

15.5. A floating object displaces its weight in liquid, so placing the boat above point A will increase the depth of the body of water just enough to compensate for the boat’s weight. Thus, the depth of point B is increased slightly so that it is at the same pressure as point A, and $p_A = p_B$.

While a column above point A contains the ship, it also contains less water than a similar column above point B, but both columns contain the same weight (or mass if we assume a uniform gravitational field).

15.6. Archimedes’ principle tells us that the displaced liquid has the same weight as the floating object, so $\rho_L V_L = \rho_B V_B$ implies $\rho_B = \rho_L (V_L/V_B)$.

where the subscripts L and B refer to liquid and block, respectively. The ratio $V_L/V_B$ is the fraction of the block that is under the liquid, so the densest block is the one for which this fraction is the largest. Thus, $\rho_A > \rho_C > \rho_B$.
You’ve heard that only 10% of an iceberg is visible above the surface of the ocean. That means 90% of the iceberg is below the surface. Therefore the density of ice is 90% the density of seawater. You can verify this by looking in Table 15.1 and finding on the Web the density of ice: $\rho_{\text{ice}} = 917 \text{ kg/m}^3$.

15.7. Archimedes’ principle states that the buoyant force on an object is equal to the weight of the fluid displaced by the object. Each object displaces exactly the same amount of fluid since each is the same volume. Therefore, the buoyant force on all three objects is the same, so $F_a = F_b = F_c$. Note that the buoyant force does not depend on the mass or location of the object.

15.8. For objects that are completely submerged the buoyant force is proportional to the volume of the object. If the densities of objects A, B, and C are the same, then the objects with greater mass (A and C) must also occupy a larger volume. Thus, A and C will experience a larger buoyant force than B.

$$F_A = F_C > F_B$$

15.9. The sphere is floating in static equilibrium, so the upward buoyant force exactly equals the sphere’s weight: $F_B = w$. But according to Archimedes’ principle, $F_B$ is the weight of the displaced liquid. That is, the weight of the missing water in B is exactly matched by the weight of the added ball. Thus, the total weights of both containers are equal.

15.10. The lower the velocity of fluid, the higher the pressure at a given point in the pipe (Bernoulli’s principle). The pressure $p$ in the horizontal liquid-containing pipe is $p = p_a + \rho gd$, where $d$ is the depth of the liquid in the vertical pipes a, b, or c and $p_a$ is the gas pressure at the liquid surface. Thus, the vertical pipes with the highest liquid level $d$ have the lowest surface pressure $p_a$, or the highest gas velocity. Therefore, the gas velocities are $v_b > v_a > v_c$.

15.11. The pressure is reduced at the chimney due to the movement of the wind above (Bernoulli’s principle). Thus, the air will flow in the window and out the chimney. Prairie dogs ventilate their burrows this way; a small breeze above their mound lowers the pressure there and allows the air in the burrow to move between openings of different types or heights.

15.12. Equation 15.34 is $F/A = Y(\Delta L/L)$. The second wire is the same material, and has length $L' = 2L$ and area $A' = \pi (2r)^2 = 4\pi r^2 = 4A$. The force required to stretch it the same length $\Delta L = 1 \text{ mm}$ is

$$F' = A' Y \frac{\Delta L}{L'} = 4A Y \frac{\Delta L}{2L} = 2 \left( A Y \frac{\Delta L}{L} \right) = 2F$$

So the force required is 4000 N.

15.13. The stress $F/A$ on the wire is proportional to the strain $\Delta L/L$ in the elastic region. The breaking point is past the elastic limit. The elastic limit is reached at a particular strain. Since the wire is the same diameter the area $A$ stays the same, so an equal force of 5000 N will also take the wire to its elastic limit.

**Exercises and Problems**

**Section 15.1 Fluids**

15.1. Solve: The volume of the liquid is

$$\rho = \frac{m}{V} \Rightarrow V = \frac{m}{\rho} = \frac{0.055 \text{ kg}}{1100 \text{ kg/m}^3} \left( \frac{10^6 \text{ mL}}{\text{m}^3} \right) = 50 \text{ mL}$$

Assess: The liquid’s density slightly higher than that of water $(1000 \text{ kg/m}^3)$, so it is reasonable that it requires slightly less than 55 mL to get a mass of 55 g.

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15.2. Solve: The volume of the helium gas in container A is equal to the volume of the liquid in container B. That is, \( V_A = V_B \). Using the definition of mass density \( \rho = m/V \), this relationship becomes

\[
\frac{m_A}{\rho_A} = \frac{m_B}{\rho_B} \Rightarrow \frac{m_{He}}{\rho_{He}} = \frac{7000 \, \text{kg/m}^3}{1260 \, \text{kg/m}^3} \Rightarrow \rho_B = 7000\rho_{He} = (7000)(0.18 \, \text{kg/m}^3) = 1260 \, \text{kg/m}^3
\]

Referring to Table 15.1, we find that the liquid is glycerine.

15.3. Model: The density of water is 1000 kg/m\(^3\).

Visualize:

Solve: Volume of water in the swimming pool is

\[
V = 6.0 \, \text{m} \times 12 \, \text{m} \times 3.0 \, \text{m} - \frac{1}{2}(6.0 \, \text{m} \times 12 \, \text{m} \times 2.0 \, \text{m}) = 144 \, \text{m}^3
\]

The mass of water in the swimming pool is

\[
m = \rho V = (1000 \, \text{kg/m}^3)(144 \, \text{m}^3) = 1.4 \times 10^5 \, \text{kg}
\]

15.4. Model: The densities of gasoline and water are given in Table 15.1.

Solve: (a) The total mass is

\[
m_{\text{total}} = m_{\text{gasoline}} + m_{\text{water}} = 0.050 \, \text{kg} + 0.050 \, \text{kg} = 0.100 \, \text{kg}
\]

The total volume is

\[
V_{\text{total}} = V_{\text{gasoline}} + V_{\text{water}} = m_{\text{gasoline}}/\rho_{\text{gasoline}} + m_{\text{water}}/\rho_{\text{water}} = \frac{0.050 \, \text{kg}}{680 \, \text{kg/m}^3} + \frac{0.050 \, \text{kg}}{1000 \, \text{kg/m}^3} = 1.24 \times 10^{-4} \, \text{m}^3
\]

\[
\rho_{\text{avg}} = \frac{m_{\text{total}}}{V_{\text{total}}} = \frac{0.100 \, \text{kg}}{1.24 \times 10^{-4} \, \text{m}^3} = 8.1 \times 10^2 \, \text{kg/m}^3
\]

(b) The average density is calculated as follows:

\[
\rho_{\text{avg}} = \frac{m_{\text{water}}V_{\text{water}} + m_{\text{gasoline}}V_{\text{gasoline}}}{V_{\text{water}} + V_{\text{gasoline}}} = \frac{(50 \, \text{cm}^3)(1000 \, \text{kg/m}^3 + 680 \, \text{kg/m}^3)}{100 \, \text{cm}^3} = 8.4 \times 10^2 \, \text{kg/m}^3
\]

Assess: The above average densities are between those of gasoline and water, which is reasonable.

Section 15.2 Pressure

15.5. Model: The density of sea water is 1030 kg/m\(^3\).

Solve: The pressure below sea level can be found from Equation 15.5 as follows:

\[
p = \rho g d = 1.013 \times 10^5 \, \text{Pa} + (1030 \, \text{kg/m}^3)(9.81 \, \text{m/s}^2)(1.1 \times 10^4 \, \text{m}) = 1.013 \times 10^5 \, \text{Pa} + 1.1103 \times 10^8 \, \text{Pa} = 1.1113 \times 10^8 \, \text{Pa} = 1.1 \times 10^3 \, \text{atm}
\]

where we have used the conversion 1 atm = 1.013 \times 10^5 \, \text{Pa}.

Assess: The pressure deep in the ocean is very large.
15.6. Visualize:

\[ p_0 = \rho g d \]

\[ p = p_0 + \rho g d \]

Solve: The pressure at the bottom of the vat is \( p = p_0 + \rho g d = 1.3 \text{ atm} \). Substituting into this equation gives

\[ 1.013 \times 10^5 \text{ Pa} + \rho (9.8 \text{ m/s}^2)(2.0 \text{ m}) = (1.3)(1.013 \times 10^5) \text{ Pa} \Rightarrow \rho = 1550.5 \text{ kg/m}^3 \]

The mass of the liquid in the vat is

\[ m = \rho V = \rho \pi (0.50 \text{ m})^2 d = (1550.5 \text{ kg/m}^3) \pi (0.50 \text{ m})^2 (2.0 \text{ m}) = 2.4 \times 10^3 \text{ kg} \]

15.7. Model: The density of water is 1000 kg/m³ and the density of ethyl alcohol is 790 kg/m³.

Solve: (a) The volume of water that has the same mass as 8.0 m³ of ethyl alcohol is

\[ V_{water} = \frac{m_{water}}{\rho_{water}} = \frac{m_{alcohol}}{\rho_{water}} \rho_{alcohol} V_{alcohol} = \left( \frac{790 \text{ kg/m}^3}{1000 \text{ kg/m}^3} \right) (8.0 \text{ m}^3) = 6.3 \text{ m}^3 \]

(b) The pressure at the bottom of the cubic tank is \( p = p_0 + \rho_{water}gd \):

\[ p = 1.013 \times 10^5 \text{ Pa} + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(6.3)^{1/3} = 1.2 \times 10^5 \text{ Pa} \]

where we have used the relation \( d = (V_{water})^{1/3} \).

15.8. Model: The density of oil is \( \rho_{oil} = 900 \text{ kg/m}^3 \) and the density of water is \( \rho_{water} = 1000 \text{ kg/m}^3 \).

Visualize:

Solve: The pressure at the bottom of the oil layer is \( p_1 = p_0 + \rho_{oil}g d_1 \), and the pressure at the bottom of the water layer is

\[ p_2 = p_1 + \rho_{water}g d_2 = p_0 + \rho_{oil}g d_1 + \rho_{water}g d_2 \]

\[ p_2 = (1.013 \times 10^5 \text{ Pa}) + (900 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.5 \text{ m}) + (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.2 \text{ m}) = 1.2 \times 10^5 \text{ Pa} \]

Assess: A pressure of \( 1.2 \times 10^5 \text{ Pa} = 1.2 \text{ atm} \) is reasonable.
15.9. Model: The density of seawater is $\rho _{\text{seawater}} = 1030 \text{ kg/m}^2$.

