Physics 41: Waves, Optics, Thermo

"If we have everlasting life, what about entropy?"
Particles & Waves

Particle

• Localized in Space: *LOCAL*
• Have Mass & Momentum
• *No Superposition*: Two particles cannot occupy the same space at the same time!
• Particles have energy.
• Particles can have any energy.

Wave

• Spread Out in Space: *NONLOCAL*
• *Superposition*: Waves add in space and show interference.
• Do not have mass or Momentum
• Waves transmit energy.
• Bound waves have discrete energy states – they are quantized.
Light Waves

Direction of wave travel

Frequency (Hz)

Wavelength (m)

Radio waves

Infrared

Ultraviolet

X-rays

Gamma rays

AM

FM

Microwaves

$10^4$

$10^8$

$10^{12}$

$10^{16}$

$10^{20}$

$10^{24}$

$4 \times 10^{14}$

$7.9 \times 10^{14}$

Visible light

Red

Violet
Sound is a *What you Hear*

The ear converts sound energy to mechanical energy to a nerve impulse which is transmitted to the brain.

The Pressure Wave sets the Ear Drum into Vibration.
Brain Waves

electroencephalogram
Figure 1. Depiction of deep water waves (water deeper than 0.5 WL).
Shock Waves!
\[ \frac{v_s}{v} = \text{Mach } \# \]
Mach 1
Electron Waves

Probability Waves in an Ocean of Uncertainty

A wave packet in a square well (an electron in a box) changing with time.
Superposition: Waves ADD in Space

Interference: Waves interfere with other waves and with themselves without any permanent damage!
Contour Map of Interference Pattern

(a) Two identical sources

\[ \Delta r = \lambda \]
\[ \Delta r = 0 \]
\[ \Delta r = \lambda \]

Intesity at \( x = 4 \) m

Copyright © 2004 Pearson Education, Inc., publishing as Addison Wesley
Importance of Simple Harmonic Oscillators

- Simple harmonic oscillators are good models of a wide variety of physical phenomena
- Molecular example
  - If the atoms in the molecule do not move too far, the forces between them can be modeled as if there were springs between the atoms
  - The potential energy acts similar to that of the SHM oscillator
Natural Frequency & Resonance

All objects have a natural frequency of vibration or oscillation. Bells, tuning forks, bridges, swings and atoms all have a natural frequency that is related to their size, shape and composition. A system being driven at its natural frequency will resonate and produce maximum amplitude and energy.
Some systems have only one natural frequency: springs, pendulums, tuning forks, satellites orbits.
Some Systems have more than one frequency they oscillate with: *Harmonics.*
Natural Frequency & Resonance

When the driving vibration matches the natural frequency of an object, it produces a *Sympathetic Vibration* - it Resonates!
Sound Waves: Mechanical Vibrations

The Ear: An Acoustic Tuner

Cilia: Acoustic Tuning Forks
Eyes: Optical Tuner
Optical Antennae: Rods & Cones

Rods: Intensity
Cones: Color
Light Waves: EM Vibrations

\[ E = E_{\text{max}} \cos (kx - \omega t) \]
\[ B = B_{\text{max}} \cos (kx - \omega t) \]

\[ c = \frac{E}{B} = \frac{1}{\sqrt{\ldots 0^\infty 0}} \]

Speed of Light in a vacuum:
186,000 miles per second
300,000 kilometers per second
3 \times 10^8 \text{ m/s}

\[ \nu = \frac{c}{n} \]
Coupled Oscillators

Molecules, atoms and particles are modeled as coupled oscillators. Waves Transmit Energy through coupled oscillators. Forces are transmitted between the oscillators like springs. Coupled oscillators make the medium.
Atoms are EM Tuning Forks

They are ‘tuned’ to particular frequencies of light energy.
Strings & Atoms are Quantized

The possible frequency and energy states of an electron in an atomic orbit or of a wave on a string are quantized.

\[ f = n \frac{v}{2l} \]

\[ E_n = n hf, \quad n = 0, 1, 2, 3, \ldots \]

\[ h = 6.626 \times 10^{-34} \text{ Js} \]
String Idea: Particles are string vibrations in resonance
DARK ENERGY
The Vibration of Nothing

The Quantum Oscillator

- Classically, at $x=0$, the energy is zero.
- Therefore, the momentum is zero too.
- But this would violate the Heisenberg uncertainty principle!
- Therefore, the quantum oscillator cannot be completely at rest. Allowed energies, $E_n$, are:

$$E_n = (n + \frac{1}{2})hf \quad (n=0,1,2,...)$$

$f$ = oscillation frequency

The minimum possible energy for the quantum oscillator is $E_0 = \frac{1}{2}hf$.