Visualize:

Solve: The pressure outside the submarine’s window is $p_{\text{out}} = p_0 + \rho _{\text{seawater}}gd$, where $d$ is the maximum safe depth for the window to withstand a force $F$. This force is $F/A = p_{\text{out}} - p_{\text{in}}$, where $A$ is the area of the window. With $p_{\text{in}} = p_0$, we simplify the pressure equation to

$$d = \frac{1 \times 10^6 \text{ N}}{\pi (0.10 \text{ m})^2 (1030 \text{ kg/m}^2)(9.8 \text{ m/s}^2)} = 3.2 \text{ km}$$

Assess: A force of $1 \times 10^6 \text{ N}$ corresponds to a pressure of $\rho = \frac{F}{A} = \frac{1 \times 10^6 \text{ N}}{\pi (0.10 \text{ m})^2} = 314 \text{ atm}$

A depth of 3 km is therefore reasonable.

15.10. Visualize:

We assume that the seal is at a radius of 5 cm. Outside the seal, atmospheric pressure presses on both sides of the cover and the forces cancel. Thus, only the 10-cm-diameter opening inside the seal is relevant, not the 20 cm diameter of the cover.

Solve: Within the 10 cm diameter area where the pressures differ, 

$$\begin{align*}
F_{\text{left}} &= p_{\text{atmos}}A \\
F_{\text{right}} &= p_{\text{gas}}A
\end{align*}$$

where $A = \pi r^2 = 7.85 \times 10^{-3} \text{ m}^2$ is the area of the opening. The difference between the forces is

$$F_{\text{left}} - F_{\text{right}} = (p_{\text{atmos}} - p_{\text{gas}})A = (101,300 \text{ Pa} - 20,000 \text{ Pa})(7.85 \times 10^{-3} \text{ m}^2) = 0.64 \text{ kN}$$

Normally, the rubber seal exerts a 0.64 kN force to the right to balance the air pressure force. To pull the cover off, an external force must pull to the right with a force $> 0.64 \text{ kN}$. 

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Section 15.3  Measuring and Using Pressure

15.11.  Model: The density of water is \( \rho = 1000 \text{ kg/m}^3 \).

Visualize: Please refer to Figure 15.16.

Solve: From the figure and the equation for hydrostatic pressure, we have

\[
P_0 + \rho gh = P_{\text{atmos}}
\]

Using \( p_0 = 0 \text{ atm} \), and \( P_{\text{atmos}} = 1.013 \times 10^5 \text{ Pa} \), we get

\[
0 \text{ Pa} + (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)h = 1.013 \times 10^5 \text{ Pa} \implies h = 10.3 \text{ m}
\]

Assess: This large value of \( h \) is due to water having a much smaller density than mercury.

15.12.  Model: Assume that the oil is incompressible. Its density is \( 900 \text{ kg/m}^3 \).

Visualize: Please refer to Figure 15.18. Because the liquid is incompressible, the volume displaced in the left cylinder of the hydraulic lift is equal to the volume displaced in the right cylinder.

Solve: Equating the two volumes,

\[
A_1 d_1 = A_2 d_2 \implies (\pi r_1^2) d_1 = (\pi r_2^2) d_2 \implies d_1 = \left( \frac{r_2}{r_1} \right)^2 d_2 = \left( \frac{0.040 \text{ m}}{0.010 \text{ m}} \right)^2 (0.20 \text{ m}) = 3.2 \text{ m}
\]

15.13.  Model: Assume that the vacuum cleaner can create zero pressure.

Solve: The gravitational force on the dog is balanced by the force resulting from the pressure difference between the atmosphere and the vacuum \( (P_{\text{hose}} = 0) \) in the hose. The force applied by the hose is

\[
F = (P_{\text{atmos}} - P_{\text{hose}}) A = P_{\text{atmos}} A = mg
\]

\[
A = \frac{(10 \text{ kg})(9.8 \text{ m/s}^2)}{1.013 \times 10^5 \text{ Pa}} = 9.7 \times 10^{-4} \text{ m}^2
\]

Since \( A = \pi (d/2)^2 \), the diameter of the hose is \( d = 2\sqrt{A/\pi} = 0.035 \text{ m} = 3.5 \text{ cm} \).

Section 15.4  Buoyancy


Visualize:
Solve: The sphere is in static equilibrium because it is neutrally buoyant. That is,
\[ \sum F_y = F_B - F_G = 0 \quad \Rightarrow \quad \rho V g - m g = 0 \quad N \]
The sphere displaces a volume of liquid equal to its own volume, \( V_s = V_s \), so
\[ \rho = \frac{m}{V} = \frac{m}{\frac{4}{3} \pi r^3} = \frac{0.0893 \text{ kg}}{\frac{4}{3} \pi (0.030 \text{ m})^3} = 7.9 \times 10^2 \text{ kg/m}^3 \]
A density of \( 790 \text{ kg/m}^3 \) in Table 15.1 identifies the liquid as ethyl alcohol.
Assess: If the density of the fluid and an object are equal, we have neutral buoyancy.

15.15. Model: The buoyant force on the cylinder is given by Archimedes’ principle.
Visualize:

\[ V_{cyl} \] is the volume of the cylinder and \( V_w \) is the volume of the water displaced by the cylinder. Note that the volume displaced is only from the part of the cylinder that is immersed in water.
Solve: The cylinder is in static equilibrium, so \( F_B = F_G \). The buoyant force is the weight \( \rho V_w g \) of the displaced water. Thus
\[ F_B = \rho V_w g = F_G = mg = \rho_{cyl} V_{cyl} g \quad \Rightarrow \quad \rho_w V_w = \rho_{cyl} V_{cyl} \quad \Rightarrow \quad \rho_{cyl} = \frac{\rho_w V_w}{V_{cyl}} \]
\[ \rho_{cyl} = \frac{(1000 \text{ kg/m}^3) A(0.040 \text{ m})}{A(0.060 \text{ m})} = 6.7 \times 10^2 \text{ kg/m}^3 \]
where \( A \) is the cross-sectional area of the cylinder.
Assess: \( \rho_{cyl} < \rho_w \) for a cylinder floating in water is an expected result.

15.16. Model: The buoyant force on the sphere is given by Archimedes’ principle.
Visualize:

Solve: The sphere is in static equilibrium. The free-body diagram on the sphere shows that
\[ \sum F_y = F_B - T - F_G = 0 \quad \Rightarrow \quad F_B = T + F_G = \frac{1}{3} F_G + F_G = \frac{4}{3} F_G \]
\[ \rho_w V_{sphere} g = \frac{4}{3} \rho_{sphere} V_{sphere} g \quad \Rightarrow \quad \rho_{sphere} = \frac{3}{4} \rho_w = \frac{3}{4} (1000 \text{ kg/m}^3) = 750 \text{ kg/m}^3 \]
15.17. Model: The buoyant force on the rock is given by Archimedes’ principle. 

Visualize:

\[ F_B^g = \rho_g V_g g \]

Solve: Because the rock is in static equilibrium, Newton’s first law gives

\[ F_{net} = T + F_B^g - (F_G)_{rock} = 0 \text{ N} \]

\[ T = \rho_{rock} V_{rock} g - \rho_{water} \left( \frac{1}{2} V_{rock} \right) g = \left( \rho_{rock} - \frac{1}{2} \rho_{water} \right) V_{rock} g = \left( \frac{m_{rock} g}{\rho_{rock}} \right) \left( 1 - \frac{\rho_{water}}{2 \rho_{rock}} \right) m_{rock} g \]

Using \( \rho_{rock} = 4800 \text{ kg/m}^3 \) and \( m_{rock} = 5.0 \text{ kg} \), we get \( T = 44 \text{ N} \).

15.18. Model: The buoyant force on the aluminum block is given by Archimedes’ principle. The density of aluminum and ethyl alcohol are \( \rho_{Al} = 2700 \text{ kg/m}^3 \) and \( \rho_{ethyl alcohol} = 790 \text{ kg/m}^3 \).

Visualize:

The buoyant force \( F_B^g \) and the tension due to the string act vertically up, and the gravitational force on the aluminum block acts vertically down. The block is submerged, so the volume of displaced fluid equals \( V_{Al} \), the volume of the block.

Solve: The aluminum block is in static equilibrium, so

\[ \sum F_y = F_B^g + T - F_G = 0 \text{ N} \Rightarrow \rho_{Al} V_{Al} g + T - \rho_{Al} V_{Al} g = 0 \text{ N} \Rightarrow T = V_{Al} g (\rho_{Al} - \rho_I) \]

\[ T = \left( 100 \times 10^{-6} \text{ m}^3 \right) \left( 9.81 \text{ m/s}^2 \right) \left( 2700 \text{ kg/m}^3 - 790 \text{ kg/m}^3 \right) = 1.9 \text{ N} \]

where we have used the conversion \( 100 \text{ cm}^3 = 100 \times (10^{-2} \text{ m})^3 = 10^{-4} \text{ m}^3 \).

Assess: The gravitational force on the aluminum block is \( \rho_{Al} V_{Al} g = 2.65 \text{ N} \). A similar order of magnitude for \( T \) is reasonable.
15.19. **Model:** The buoyant force on the steel cylinder is given by Archimedes’ principle.

**Visualize:**

The length of the cylinder above the surface of mercury is \( d \).

**Solve:** The cylinder is in static equilibrium with \( F_B = F_G \). Thus

\[
F_B = \rho_{\text{Hg}} V_{\text{Hg}} g = F_G = mg = \rho_{\text{cyl}} V_{\text{cyl}} g \quad \Rightarrow \quad \rho_{\text{Hg}} V_{\text{Hg}} = \rho_{\text{cyl}} V_{\text{cyl}} \quad \Rightarrow \quad \rho_{\text{Hg}} A(0.20 \text{ m} - d) = \rho_{\text{cyl}} A(0.20 \text{ m})
\]

\[
d = 0.20 \text{ m} - \frac{\rho_{\text{cyl}}}{\rho_{\text{Hg}}} (0.20 \text{ m}) = (0.20 \text{ m}) \left( 1 - \frac{7900 \text{ kg/m}^3}{13,600 \text{ kg/m}^3} \right) = 0.084 \text{ m} = 8.4 \text{ cm}
\]

That is, the length of the cylinder above the surface of the mercury is 8.4 cm.

15.20. **Model:** The buoyant force is determined by Archimedes’ principle. Ignore any compression the air in the beach ball may undergo as a result of submersion.

**Solve:** The mass of the beach ball is negligible, so the force needed to push it below the water is equal to the buoyant force.

\[
F_B = \rho_w \left( \frac{4}{3} \pi R^3 \right) g = (1000 \text{ kg/m}^3) \left( \frac{4}{3} \pi (0.30 \text{ m})^3 \right) (9.8 \text{ m/s}^2) = 1.1 \text{ kN}
\]

**Assess:** It would take a 113 kg (250 lb) person to push the ball below the water. Two people together could do it. This seems about right.

15.21. **Model:** The buoyant force on the sphere is given by Archimedes’ principle.

**Visualize:**

**Solve:** For the Styrofoam sphere and the mass not to sink, the sphere must be completely submerged and the buoyant force \( F_B \) must be equal to the sum of the gravitational force on the Styrofoam sphere and the attached mass. Neglecting the volume of the hanging mass, the volume of displaced water equals the volume of the sphere, so

\[
F_B = \rho_{\text{water}} V_{\text{water}} g = (1000 \text{ kg/m}^3) \left( \frac{4}{3} \pi (0.25 \text{ m})^3 \right) (9.8 \text{ m/s}^2) = 641.4 \text{ N}
\]

\[
(F_{G})_{\text{Styrofoam}} = \rho_{\text{Styrofoam}} V_{\text{Styrofoam}} g = (150 \text{ kg/m}^3) \left[ \frac{4}{3} \pi (0.25 \text{ m})^3 \right] (9.8 \text{ m/s}^2) = 96.2 \text{ N}
\]

Because \( (F_{G})_{\text{Styrofoam}} + mg = F_B \),

\[
m = \frac{F_B - (F_{G})_{\text{Styrofoam}}}{g} = \frac{641.4 \text{ N} - 96.2 \text{ N}}{9.8 \text{ m/s}^2} = 55.6 \text{ kg}
\]

To two significant figures, the mass is 56 kg.
Section 15.5 Fluid Dynamics

15.22. Model: Treat the water as an ideal fluid. The hose is a flow tube, so the equation of continuity applies.
Solve: The volume flow rate is
\[ Q = \frac{600 \text{ L}}{8.0 \text{ min}} = \frac{600 \times 10^{-3} \text{ m}^3}{8.0 \times 60 \text{ s}} = 1.25 \times 10^{-3} \text{ m}^3/\text{s} \]
Using the definition \( Q = \nu A = \pi (d/2)^2 \), we get
\[ d = \sqrt{\frac{4Q}{\pi \nu}} = \sqrt{\frac{4(1.25 \times 10^{-3} \text{ m}^3/\text{s})}{\pi (4.0 \text{ m/s})}} = 2.0 \text{ cm} \]
Assess: This is a reasonable diameter for a typical garden hose.

15.23. Model: Treat the water as an ideal fluid. The pipe itself is a flow tube, so the equation of continuity applies.
Visualize:

Note that \( A_1, A_2, \) and \( A_3 \) and \( v_1, v_2, \) and \( v_3 \) are the cross-sectional areas and the speeds in the first, second, and third segments of the pipe, respectively.
Solve: (a) The equation of continuity is
\[ A_1 v_1 = A_2 v_2 = A_3 v_3 \Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2 = \pi r_3^2 v_3 \Rightarrow r_1^2 v_1 = r_2^2 v_2 = r_3^2 v_3 \]
\[ (0.0050 \text{ m})^2 (4.0 \text{ m/s}) = (0.010 \text{ m})^2 v_2 = (0.0025 \text{ m})^2 v_3 \]
\[ v_2 = \left( \frac{0.0050 \text{ m}}{0.010 \text{ m}} \right)^2 (4.0 \text{ m/s}) = 1.0 \text{ m/s} \]
\[ v_3 = \left( \frac{0.0050 \text{ m}}{0.0025 \text{ m}} \right)^2 (4.0 \text{ m/s}) = 16 \text{ m/s} \]
(b) The volume flow rate through the pipe is
\[ Q = A_1 v_1 = \pi (0.0050 \text{ m})^2 (4.0 \text{ m/s}) = 3.1 \times 10^{-4} \text{ m}^3/\text{s} \]

15.24. Model: Treat the water as an ideal fluid so that the flow in the tube follows the continuity equation.
Visualize:

Solve: The equation of continuity is
\( v_0 A_0 = v_1 A_1 \), where \( A_0 = L^2 \) and \( A_1 = \pi \left( \frac{L}{2} \right)^2 \). The above equation simplifies to
\[ v_0 L^2 = v_1 \pi \left( \frac{L}{2} \right)^2 \Rightarrow v_1 = \left( \frac{4}{\pi} \right) v_0 = 1.27 v_0 \]
15.25. Model: Treat the oil as an ideal fluid obeying Bernoulli’s equation. Consider the path connecting point 1 in the lower pipe with point 2 in the upper pipe a streamline.

Visualize: Please refer to Figure EX15.25.

Solve: Bernoulli’s equation is

\[ p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 = p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 \]

Using \( p_1 = 200 \text{ kPa} = 2.0 \times 10^5 \text{ Pa}, \quad \rho = 900 \text{ kg/m}^3, \quad y_2 - y_1 = 10.0 \text{ m}, \quad v_1 = 2.0 \text{ m/s}, \quad \text{and} \quad v_2 = 3.0 \text{ m/s}, \) we get \( p_2 = 1.096 \times 10^5 \text{ Pa} = 110 \text{ kPa}. \)

Section 15.6 Elasticity

15.26. Model: Turning the tuning screws on a guitar string creates tensile stress in the string.

Solve: The tensile stress in the string is given by \( T/A \), where \( T \) is the tension in the string and \( A \) is the cross-sectional area of the string. From the definition of Young’s modulus,

\[ Y = \frac{T}{\Delta L/L} \Rightarrow \Delta L = \frac{T(L)}{AF} \]

Using \( T = 2000 \text{ N}, \quad L = 0.80 \text{ m}, \quad A = \pi(0.00050 \text{ m})^2, \) and \( Y = 20 \times 10^{10} \text{ N/m}^2 \) (from Table 15.3), we obtain \( \Delta L = 0.010 \text{ m} = 1.0 \text{ cm}. \)

Assess: 1.0 cm is a large stretch for a length of 80 cm, but 2000 N is a large tension.

15.27. Model: The dangling mountain climber creates tensile stress in the rope.

Solve: Young’s modulus for the rope is

\[ Y = \frac{F/A}{\Delta L/L} = \frac{\text{stress}}{\text{strain}} \]

The tensile stress is

\[ \frac{(70 \text{ kg})(9.8 \text{ m/s}^2)}{\pi (0.0050 \text{ m})^2} = 8.734 \times 10^6 \text{ Pa} \]

and the strain is 0.080 m/50 m = 0.00160. Dividing the two quantities yields \( Y = 5.5 \times 10^9 \text{ N/m}^2 \).

15.28. Model: The hanging mass creates tensile stress in the wire.

Solve: The force \( (F) \) pulling on the wire, which is simply the gravitational force \( (mg) \) on the hanging mass, produces tensile stress given by \( F/A \), where \( A \) is the cross-sectional area of the wire. The fractional change in the wire’s length is \( \Delta L/L = 0.010 \). From the definition of Young’s modulus, we have

\[ Y = \frac{mg/A}{\Delta L/L} \Rightarrow \frac{m}{\pi(0.50 \times 10^{-3} \text{ m})^2(7 \times 10^{10} \text{ N/m}^2)(0.010)}{9.8 \text{ m/s}^2} = 60 \text{ kg} \]

15.29. Model: The load supported by a concrete column creates compressive stress in the concrete column.

Solve: The gravitational force on the load produces tensile stress given by \( F/A \), where \( A \) is the cross-sectional area of the concrete column and \( F \) equals the gravitational force on the load. From the definition of Young’s modulus,

\[ Y = \frac{F/A}{\Delta L/L} \Rightarrow \Delta L = \frac{\pi(0.25 \text{ m})^2}{3 \times 10^{10} \text{ N/m}^2} = 1 \text{ mm} \]

Assess: A compression of 1.0 mm of the concrete column by a load of approximately 200 tons is reasonable.

15.30. Model: Water is almost incompressible and it applies a volume stress.

Solve: (a) The pressure at a depth of 5000 m in the ocean is

\[ p = p_0 + \rho_{\text{sea water}} g (5000 \text{ m}) = 1.013 \times 10^5 \text{ Pa} + (1030 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(5000 \text{ m}) = 5.057 \times 10^7 \text{ Pa} = 5.1 \times 10^7 \text{ Pa} \]
(b) Using the bulk modulus of water,
\[ \frac{\Delta V}{V} = 2 \frac{p}{B} = 2 \frac{5.057 \times 10^7 \text{ Pa}}{0.2 \times 10^6 \text{ Pa}} = 2 \text{ 0.025} \]

(c) The volume of a mass of water decreases from \( V \) to \( 0.975V \). Thus the water’s density increases from \( \rho \) to \( \rho/0.975 \). The new density is
\[ \rho_{5000 \text{ m}} = \frac{1030 \text{ kg/m}^3}{0.975} = 1056 \text{ kg/m}^3 \]

15.31. Solve: The pressure \( p \) at depth \( d \) in a fluid is \( p = p_0 + \rho gd \). Using \( 1.29 \text{ kg/m}^3 \) for the density of air,
\[ p_{\text{bottom}} = p_{\text{top}} + p_{\text{air}} gd \implies p_{\text{bottom}} - p_{\text{top}} = (1.29 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(16 \text{ m}) = 202 \text{ Pa} = 1.99 \times 10^{-3} \text{ atm} \]
Assuming \( p_{\text{bottom}} = 1 \text{ atm} \),
\[ \frac{p_{\text{bottom}} - p_{\text{top}}}{1 \text{ atm}} = \frac{1.99 \times 10^{-3} \text{ atm}}{1 \text{ atm}} = 0.20, \]

15.32. Model: We assume that there is a perfect vacuum inside the cylinders with \( p = 0 \text{ Pa} \). We also assume that the atmospheric pressure in the room is 1 atm.
Visualize: Please refer to Figure P15.32.
Solve: (a) The flat end of each cylinder has an area \( A = \pi r^2 = \pi (0.30 \text{ m})^2 = 0.283 \text{ m}^2 \). The force on each end is thus
\[ F_{\text{atm}} = p_0 A = (1.013 \times 10^5 \text{ Pa})(0.283 \text{ m}^2) = 2.86 \times 10^4 \text{ N} \]
To two significant figures, the force on each end is \( 2.9 \times 10^4 \text{ N} \).
(b) The net vertical force on the lower cylinder when it is on the verge of being pulled apart is
\[ \sum F_y = F_{\text{atm}} - (F_G)_{\text{players}} = 0 \text{ N} \implies (F_G)_{\text{players}} = F_{\text{atm}} = 2.86 \times 10^4 \text{ N} \]
number of players = \( \frac{2.86 \times 10^4 \text{ N}}{(100 \text{ kg})(9.8 \text{ m/s}^2)} = 29.2 \)
That is, 30 players are needed to pull the two cylinders apart.