This is called the zero point energy.
Chapter 14
Simple Harmonic Motion
Our Task: Springs and Pendulums Obey Hooke’s Law and exhibit Oscillatory Motion. Find the equations of motion: Position vs Time: Sinusoidal
We want to describe the motion of oscillating systems and find the natural frequency of objects and systems.

If you know the natural frequency of an object, the frequency it can oscillate or vibrate with, then you know everything about it, most importantly it’s ENERGY and the MUSIC it makes!

Use Hooke’s Law!
Review: Hooke’s Law

*Ut tensio, sic vis* - as the extension, so is the force

Hooke’s Law describes the *elastic* response to an applied force. Elasticity is the property of an object or material which causes it to be restored to its original shape after distortion.

An elastic system displaced from equilibrium oscillates in a simple way about its equilibrium position with *

*Simple Harmonic Motion.*
Hooke’s Law

It takes twice as much force to stretch a spring twice as far.
The linear dependence of displacement upon stretching force:

\[ F_{\text{applied}} = kx \]
Hooke’s Law

Stress is directly proportional to strain.

\[ F_{\text{applied}}(\text{stress}) = kx(\text{strain}) \]
Hooke’s Law: \( F = -kx \)
Hooke’s Law: $F = -kx$
Hooke’s Law: \[ F = -kx \]
Hooke’s Law: \( F = -kx \)
Hooke’s Law: \[ F = -kx \]
Review: Energy in a Mass-Spring

\[ K = \frac{1}{2} mv^2 \]
\[ U = \frac{1}{2} kx^2 \]
\[ E = \frac{1}{2} kA^2 \]
Review: Circular Motion

\[ x(t) = A \cos \omega t \]

\[ v(t) = -A\omega \sin \omega t \]

\[ a(t) = -A\omega^2 \cos \omega t \]

\[ \theta(t) = \omega t, \quad v_t = R\omega = \frac{2\pi R}{T}, \quad a_t = R\alpha, \quad a_c = \frac{v^2}{R} = \omega^2 R \]
- Figure (a) shows a “shadow movie” of a ball made by projecting a light past the ball and onto a screen.
- As the ball moves in uniform circular motion, the shadow moves with simple harmonic motion.
- The block on a spring in figure (b) moves with the same motion.
Review Terms:

**Amplitude:** \[ [A] = m \]

**Period:** \[ T = \text{time / cycle}, \ [T] = \text{sec} \]

**Frequency:** \[ f = \frac{1}{T} \ (\# \text{cycles / sec}), \ [f] = \text{Hz} \]

**Angular Frequency:** \[ \omega = 2\pi f = \frac{2\pi}{T}, \ [\omega] = \text{rad / s} \]
We want to describe the motion of oscillating systems and find the natural frequency of objects and systems.

If you know the natural frequency of an object, the frequency it can oscillate or vibrate with, then you know everything about it, most importantly it’s ENERGY and the MUSIC it makes!

Finding the energy states of systems is pretty much the goal of most of physics!
Some systems have only ONE natural frequency and energy state: springs, pendulums, tuning forks, satellites orbits.
System 1: Ideal Mass-Spring System.

Is there any difference in the resulting motion between a horizontal and vertical system?
In the absence of any retarding forces the motion for a mass hanging from a spring is the same as for horizontal SHM, but the equilibrium position is affected. The role of gravity is to determine where the equilibrium position is but it doesn’t affect the SHM around the equilibrium position.
Position Equation for SHM

$$x(t) = A \cos (\omega t + \phi_0)$$

- $A$ is the amplitude of the motion
- The phase is: $(\omega t + \phi_0)$
- $\omega$ is called the angular frequency (rad/s)
- $\phi_0$ is the phase constant or the initial phase angle at $t=0$
- $A$, $\omega$, $\phi$ are all constants
The Phase Constant

- What if an object in SHM is not initially at rest at $x = A$ when $t = 0$?
- Then we may still use the cosine function, but with a phase constant measured in radians.
- In this case, the two primary kinematic equations of SHM are:

$$x(t) = A \cos(\omega t + \phi_0)$$
$$v_x(t) = -\omega A \sin(\omega t + \phi_0) = -v_{\text{max}} \sin(\omega t + \phi_0)$$
Oscillations described by the phase constants $\phi_0 = \pi/3 \text{ rad}$, $-\pi/3 \text{ rad}$, and $\pi \text{ rad}$.