15.33. Model: Assume that the oil is incompressible and its density is \( 900 \text{ kg/m}^3 \).
Visualize: Please refer to Figure P15.33.
Solve: (a) The pressure at depth \( d \) in a fluid is \( p = p_0 + \rho gd \). Here, pressure \( p_0 \) at the top of the fluid is due both to the atmosphere and to the gravitational force on the piston. That is, \( p_0 = p_{\text{atm}} + (F_G)p/A \). At point A,
\[ p_A = p_{\text{atm}} + \frac{(F_G)p}{A} + \rho g(1.00 \text{ m} - 0.30 \text{ m}) \]
\[ = 1.013 \times 10^5 \text{ Pa} + \frac{(10 \text{ kg})(9.8 \text{ m/s}^2)}{\pi (0.020 \text{ m})^2} + (900 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.70 \text{ m}) = 185,460 \text{ Pa} \]
\[ F_A = p_A A = (185,460 \text{ Pa})\pi (0.10 \text{ m})^2 = 5.8 \text{ kN} \]
(b) In the same way,
\[ p_B = p_{\text{atm}} + \frac{(F_G)p}{A} + \rho g(1.30 \text{ m}) = 190,752 \text{ Pa} \implies F_B = 6.0 \text{ kN} \]
Assess: \( F_B \) is larger than \( F_A \), because \( p_B \) is larger than \( p_A \).
15.34. **Model:** Assume that blood is incompressible.

**Solve:** When lying down, the pressure at both the heart and the brain is at 118 mm Hg (gauge pressure) or 118 mm Hg + 760 mm Hg = 878 mm Hg absolute pressure. Upon standing up, the pressure in the heart remains at 878 mm Hg, but the pressure in the brain is reduced according to the hydrostatic pressure equation:

\[ p_H = p_B + \rho g h \]

where B and H refer to brain and heart, respectively. Inserting \( h = 0.40 \text{ m} \) and solving for \( p_B \) gives

\[
\begin{align*}
 p_B &= p_H - \rho g h \\
     &= 878 \text{ mm Hg} - (1060 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.40 \text{ m}) \left( \frac{760 \text{ mm Hg}}{101.3 \text{ kPa}} \right) \\
     &= 847 \text{ mm Hg}
\end{align*}
\]

In gauge pressure, the blood pressure in the brain is

\[ p_B = 847 \text{ mm Hg} - 760 \text{ mm Hg} = 87 \text{ mm Hg}. \]

**Assess:** This blood pressure is below the 90 mm Hg that can cause fainting, which explains why people whose blood vessels do not constrict to compensate for this effect feel faint when standing from a prone position.

15.35. **Model:** The tire flattens until the pressure force against the ground balances the upward normal force of the ground on the tire.

**Solve:** The area of the tire in contact with the road is

\[ A = (0.15 \text{ m})(0.13 \text{ m}) = 0.0195 \text{ m}^2. \]

The normal force on each tire is

\[ F_n = (1500 \text{ kg})(9.8 \text{ m/s}^2) = 3675 \text{ N}. \]

Thus, the pressure inside each tire is

\[ p_{\text{inside}} = \frac{F_n}{A} = \frac{3675 \text{ N}}{0.0195 \text{ m}^2} = 188,500 \text{ Pa} = 1.86 \text{ atm} \times \frac{14.7 \text{ psi}}{1 \text{ atm}} = 27 \text{ psi}. \]

15.36. **Visualize:**

**Solve:** (a) Because the patient’s blood pressure is 140/100, the minimum fluid pressure needs to be 100 mm of Hg above atmospheric pressure. Since 760 mm of Hg is equivalent to 1 atm and 1 atm is equivalent to \( 1.013 \times 10^5 \text{ Pa} \), the minimum pressure is

\[ 100 \text{ mm} = 1.333 \times 10^4 \text{ Pa}. \]

The excess pressure in the fluid is due to force \( F \) pushing on the internal 6.0-mm-diameter piston that presses against the liquid. Thus, the minimum force the nurse needs to apply to the syringe is

\[ F = \text{fluid pressure} \times \text{area of plunger} = (1.333 \times 10^4 \text{ Pa}) \left( \pi (0.0030 \text{ m})^2 \right) = 0.38 \text{ N}. \]

(b) The flow rate is \( Q = \nu A \), where \( \nu \) is the flow speed of the medicine and \( A \) is the cross-sectional area of the needle. Thus,

\[
\nu = \frac{Q}{A} = \frac{2.0 \times 10^{-6} \text{ m}^3/2.0 \text{ s}}{\pi (0.125 \times 10^{-3} \text{ m})^2} = 20 \text{ m/s}
\]

**Assess:** Note that the pressure in the fluid is due to \( F \) that is not dependent on the size of the plunger pad. Also note that the syringe is not drawn to scale.
15.37. Solve: The fact that atmospheric pressure at sea level is 101.3 kPa = 101,300 N/m² means that the weight of the atmosphere over each square meter of surface is 101,300 N. Thus the mass of air over each square meter is \(m = (101,300 \text{ N})/g = (101,300 \text{ N})/(9.80 \text{ m/s}^2) = 10,340 \text{ kg per m}^2\). Multiplying by the earth’s surface area will give the total mass. Using \(R_e = 6.27 \times 10^6 \text{ m}\) for the earth’s radius, the total mass of the atmosphere is
\[
M_{\text{air}} = A_{\text{earth}} m = (4\pi R_e^2) m = 4\pi(6.37 \times 10^6 \text{ m})^2(10,340 \text{ kg/m}^2) = 5.27 \times 10^{18} \text{ kg}
\]

15.38. Visualize: Let \(d\) be the atmosphere’s thickness, \(p\) the atmospheric pressure on the earth’s surface, and \(p_0(= 0 \text{ atm})\) the pressure beyond the earth’s atmosphere.
Solve: The pressure at a depth \(d\) in a fluid is \(p = p_0 + \rho gd\). This equation becomes
\[
1 \text{ atm} = 0 \text{ atm} + \rho_{\text{air}} gd \implies \frac{1 \text{ atm}}{\rho_{\text{air}} g} = \frac{1.013 \times 10^5 \text{ Pa}}{(1.3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 8.0 \text{ km}
\]

15.39. Model: Oil is incompressible and has a density \(900 \text{ kg/m}^3\).
Visualize: Please refer to Figure P15.39.
Solve: (a) The pressure at point A, which is 0.50 m below the open oil surface, is
\[
p_A = p_0 + \rho_{\text{oil}}(1.00 \text{ m} - 0.50 \text{ m}) = 101,300 \text{ Pa} + (900 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.50 \text{ m}) = 106 \text{ kPa}
\]
(b) The pressure difference between A and B is
\[
p_B - p_A = (p_0 + \rho_{\text{oil}}d_B) - (p_0 + \rho_{\text{oil}}d_A) = \rho_{\text{oil}}(d_B - d_A) = (900 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.50 \text{ m}) = 4.4 \text{ kPa}
\]
Pressure depends only on depth, and C is the same depth as B. Thus \(p_C - p_A = 4.4 \text{ kPa}\) also, even though C isn’t directly under A.

15.40. Model: Assume that oil is incompressible and its density is \(\rho = 900 \text{ kg/m}^3\).
Visualize: Please refer to Figure P15.40.
Solve: (a) The hydraulic lift is in equilibrium and the pistons on the left and the right are at the same level. Therefore, Equation 15.10 simplifies to
\[
\frac{F_{\text{left piston}}}{A_{\text{left piston}}} = \frac{F_{\text{right piston}}}{A_{\text{right piston}}} \implies \frac{(F_G)_\text{student}}{\pi (r_{\text{student}})^2} = \frac{(F_G)_\text{elephant}}{\pi (r_{\text{elephant}})^2}
\]
\[
r_{\text{student}} = \sqrt{\frac{(F_G)_\text{student}}{(F_G)_\text{elephant}}} r_{\text{elephant}} = \sqrt{\frac{(70 \text{ kg})g}{(1200 \text{ kg})g}} (1.0 \text{ m}) = 0.2415 \text{ m}
\]
The diameter of the piston the student is standing on is therefore \(2 \times 0.2415 \text{ m} = 0.48 \text{ m}\).
(b) With the second student added, the pistons are no longer at the same height. Therefore, Equation 15.10 gives
\[
\frac{F_{\text{right piston}}}{A_{\text{right piston}}} = \frac{F_{\text{left piston}}}{A_{\text{left piston}}} - \rho gh
\]
With \(F_{\text{left piston}} = m_s g + (70 \text{ kg})g\) and \(h = 0.35 \text{ m}\), we can solve for the second students mass \(m_s\):
\[
m_s = m_{\text{elephant}} A_{\text{left piston}} - 70 \text{ kg} + \rho h A_{\text{left piston}}
\]
\[
= \frac{(1200 \text{ kg})(0.2415 \text{ m})^2}{(1.0 \text{ m})^2} - 70 \text{ kg} + (900 \text{ kg/m}^3)(0.35 \text{ m})\pi(0.2415 \text{ m})^2 = 58 \text{ kg}
\]
15.41. Model: Assume that the oil is incompressible and its density is 900 kg/m³.

Visualize:

\[ p_1 = \frac{F_1}{A_1} = p_0 + \frac{F_2}{A_2} + \rho gh \]

\[ F_1 = F_2 + \rho gh \]

With \( F_1 - m_1g, F_2 = 4m_2g, A_1 = \pi r_1^2 \), and \( A_2 = \pi r_2^2 \), we have

\[ m_1 g = \frac{4m_2 g}{\pi r_2^2} + \rho gh \Rightarrow r_2 = \left( \frac{4m_2 g}{\pi} \right)^{1/2} \left( \frac{m_1 g}{\pi r_1^2} - \rho gh \right)^{-1/2} \]

Using \( m_1 = 55 \text{ kg}, m_2 = 110 \text{ kg}, r_1 = 0.08 \text{ m}, \rho = 900 \text{ kg/m}^3 \), and \( h = 1.0 \text{ m} \), the calculation yields \( r_2 = 0.276 \text{ m} \).

Assess: Both pistons are too small to hold the people as shown, but the ideas are correct.

15.42. Model: Water and mercury are incompressible and immiscible liquids.

Visualize:

The water in the left arm floats on top of the mercury and presses the mercury down from its initial level. Because points 1 and 2 are level with each other and the fluid is in static equilibrium, the pressure at these two points must be equal. If the pressures were not equal, the pressure difference would cause the fluid to flow, violating the assumption of static equilibrium.