The starting point of the oscillation is shown on the circle and on the graph.

The graphs each have the same amplitude and period. They are shifted relative to the $\phi_0 = 0 \text{ rad}$ graphs of Figure 14.5 because they have different initial conditions.
The quantity ($\omega t + \phi_0$) is called the **phase** of the wave, denoted $\phi$.

The **phase constant** $\phi_0$ is the initial phase angle at $t=0$.

The **phase difference** $\Delta \phi$ between two points on a wave depends on only the ratio of their separation $\Delta x$ to the wavelength $\lambda$. 

![Graph showing phase values over time]
The figure shows four oscillators at $t = 0$. For which is the phase constant $\phi_0 = -\pi / 4$?
The figure shows four oscillators at $t = 0$. For which is the phase constant $\phi_0 = -\pi / 4$?

Initial conditions:
$x = 0.71A$
$v_x > 0$
This is the position graph of a mass oscillating on a horizontal spring. What is the phase constant $\phi_0$?

A. $-\pi/2$ rad.
B. 0 rad.
C. $\pi/2$ rad.
D. $\pi$ rad.
E. None of these.
This is the position graph of a mass oscillating on a horizontal spring. What is the phase constant $\phi_0$?

A. $-\pi/2$ rad.
B. 0 rad.
C. $\pi/2$ rad.
D. $\pi$ rad.
E. None of these.

Initial conditions:
$x = 0$
$v_x > 0$
This is the position graph of a mass oscillating on a horizontal spring. What is the phase constant $\phi_0$?

A. $-\pi/2$ rad.
B. 0 rad.
C. $\pi/2$ rad.
D. $\pi$ rad.
E. None of these.
This is the position graph of a mass oscillating on a horizontal spring. What is the phase constant $\phi_0$?

A. $-\pi/2$ rad.
B. 0 rad.
C. $\pi/2$ rad.
D. $\pi$ rad.
E. None of these.

Initial conditions:
$x = -A$
$v_x = 0$
Simple Harmonic Motion Equations

\[ x(t) = A \cos(\omega t + \phi_0) \]

\[ v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi_0) \]

\[ a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi_0) \]

Notice: \[ a = -\omega^2 x \]

In SHM, the acceleration is proportional to the negative of the displacement.
Consider a mass $m$ oscillating on a horizontal spring with no friction.

The spring force is:

$$(F_{sp})_x = -k \Delta x$$

Since the spring force is the net force, Newton’s second law gives:

$$(F_{net})_x = (F_{sp})_x = -kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

Since $a_x = -\omega^2 x$, the angular frequency must be $\omega = \sqrt{\frac{k}{m}}$. 
A mass oscillates on a horizontal spring with period $T = 2.0\, \text{s}$. If the amplitude of the oscillation is doubled, the new period will be

A. 1.0 s.
B. 1.4 s.
C. 2.0 s.
D. 2.8 s.
E. 4.0 s.
A mass oscillates on a horizontal spring with period $T = 2.0 \text{ s}$. If the amplitude of the oscillation is doubled, the new period will be

A. 1.0 s.
B. 1.4 s.
C. 2.0 s.
D. 2.8 s.
E. 4.0 s.

Correct answer: C. 2.0 s.
A mass oscillates on a horizontal spring. It’s velocity is $v_x$ and the spring exerts force $F_x$. At the time indicated by the arrow,

A. $v_x$ is + and $F_x$ is +.
B. $v_x$ is + and $F_x$ is −.
C. $v_x$ is − and $F_x$ is 0.
D. $v_x$ is 0 and $F_x$ is +.
E. $v_x$ is 0 and $F_x$ is −.
A mass oscillates on a horizontal spring. It’s velocity is $v_x$ and the spring exerts force $F_x$. At the time indicated by the arrow,