Solve: The pressure at point 1 is due to water of depth \( d_w = 10 \text{ cm} \):

\[ p_1 = p_{\text{atmos}} + \rho_w g d_w \]

Because mercury is incompressible, the mercury in the left arm goes down a distance \( h \) while the mercury in the right arm goes up a distance \( h \). Thus, the pressure at point 2 is due to mercury of depth \( d_{\text{Hg}} = 2h \):

\[ p_2 = p_{\text{atmos}} + \rho_{\text{Hg}} g d_{\text{Hg}} = p_{\text{atmos}} + 2 \rho_{\text{Hg}} gh \]

Equating \( p_1 \) and \( p_2 \) gives

\[ p_{\text{atmos}} + \rho_w g d_w = p_{\text{atmos}} + 2 \rho_{\text{Hg}} gh \Rightarrow h = \frac{1}{2} \frac{\rho_w}{\rho_{\text{Hg}}} d_w = \left( \frac{1}{2} \right) \left( \frac{1000 \text{ kg/m}^3}{13,600 \text{ kg/m}^3} \right) (10 \text{ cm}) = 3.7 \text{ mm} \]

The mercury in the right arm rises 3.7 mm above its initial level.
15.43. **Model:** Glycerin and ethyl alcohol are incompressible and do not mix.
**Visualize:**

**Solve:** The alcohol in the left arm floats on top of the denser glycerin and presses the glycerin down distance $h$ from its initial level. This causes the glycerin to rise distance $h$ in the right arm. Points 1 and 2 are level with each other and the fluids are in static equilibrium, so the pressures at these two points must be equal:

$$p_1 = p_2 \implies p_0 + \rho_{eth}gd_{eth} = p_0 + \rho_{gly}gd_{gly} \implies \rho_{eth}g(20 \text{ cm}) = \rho_{gly}g(2h)$$

$$h = \frac{1}{2} \rho_{eth}(20 \text{ cm}) = \left(\frac{1}{2}\right)\left(\frac{790 \text{ kg/m}^3}{1260 \text{ kg/m}^3}\right)(20 \text{ cm}) = 6.27 \text{ cm}$$

You can see from the figure that the difference between the top surfaces of the fluids is

$$\Delta y = 20 \text{ cm} - 2h = 20 \text{ cm} - (6.27 \text{ cm}) = 7.46 \text{ cm} \approx 7.5 \text{ cm}$$

15.44. **Model:** Assume the liquid is incompressible.
**Visualize:**

**Solve:** (a) Can 2 has moved down with respect to Can 1 since the water level in Can 2 has risen. Since the total volume of water stays constant, the water level in Can 1 has fallen by the same amount. The water level is equalized in the two cans at the middle of the height change, so the change in height of the water is half the relative change in height of the cans. Can 2 has moved relative to Can 1 (6.5 cm − 5.0 cm) × 2 = 3.0 cm down.

(b) The water level in Can 1 has fallen by the same amount. The new level is 5.0 − 1.5 cm = 3.5 cm.
**Assess:** The two cans are an inexpensive method of measuring relative changes in height.

15.45. **Model:** The water is in hydrostatic equilibrium.
**Visualize:**

**Solve:** (a) The force of the liquid on the bottom is the pressure $p = \rho gD$ times the area $A = WL$. This gives

$$F = pA = \rho gDWL$$
(b) The pressure on the front panel of the aquarium can be found by integrating the force on a thin strip $dy$ (see figure above). Using $F = pA$, we find

$$dF = \rho gyLdy$$

$$F = \int_0^D \rho gyLdy = \frac{1}{2} \rho gD^2 L$$

(c) For a 100-cm-long, 35-cm-wide, 40-cm-deep aquarium filled with water, the force from the liquid on the front window is

$$F = \frac{1}{2} (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.40 \text{ m})^2(1.0 \text{ m}) = 784 \text{ N} \approx 0.78 \text{ kN}$$

The force on the bottom is

$$F = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.40 \text{ m})(0.35 \text{ m})(1.0 \text{ m}) = 1372 \text{ N} \approx 1.4 \text{ kN}$$

15.46. Visualize:

The figure shows a small column of air of thickness $dz$, of cross-sectional area $A = 1 \text{ m}^2$, and of density $\rho(z)$. The column is at a height $z$ above the surface of the earth.

**Solve:** (a) The atmospheric pressure at sea level is $1.013 \times 10^5 \text{ Pa}$. That is, the weight of the air column with a $1 \text{ m}^2$ cross section is $1.013 \times 10^5 \text{ N}$. Consider the weight of a $1 \text{ m}^2$ slice of thickness $dz$ at a height $z$. This slice has volume $dV = Adz = (1 \text{ m}^2)dz$, so its weight is $dw = \rho g dV = \rho g (1 \text{ m}^2)dz = \rho_0 e^{-z/z_0} g (1 \text{ m}^2)dz$. The total weight of the $1 \text{ m}^2$ column is found by adding all the $dw$. Integrating from $z = 0$ to $z = \infty$,

$$w = \int_0^\infty \rho_0 g (1 \text{ m}^2) e^{-z/z_0} dz = \left[ -\rho_0 g (1 \text{ m}^2) z_0 \right]_{0}^{\infty} = \rho_0 g (1 \text{ m}^2) z_0$$

Because $w = 101,300 \text{ N} = \rho_0 g (1 \text{ m}^2) z_0$,

$$z_0 = \frac{101,300 \text{ N}}{(1.28 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.0 \text{ m}^2)} = 8.1 \times 10^3 \text{ m}$$

(b) Using the density at sea level from Table 15.1,

$$\rho = (1.28 \text{ kg/m}^3) e^{-z/(8.08 \times 10^7 \text{ m})} = (1.28 \text{ kg/m}^3) e^{-1600 \text{ m}/(8.08 \times 10^7 \text{ m})} = 1.05 \text{ kg/m}^3$$

This is 82% of $\rho_0$. 

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15.47. **Model:** Ignore the change in water density over lengths comparable to the size of the fish.

**Visualize:**

Solve: The buoyant force is the force of gravity on the volume of water displaced by the fish times, or

\[ F_B = \rho_w (V_F + V_b)g \]

where the subscripts w, F, and b indicate water, fish, and bladder, respectively. The force due to gravity on the fish is

\[ F_G = \rho_F V_F g + \rho_{air} V_b g \]

For neutral buoyancy, the two forces must have equal magnitude. Equating them and solving for the \( V_b \) gives

\[
V_b = \frac{\rho_w - \rho_F}{\rho_{air} - \rho_w} V_F = \frac{1000 \text{ kg/m}^3 - 1080 \text{ kg/m}^3}{1.19 \text{ kg/m}^3 - 1000 \text{ kg/m}^3} V_F = 0.0801 V_F
\]

so the fish needs to increase its volume by 8.01%.

15.48. **Model:** The buoyant force on the ceramic statue is given by Archimedes’ principle.

**Visualize:**

Solve: The gravitational force on the statue is the 28.4 N registered on the scale in air. In water, the gravitational force on the statue is balanced by the sum of the buoyant force \( F_B \) and the spring’s force on the statue; that is,

\[
(F_G)_{statue} = F_B + F_{spring on statue} \quad \Rightarrow \quad 28.4 \text{ N} = \rho_w V_{statue} g + 17.0 \text{ N} \quad \Rightarrow \quad V_{statue} = \frac{11.4 \text{ N}}{g \rho_w} = \frac{m_{statue}}{\rho_{statue}}
\]

\[
\rho_{statue} = \frac{(m_{statue} g \rho_w)}{(11.4 \text{ N})} = \frac{(28.4 \text{ N})(1000 \text{ kg/m}^3)}{(11.4 \text{ N})} = 2.49 \times 10^3 \text{ kg/m}^3
\]
15.49. **Model:** The buoyant force on the cylinder is given by Archimedes’ principle.

**Visualize:**

![Diagram of a cylinder floating in water with forces labeled.]

**Solve:** (a) Initially, as it floats, the cylinder is in static equilibrium, with the buoyant force balancing the gravitational force on the cylinder. The volume of displaced liquid is \( Ah \), so

\[
F_B = \rho_{\text{liq}} (Ah) g = F_G
\]

Force \( F \) pushes the cylinder down distance \( x \), so the submerged length is \( h + x \) and the volume of displaced liquid is \( A(h + x) \). The cylinder is again in equilibrium, but now the buoyant force balances both the gravitational force and force \( F \). Thus

\[
F_B = \rho_{\text{liq}} [A(h + x)] g = F_G + F
\]

Since \( \rho_{\text{liq}} (Ah) g = F_G \), we’re left with

\[
F = \rho_{\text{liq}} Agx
\]

(b) The amount of work \( dW \) done by force \( F \) to push the cylinder from \( x \) to \( x + dx \) is \( dW = Fdx = (\rho_{\text{liq}} Agx)dx \). To push the cylinder from \( x_i = 0 \) m to \( x_f = 10 \) cm = 0.10 m requires work

\[
W = \int Fdx = \rho_{\text{liq}} Ag \int xdx = \frac{1}{2} \rho_{\text{liq}} Ag (x_f^2 - x_i^2)
\]

\[
= \frac{1}{2} (1000 \text{ kg/m}^3) \pi (0.020 \text{ m})^2 (9.8 \text{ m/s}^2)(0.10 \text{ m})^2 = 0.62 \text{ J}
\]

15.50. **Model:** The buoyant force on the cylinder is given by Archimedes’ principle.

**Visualize:**

![Diagram of two cylinders in liquids with forces labeled.]

Let \( d_1 \) be the length of the cylinder in the less-dense liquid with density \( \rho_1 \), and \( d_2 \) be the length of the cylinder in the more-dense liquid with density \( \rho_2 \).

**Solve:** The cylinder is in static equilibrium, so

\[
\sum F_y = F_{B1} + F_{B2} - F_G = 0 \text{ N} \Rightarrow \rho_1 (Ad_1)g + \rho_2 (Ad_2)g = \rho A(d_1 + d_2)g
\]

\[
\frac{\rho_1}{\rho_2} d_1 + d_2 = \frac{\rho}{\rho_2} (d_1 + d_2)
\]

Since \( l = d_1 + d_2 \Rightarrow d_1 = l - d_2 \), we can simplify the above equation to obtain

\[
d_2 = \left( \frac{\rho - \rho_1}{\rho_2 - \rho_1} \right) l
\]
15.51. **Model:** The buoyant force on the cylinder is given by Archimedes’ principle.

**Visualize:**

**Solve:** The tube is in static equilibrium, so

\[
\sum F_y = F_B - (F_G)_{tube} - (F_G)_{Pb} = 0 \text{ N} \implies \rho_{\text{liquid}} A (0.25 \text{ m}) g = (0.030 \text{ kg}) g + (0.250 \text{ kg}) g
\]

\[
\rho_{\text{liquid}} = \frac{(0.280 \text{ kg})}{\pi (0.020 \text{ m})^2 (0.25 \text{ m})} = 8.9 \times 10^2 \text{ kg/m}^3
\]

**Assess:** This is a reasonable value for a liquid.

15.52. **Model:** Archimedes’ Principle determines the buoyant force.

**Visualize:**

**Solve:** The plastic hemisphere will hold the most weight when its rim is at the surface of the water. The buoyant force balances the gravitational force on the bowl and rock.

\[
\sum F_y = F_B - (F_G)_{\text{rock}} - (F_G)_{\text{bowl}} = 0 \text{ N}
\]

Thus

\[
mg = p_w V_{\text{bowl}} g - m_{\text{bowl}} g = (1000 \text{ kg/m}^3) \left( \frac{4}{3} \pi (0.040 \text{ m})^3 \right) (9.8 \text{ m/s}^2) - (0.021 \text{ kg})(9.8 \text{ m/s}^2) \implies m = 0.16 \text{ kg}
\]

**Assess:** Putting a rock as big as 160 g in an 8-cm-diameter bowl before it sinks is reasonable.
15.53. **Model:** The buoyant force is determined by Archimedes’ principle. The spring is ideal.

**Visualize:**

![Diagram of a cylinder submerged in water with a spring above it.](image)

**Solve:** The spring is stretched by the same amount that the cylinder is submerged. The buoyant force and spring force balance the gravitational force on the cylinder.

\[
\sum F_y = F_B + F_s - mg = 0 \text{ N}
\]

\[
p_v Ayg + ky = mg
\]

\[
y = \frac{mg}{p_v Ayg + k} = \frac{(1.0 \text{ kg})(9.8 \text{ m/s}^2)}{(1000 \text{ kg/m}^3)\pi(0.025 \text{ m})^2(9.8 \text{ m/s}^2) + 35 \text{ N/m}}
\]

\[
y = 0.181 \text{ m} = 18 \text{ cm}
\]

**Assess:** This is difficult to assess because we don’t know the height \( h \) of the cylinder and can’t calculate it without the density of the metal material.

15.54. **Model:** The buoyant force is determined by Archimedes’ principle. The cross-sectional area of the boat is denoted by \( A \).

**Visualize:**

![Diagram of a boat floating in water with a spring above it.](image)

**Solve:** For the boat to have neutral buoyancy, the buoyant force must have the same magnitude as the force due to gravity:

\[
F_B = F_g \Rightarrow \rho dA g = (M + nm) g
\]

where \( \rho \) is the density of the liquid, \( M \) is the boat’s mass, and \( n \) is an integer. Rearranging this expression gives

\[
nm = \rho dA - M
\]

If we plot the mass \( nm \) added to the boat as a function of the depth \( d \), the \( y \)-intercept will be the negative of the boat’s mass \( M \) and the slope \( s \) will be related to the liquid’s density as \( s = \rho / A \).
From the fit to the data we find $M = 29 \text{ g}$ and $s = 2.667 \text{ kg/m}$. This gives a density of

$$
\rho = \frac{s}{A} = \frac{2.667 \text{ kg/m}}{25 \times 10^{-4} \text{ m}^2} = 1067 \text{ kg/m}^3 = 1.1 \times 10^3 \text{ kg/m}^3
$$

Assess: The mass of the boat is some 6 times greater than the mass of the blocks, which seems reasonable. The density of the liquid is slightly greater than the density of water, which is also a reasonable result.

15.55. **Model:** The buoyant force on the can is given by Archimedes’ principle.

**Visualize:**

The length of the can above the water level is $d$, the length of the can is $L$, and the cross-sectional area of the can is $A$.

**Solve:** The can is in static equilibrium, so

$$
\sum F_y = F_B - (F_G)_{can} - (F_G)_{water} = 0 \
\Rightarrow 0 = \rho_{water} A (L - d) g = (0.020 \text{ kg/g}) g + m_{water} g
$$

The mass of the water in the can is

$$
m_{water} = \rho_{water} \left( \frac{V_{can}}{2} \right) = (1000 \text{ kg/m}^3) \frac{355 \times 10^{-6} \text{ m}^3}{2} = 0.1775 \text{ kg}
$$

$$
\rho_{water} A (L - d) = 0.020 \text{ kg} + 0.1775 \text{ kg} = 0.1975 \text{ kg} \Rightarrow d - L = 2 \frac{0.1975 \text{ kg}}{\rho_{water} A} = 0.0654 \text{ m}
$$

Because $V_{can} = \pi (0.031 \text{ m})^2 L = 355 \times 10^{-6} \text{ m}^3$, $L = 0.1176 \text{ m}$. Using this value of $L$, we get $d = 0.0522 \text{ m} = 5.2 \text{ cm}$.

**Assess:** $d/L = 5.22 \text{ cm}/11.76 \text{ cm} = 0.444$, thus 44.4% of the length of the can is above the water surface. This is reasonable.
15.56. **Model:** The buoyant force on the boat is given by Archimedes’ principle.

**Visualize:**

![Diagram of a boat with dimensions 5 m x 2 cm x 10 m, showing Side 1, Side 2, Bottom, and 0.50 cm depth.]

The minimum height of the boat that will enable the boat to float in perfectly calm water is \( h \).

**Solve:** The boat barely floats if the water comes completely to the top of the sides. In this case, the volume of displaced water is the volume of the boat. Archimedes’ principle in equation form is \( F_B = \rho_w V_{boat}g \). For the boat to float \( F_B = (F_G)_{boat} \). Let us first calculate the gravitational force on the boat:

\[
(F_G)_{boat} = (F_G)_{bottom} + 2(F_G)_{side_1} + 2(F_G)_{side_2},
\]

where

\[
(F_G)_{bottom} = \rho_{steel} V_{bottom}g = (7900 \text{ kg/m}^3)(5.0 \times 10 \times 0.020 \text{ m}^3)g = 7900g \text{ N}
\]

\[
(F_G)_{side_1} = \rho_{steel} V_{side_1}g = (7900 \text{ kg/m}^3)(5.0 \times h \times 0.0050 \text{ m}^3)g = 197.5gh \text{ N}
\]

\[
(F_G)_{side_2} = \rho_{steel} V_{side_2}g = (7900 \text{ kg/m}^3)(10 \times h \times 0.0050 \text{ m}^3)g = 395gh
\]

\[
(F_G)_{boat} = [7900g + 2(197.5gh) + 2(395gh)] N = (7900 + 1185)gh \text{ N}
\]

Going back to the Archimedes’ equation and remembering that \( h \) is in meters, we obtain

\[
\rho_w V_{boat}g = (7900 + 1185)gh \Rightarrow (1000)[10 \times 5.0 \times (h + 0.020)] = 7900 + 1185h
\]

\[ h = 14 \text{ cm} \]

15.57. **Model:** The two pipes are identical.

**Visualize:**

![Diagram of a conical pipe with dimensions 3.0 m x 3.0 m x 3.0 m and a flow rate of 3.0 x 10^6 L/min.]

**Solve:** The water speed is the same in both pipes. The flow rate is

\[
Q = 3.0 \times 10^6 \text{ L/min} = 2(vA)
\]

\[
v = \frac{Q}{2A} = \frac{(3.0 \times 10^6 \text{ L/min})(10^{-3}) \left( \frac{1}{60} \right)}{2\pi(1.5 \text{ m})^2} = 3.5 \text{ m/s}
\]
15.58. **Model:** Treat the liquid as an ideal liquid obeying Bernoulli’s equation (Equation 15.27).

**Visualize:**

![Diagram](image)

**Solve:** (a) Bernoulli’s equation gives

\[ p_0 + \frac{1}{2} \rho v_0^2 + \rho g y_0 = p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 \]

Because the pipe expands equally in the positive and negative \( y \) directions, the contributions from the gravitational-potential-energy term cancel by symmetry, so Bernoulli’s equation reduces to

\[ p_0 + \frac{1}{2} \rho v_0^2 = p_1 + \frac{1}{2} \rho v_1^2 \]

Expressing \( p_1 \) in terms of \( p_0 \) gives

\[ p_1 = p_0 + \frac{1}{2} \rho (v_0^2 - v_1^2) \]

Finally, we can express the velocity \( v_1 \) in terms of \( v_0 \) using the continuity equation (Equation 15.17):

\[ v_0 A_0 = v_1 A_1 \Rightarrow v_1 = v_0 \frac{d_0^2}{d_1^2} \]

Inserting this into the expression for \( p_0 \) gives

\[ p_1 = p_0 + \frac{1}{2} \rho \left( v_0^2 - \frac{d_0^4}{d_1^4} \right) \]

(b) Inserting the given values gives

\[ p_1 = 50 \text{ kPa} + \frac{1}{2} \left( \frac{1000 \text{ kg/m}^3 \cdot (10.0 \text{ m/s})^2}{20.0 \text{ cm}^2} \right) \left[ 1 - \frac{16.8 \text{ cm}}{20.0 \text{ cm}} \right] = 50 \text{ kPa} + 25.1 \text{ kPa} = 75 \text{ kPa} \]

**Assess:** We find that the pressure increases where the velocity has decreased, which is in accordance with Bernoulli’s equation.

15.59. **Solve:** Treat the sap as incompressible and assume sap is almost entirely made of water.

**Solve:** To replace the water lost to the atmosphere, the vessels must supply 110 g/h of sap. The sap provided by each vessel is \( \rho \pi (d/2)^2 \), so for \( N = 2000 \) sap vessels, the flow rate in each vessel is

\[ N \rho \pi \frac{(d/2)^2}{2} = 0.110 \text{ kg/h} \Rightarrow v = \frac{4(0.110 \text{ kg/h})(1 \text{ h}/3600 \text{ s})}{\pi(1040 \text{ kg/m}^3)(2000)(100 \times 10^{-6} \text{ m})^2} = 187 \text{ mm/s} \]

15.60. **Model:** Treat the water as an ideal fluid obeying Bernoulli’s equation. A streamline begins in the bigger size pipe and ends at the exit of the narrower pipe.

**Visualize:** Please see Figure P15.60. Let point 1 be beneath the standing column and point 2 be where the water exits the pipe.

**Solve:** (a) The pressure of the water as it exits into the air is \( p_2 = P_{\text{atmos}} \).

(b) Bernoulli’s equation, Equation 15.26, relates the pressure, water speed, and heights at points 1 and 2:

\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \Rightarrow p_1 - p_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) + \rho (y_2 - y_1) \]

From the continuity equation,

\[ v_1 A_1 = v_2 A_2 = (4 \text{ m/s})(5 \times 10^{-4} \text{ m}^2) \Rightarrow v_1 (10 \times 10^{-4} \text{ m}^2) = 20 \times 10^{-4} \text{ m}^3/\text{s} \Rightarrow v_1 = 2.0 \text{ m/s} \]

Substituting into Bernoulli’s equation,

\[ p_1 - p_2 = p_1 - P_{\text{atmos}} = \frac{1}{2} \left( 1000 \text{ kg/m}^3 \right) (4.0 \text{ m/s})^2 - (2.0 \text{ m/s})^2 \Rightarrow 6000 \text{ Pa} + 39,200 \text{ Pa} = 45 \text{ kPa} \]
But \( p_1 - p_2 = \rho gh \), where \( h \) is the height of the standing water column. Thus
\[
h = \frac{45 \times 10^3 \text{ Pa}}{(1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 4.6 \text{ m}
\]

15.61. **Model:** Treat the water as an ideal fluid obeying Bernoulli’s equation. A streamline begins at the faucet and continues down the stream.

**Visualize:**

![Diagram of water flow](image)

The pressure at point 1 is \( p_1 \) and the pressure at point 2 is \( p_2 \). Both \( p_1 \) and \( p_2 \) are atmospheric pressure. The velocity and the area at point 1 are \( v_1 \) and \( A_1 \) and they are \( v_2 \) and \( A_2 \) at point 2. Let \( d \) be the distance of point 2 below point 1.

**Solve:** The flow rate is
\[
Q = v_1 A_1 = \frac{2.0 \times 1000 \times 10^{-6} \text{ m}^3}{10 \text{ s}} = 2.0 \times 10^{-4} \text{ m}^3/\text{s} \quad \Rightarrow \quad v_1 = \frac{2.0 \times 10^{-4} \text{ m}^3/\text{s}}{\pi (0.0080 \text{ m})^2} = 1.0 \text{ m/s}
\]

Bernoulli’s equation at points 1 and 2 is
\[
p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \quad \Rightarrow \quad \rho g d = \frac{1}{2} \rho (v_2^2 - v_1^2)
\]

From the continuity equation,
\[
v_1 A_1 = v_2 A_2 \quad \Rightarrow \quad (1.0 \text{ m/s})\pi (8.0 \times 10^{-3} \text{ m})^2 = v_2 \pi (5.0 \times 10^{-3} \text{ m})^2 \quad \Rightarrow \quad v_2 = 2.56 \text{ m/s}
\]

Going back to Bernoulli’s equation, we have
\[
\rho g d = \frac{1}{2}(2.56 \text{ m/s})^2 - (1.0 \text{ m/s})^2 \quad \Rightarrow \quad d = 0.283 \text{ m} = 28 \text{ cm}
\]

15.62. **Model:** Treat the air as an ideal fluid obeying Bernoulli’s equation.

**Solve:**

(a) The pressure above the roof is lower due to the higher velocity of the air.

(b) Bernoulli’s equation, with \( y_{\text{inside}} = y_{\text{outside}} \) is
\[
p_{\text{inside}} = p_{\text{outside}} + \frac{1}{2} \rho \text{air} v^2 \quad \Rightarrow \quad \Delta p = \frac{1}{2} \rho \text{air} v^2 = \frac{1}{2} (1.28 \text{ kg/m}^3) \left( \frac{130 \times 1000 \text{ m}}{3600 \text{ s}} \right)^2 = 835 \text{ Pa}
\]

The pressure difference is 0.83 kPa

(c) The force on the roof is \( (\Delta p)A = (835 \text{ Pa})(6.0 \text{ m} \times 15.0 \text{ m}) = 7.5 \times 10^4 \text{ N} \). The roof will blow up, because pressure inside the house is greater than pressure on the top of the roof.

15.63. **Model:** The ideal fluid obeys Bernoulli’s equation.

**Visualize:** Please refer to Figure P15.63. There is a streamline connecting point 1 in the wider pipe on the left with point 2 in the narrower pipe on the right. The air speeds at points 1 and 2 are \( v_1 \) and \( v_2 \) and the cross-sectional area of the pipes at these points are \( A_1 \) and \( A_2 \). Points 1 and 2 are at the same height, so \( y_1 = y_2 \).

**Solve:** The volume flow rate is
\[
Q = A_1 v_1 = A_2 v_2 = 1200 \times 10^{-6} \text{ m}^3/\text{s} \quad \text{Thus}
\]
\[
\rho_1 = \frac{1200 \times 10^{-6} \text{ m}^3/\text{s}}{\pi (0.0200 \text{ m})^2} = 95.49 \text{ m/s} \\
\rho_2 = \frac{1200 \times 10^{-6} \text{ m}^3/\text{s}}{\pi (0.010 \text{ m})^2} = 3.82 \text{ m/s}
\]

Now we can use Bernoulli’s equation to connect points 1 and 2:

\[
p_1 + \frac{1}{2} \rho_1 v_1^2 + \rho_1 g y_1 = p_2 + \frac{1}{2} \rho_2 v_2^2 + \rho_2 g y_2
\]

\[
p_1 - p_2 = \frac{1}{2} \rho_1 (v_1^2 - v_2^2) + \rho g (y_2 - y_1) = \frac{1}{2} (1.28 \text{ kg/m}^3) [(95.49 \text{ m/s})^2 - (3.82 \text{ m/s})^2] + 0 \text{ Pa} = 5.83 \text{ kPa}
\]

Because the pressure above the mercury surface in the right tube is \( p_2 \) and in the left tube is \( p_1 \), the difference in the pressures \( p_1 \) and \( p_2 \) is \( \rho_\text{Hg} gh \). That is,

\[
p_1 - p_2 = \rho_\text{Hg} gh \]

Thus \( p_1 = 13,328 \text{ Pa} \), \( p_2 = 13,328 \text{ Pa} \), and \( y_1 = y_2 \).

**15.64. Model:** The ideal fluid (i.e., air) obeys Bernoulli’s equation.

**Visualize:** Please refer to Figure P15.64. There is a streamline connecting points 1 and 2. The air speeds at points 1 and 2 are \( v_1 \) and \( v_2 \), and the cross-sectional areas of the pipes at these points are \( A_1 \) and \( A_2 \). Points 1 and 2 are at the same height, so \( y_1 = y_2 \).

**Solve:** (a) The height of the mercury is 10 cm. So, the pressure at point 2 is larger than at point 1 by

\[
\rho_\text{Hg} g (0.10 \text{ m}) = (13,600 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(0.10 \text{ m}) = 13,328 \text{ Pa}
\]

Using Bernoulli’s equation,

\[
p_1 + \frac{1}{2} \rho_\text{air} v_1^2 + \rho_\text{air} g y_1 = p_2 + \frac{1}{2} \rho_\text{air} v_2^2 + \rho_\text{air} g y_2
\]

\[
v_1^2 - v_2^2 = \frac{2(p_2 - p_1)}{\rho_\text{air}} = \frac{2(13,328 \text{ Pa})}{(1.28 \text{ kg/m}^3)} = 20,825 \text{ m}^2/\text{s}^2
\]

From the continuity equation, we can obtain another equation connecting \( v_1 \) and \( v_2 \):

\[
A_1 v_1 = A_2 v_2 \Rightarrow v_1 = \frac{A_2}{A_1} v_2 = \frac{\pi (0.0050 \text{ m})^2}{\pi (0.0010 \text{ m})^2} v_2 = 25 v_2
\]

Substituting \( v_1 = 25 v_2 \) in the Bernoulli equation, we get

\[
(25 v_2)^2 - v_2^2 = 20,825 \text{ m}^2/\text{s}^2 \Rightarrow v_2 = 5.78 \text{ m/s}
\]

Thus \( v_2 = 5.8 \text{ m/s} \) and \( v_1 = 25 v_2 = 1.4 \times 10^2 \text{ m/s} \).

**15.65. Model:** Treat the water as an ideal fluid that obeys Bernoulli’s equation. There is a streamline from the top of the water to the hole.

**Visualize:** The top of the water (at \( y_1 = h \)) and the hole (at \( y_2 = y \)) are at atmospheric pressure. The speed of the water at the top is zero because the tank is kept filled.
Solve: (a) Bernoulli’s equation connecting the two points gives

\[ 0 + \rho gh = \frac{1}{2} \rho v^2 + \rho gy \quad \Rightarrow \quad v = \sqrt{2g(h-y)} \]

(b) For a particle shot horizontally from a height \( y \) with speed \( v \), the range can be found using kinematic equations. For the \( y \)-motion, using \( t_0 = 0 \) s, we have

\[ y_1 = y_0 + v_{0y}(t_1-t_0) + \frac{1}{2} a_y(t_1-t_0)^2 \quad \Rightarrow \quad y = y_0 + (0 \text{ m/s})t_1 + \frac{1}{2}(-g)t_1^2 \quad \Rightarrow \quad t_1 = \sqrt{2y/g} \]

For the \( x \)-motion,

\[ x_1 = x_0 + v_{0x}(t_1-t_0) + \frac{1}{2} a_x(t_1-t_0)^2 \quad \Rightarrow \quad x = x_0 + vt_1 + 0 \text{ m} \quad \Rightarrow \quad x = v\sqrt{2y/g} \]

(c) Combining the results of (a) and (b), we obtain

\[ x = \sqrt{2g(h-y)} \sqrt{2y/g} = \sqrt{4y(h-y)} \]

To find the maximum range relative to the vertical height,

\[ \frac{dx}{dy} = 0 \quad \Rightarrow \quad \frac{1}{\sqrt{4y(h-y)}} [4(h-y) - 4y] = 0 \quad \Rightarrow \quad y = \frac{h}{2} \]

With \( y = \frac{1}{2}h \), the maximum range is

\[ x_{\text{max}} = \sqrt{4\left(\frac{h}{2}\right)\left(h - \frac{h}{2}\right)} = h \]

15.66. Model: Treat the water as an ideal fluid that obeys Bernoulli’s equation. There is a streamline from the top of the water to the hole.

Visualize: The top of the water and the hole are at atmospheric pressure. The speed of the water at the top is not zero because the tank is emptying.
15.67. **Model:** The aquarium creates tensile stress.

**Solve:** Weight of the aquarium is

\[ F_g = mg = \rho_{\text{water}} V g = (1000 \text{ kg/m}^3)(10 \text{ m}^3)(9.8 \text{ m/s}^2) = 9.8 \times 10^4 \text{ N} \]

where we have used the conversion 1 L = 10^{-3} m^3. The weight supported by each wood post is \( \frac{1}{4}(9.8 \times 10^4 \text{ N}) = 2.45 \times 10^4 \text{ N} \). The cross-sectional area of each post is \( A = (0.040 \text{ m}^2) = 1.6 \times 10^{-3} \text{ m}^2 \). Young’s modulus for the wood is

\[ Y = 1 \times 10^{10} \text{ N/m}^2 = \frac{F/A}{\Delta L/L} = \frac{FL}{AY} \]

\[ \Delta L = \frac{FL}{AY} = \frac{(2.45 \times 10^4 \text{ N})(0.80 \text{ m})}{(1.6 \times 10^{-3} \text{ m}^2)(1 \times 10^{10} \text{ N/m}^2)} = 1.23 \times 10^{-3} \text{ m} = 1 \text{ mm} \]

**Assess:** A compression of 1 mm due to a weight of 2.45 \times 10^4 \text{ N} is reasonable.

15.68. **Model:** The weight of the person creates tensile stress in the disk cartilage.

**Solve:** The cross-sectional area of each disk of cartilage is \( \pi r^2 \). Inserting this into Equation 15.34 gives

\[ F/A = Y \frac{\Delta L}{L} = \frac{FL}{AY} = \frac{mgL}{YA} = \frac{(33 \text{ kg})(9.8 \text{ m/s}^2)(0.0050 \text{ m})}{(1 \times 10^6 \text{ N/m}^2)(\pi (0.020 \text{ m})^2)} = 1.3 \text{ mm} \]

**Assess:** The cartilage experiences a significant compressive axial strain (~25%).

15.69. **Model:** Pressure applies a volume stress to water in the cylinder.

**Solve:** The volume strain of water due to the pressure applied is

\[ \frac{\Delta V}{V} = -\frac{p}{B} = -\frac{2 \times 10^6 \text{ Pa}}{0.2 \times 10^{10} \text{ Pa}} = -1 \times 10^{-3} \]

\[ \Delta V = V' - V = -(1 \times 10^{-3})(1.30 \text{ m}^3) = -1.30 \times 10^{-3} \text{ m}^3 \approx -1 \text{ L} \]

As the safety plug on the top of the cylinder bursts, the water comes back to atmospheric pressure. To the precision of the data (one significant figure), one liter of water comes out.

15.70. **Model:** Air is an ideal gas and obeys Boyle’s law.

**Visualize:** Please refer to Figure CP15.70. The quantity \( h \) is the length of the air column when the mercury fills the cylinder to the top. \( A \) is the cross-sectional area of the cylinder.

**Solve:** For the column of air, Boyle’s law is \( p_0 V_0 = p_1 V_1 \), where \( p_0 \) and \( V_0 \) are the pressure and volume before any mercury is poured, and \( p_1 \) and \( V_1 \) are the pressure and volume when mercury fills the cylinder above the air. Using \( p_1 = p_0 + \rho_{\text{Hg}} g (1.0 \text{ m} - h) \), Boyle’s law becomes

\[ p_0 V_0 = [p_0 + \rho_{\text{Hg}} g (1.0 \text{ m} - h)] V_1 \]

\[ p_0 (1.0 \text{ m} - h) = \rho_{\text{Hg}} g (1.0 \text{ m} - h) h \]

\[ h = \frac{p_0}{\rho_{\text{Hg}} g} (1.013 \times 10^5 \text{ Pa}) = 0.76 \text{ m} = 76 \text{ cm} \]

\[ h = \frac{p_0}{\rho_{\text{Hg}} g} (13,600 \text{ kg/m}^3)(9.8 \text{ m/s}^2) \]
15.71. Model: The buoyant force on the cone is given by Archimedes’ principle. 

Visualize: 

![Diagram of a cone with dimensions labeled]

Solve: It may seem like we need the formula for the volume of a cone. You can use that formula if you know it, but it isn’t essential. The volume is clearly the area of the base multiplied by the height multiplied by some constant. That is, the cone shown above has \( V = aAl \) where \( a \) is some constant. But the radius of the base is \( r = ltan \alpha \), where \( \alpha \) is the angle of the apex of the cone, and \( A = \pi r^2 \), making \( A \) proportional to \( l^2 \). Thus the volume of a cone of height \( l \) is \( V = cl^3 \), where \( c \) is a constant. Because the cone is floating in static equilibrium, we must have \( F_B = F_G \).

The cone’s density is \( \rho_0 \), so the gravitational force on it is \( F_G = \rho_0 Vg = \rho_0 cl^3 g \). The buoyant force is the gravitational force on the displaced fluid. The volume of displaced fluid is the full volume of the cone minus the volume of the cone of height \( h \) above the water, or \( V_{disp} = cl^3 - ch^3 \). Thus \( F_B = \rho_1 V_{disp}g = \rho_1 c(l^3 - h^3)g \), and the equilibrium condition is

\[
F_B = F_G \Rightarrow \rho_1 c(l^3 - h^3)g = \rho_0 cl^3 g \Rightarrow \rho_1 \left( 1 - \frac{h^3}{l^3} \right) = \rho_0 \Rightarrow h = \left( 1 - \frac{\rho_0}{\rho_1} \right)^{1/3}
\]

15.72. Model: The grinding wheel is a uniform disk. We will use the model of kinetic friction and hydrostatics. 

Visualize: Please refer to Figure CP15.72.

Solve: This is a three-part problem. First find the desired angular acceleration, then use that to find the force applied by each brake pad, then finally the needed oil pressure. The angular acceleration required to stop the wheel is found using rotational kinematics.

\[
\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 \text{ rad/s} - 900 \times 2\pi \times (1/60) \text{ rad/s}}{5.0 \text{ s}} = -18.85 \text{ rad/s}^2
\]

Each brake pad applies a frictional force \( f_k = \mu_k n \) to the wheel. The normal force is equal to the force applied by the piston by Newton’s third law. Rotational dynamics can be used to find the magnitude of the force. \( f_k \) is applied 12 cm from the rotation axis on both sides of the disk.

\[
\tau_{net} = I\alpha
\]

\[
2 \times 2 f_k (0.12 \text{ m}) = \left( \frac{1}{2} MR^2 \right) \alpha
\]

\[
n = \frac{MR^2 \alpha}{2 \times 4 \mu_k (0.12 \text{ m})} = \frac{(15 \text{ kg})(0.13 \text{ m})^2(-18.85 \text{ rad/s}^2)}{2 \times 4(0.60)(0.12 \text{ m})} = 16.6 \text{ N}
\]

The oil pressure required to generate this much force at each brake pad is

\[
p = \frac{F}{A} = \frac{16.6 \text{ N}}{\pi(0.010 \text{ m})^2} = 53 \text{ kPa}
\]

relative to atmospheric pressure. The absolute pressure is 53 kPa + 101.3 kPa = 154.3 kPa = 1.5 \times 10^5 \text{ Pa} to two significant figures.

Assess: The required oil pressure is about half an atmosphere, which is quite reasonable.
15.73. Model: The buoyant force on the cylinder is given by Archimedes’ principle.
Visualize:

\[ F_A = \rho_l A h g \]

\[ F_B = \rho_0 A h g \]

(a) Now the volume of the displaced liquid is \( A(h - y). \) Applying Newton’s second law in the \( y \)-direction gives

\[ \sum F_y = -F_G + F_B = -\rho_0 A h g + \rho_l A(h - y) g \]

Using \( \rho_l A h g = \rho_0 A h g \) from part (a), we find

\[ (F_{net})_y = -\rho_l A g y \]

(c) The result in part (b) is \( F = -ky, \) where \( k = \rho_l A g. \) This is Hooke’s law.
(d) Since the cylinder’s equation of motion is determined by Hooke’s law, the angular frequency for the resulting simple harmonic motion is \( \omega = \sqrt{k/m}, \) and the period is

\[ T = \frac{2\pi}{\omega} = T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{\rho_l A g}} = 2\pi \sqrt{\frac{h}{g}} \]

where we have used the expression for \( h \) from part (a).
(e) The oscillation period for the 100-m-tall iceberg \( (\rho_{ice} = 917 \text{ kg/m}^3) \) in sea water is

\[ T = 2\pi \sqrt{\frac{h}{g}} = 2\pi \sqrt{\frac{\rho_{ice}}{\rho_l g}} = 2\pi \sqrt{\frac{(917 \text{ kg/m}^3)(100 \text{ m})}{(1030 \text{ kg/m}^3)(9.80 \text{ m/s}^2)}} = 18.9 \text{ s} \]

15.74. Model: A streamline connects every point on the surface of the liquid to a point in the drain. The drain diameter is much smaller than the tank diameter \( (r \ll R). \)
Visualize:
Solve: The pressures at the surface and drain (points 1 and 2) are equal to one atmosphere. When the liquid is a depth \( y \), Bernoulli’s principle connecting points 1 and 2 is

\[
p_1 + \frac{1}{2} \rho v_1^2 + \rho g y = p_2 + \frac{1}{2} \rho v_2^2 + \rho g(0)
\]

\[
v_2^2 = v_1^2 + 2gy
\]

The flow rate through the drain is the same as through a horizontal layer in the tank:

\[
Q = v_1 A_1 = v_2 A_2 \quad \Rightarrow \quad v_1 = v_2 \frac{A_2}{A_1}
\]

Thus the velocity of the liquid through the drain is

\[
v_2^2 = \left( v_2 \frac{A_2}{A_1} \right)^2 + 2gy \quad \Rightarrow \quad v_2 = \sqrt{\frac{2gy}{1 - (r/R)^4}}
\]

Since \( r \ll R \), \( v_2 \approx \sqrt{2gy} \), and the flow rate through the drain is

\[
Q \approx \pi r^2 \sqrt{2gy}
\]

The flow rate gives the volume of liquid flowing out per unit time. The inverse gives the time needed for a unit volume of liquid to flow out. For the volume of liquid to decrease by \( dV \) requires a time

\[
dt = \frac{dV}{Q} = \frac{-R^2 dy}{\pi r^2 \sqrt{2gy}} = \frac{-R^2}{r^2 \sqrt{2g}} \frac{dy}{\sqrt{y}}
\]

Note that \( dV < 0 \) implies the volume of liquid is decreasing. Integrating both sides from the initial condition \((t = 0, y = d)\) to the final condition \((t, y = 0)\) yields

\[
t = -\frac{R^2}{r^2 \sqrt{2g}} \int_0^d y^{-1/2} dy = -\frac{2R^2}{r^2 \sqrt{2g}} y^{1/2} \bigg|_0^d = \frac{R^2}{r^2} \sqrt{\frac{2d}{g}}
\]

Assess: The time for the tank to drain depends on the ratio of the cross-sectional areas of the tank to drain, which makes sense, as well as on the strength of the acceleration due to gravity. Note that, if \( g \) were larger (say we were on Jupiter), the time to drain would be shorter, whereas if the tank were taller (larger \( d \)), the time would be longer.