A. $v_x$ is + and $F_x$ is +.
B. $v_x$ is + and $F_x$ is −.
C. $v_x$ is − and $F_x$ is 0.
D. $v_x$ is 0 and $F_x$ is +.
E. $v_x$ is 0 and $F_x$ is −.
A mass oscillates on a horizontal spring. It’s velocity is \( v_x \) and the spring exerts force \( F_x \). At the time indicated by the arrow,

A. \( v_x \) is + and \( F_x \) is +.
B. \( v_x \) is + and \( F_x \) is −.
C. \( v_x \) is − and \( F_x \) is 0.
D. \( v_x \) is 0 and \( F_x \) is +.
E. \( v_x \) is 0 and \( F_x \) is −.
A mass oscillates on a horizontal spring. It’s velocity is $v_x$ and the spring exerts force $F_x$. At the time indicated by the arrow,

A. $v_x$ is + and $F_x$ is +.
B. $v_x$ is + and $F_x$ is −.
C. $v_x$ is − and $F_x$ is 0.
D. $v_x$ is 0 and $F_x$ is +.
E. $v_x$ is 0 and $F_x$ is −.
A block of mass $m$ oscillates on a horizontal spring with period $T = 2.0 \text{ s}$. If a second identical block is glued to the top of the first block, the new period will be

A. 1.0 s.
B. 1.4 s.
C. 2.0 s.
D. 2.8 s.
E. 4.0 s.
A block of mass $m$ oscillates on a horizontal spring with period $T = 2.0 \text{ s}$. If a second identical block is glued to the top of the first block, the new period will be

A. 1.0 s.
B. 1.4 s.
C. 2.0 s.
D. 2.8 s. $T \propto \sqrt{m}$
E. 4.0 s.
Two identical blocks oscillate on different horizontal springs. Which spring has the larger spring constant?

A. The red spring.
B. The blue spring.
C. There’s not enough information to tell.
Two identical blocks oscillate on different horizontal springs. Which spring has the larger spring constant?

A. The red spring.
B. The blue spring.
C. There’s not enough information to tell.
A block oscillates on a vertical spring. When the block is at the lowest point of the oscillation, it’s acceleration $a_y$ is

A. Negative.
B. Zero.
C. Positive.
A block oscillates on a vertical spring. When the block is at the lowest point of the oscillation, its acceleration $a_y$ is

A. Negative.

B. Zero.

✓ C. Positive.
We want to describe the motion of oscillating systems and find the natural frequency of objects and systems.

If you know the natural frequency of an object, the frequency it can oscillate or vibrate with, then you know everything about it, most importantly it’s ENERGY and the MUSIC it makes!

Finding the energy states of systems is pretty much the goal of most of physics!
Some systems have only ONE natural frequency and energy state: springs, pendulums, tuning forks, satellites orbits.
Some Systems have more than one frequency they oscillate with: *Harmonics.*
YESTERDAY……

Mass-Spring Systems that obey Hooke’s Law exhibit Simple Harmonic Motion.
Position vs Time: Sinusoidal

\[ F = ma = -kx \]
\[ = m(-\omega^2 x) = -kx \]
\[ k = m\omega^2 \]
\[ \omega = \sqrt{\frac{k}{m}} \]

Simple Harmonic Motion
\[ \omega = \sqrt{\frac{k}{m}} \quad T = \frac{2\pi}{\omega} \]

Both the angular frequency and period depend only on how stiff the spring is and how much inertia there is.

Does the period depend on the displacement, \( x \)?

\[ T = 2\pi \sqrt{\frac{m}{k}} \]

Let’s do it again and then prove that there is only one energy state and find what it is!
Position Equation for SHM

\[ x(t) = A \cos(\omega t + \phi) \]

- \( A \) is the amplitude of the motion
- \( \omega \) is called the angular frequency
  - Units are rad/s
- \( \phi \) is the phase constant or the initial phase angle
- \( A, \omega, \phi \) are all constants
Motion Equations for ANY Simple Harmonic Motion

\[ x(t) = A \cos(\omega t + \phi) \]
\[ v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \]
\[ a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi) \]

Notice:

\[ a = -\omega^2 x \]
Energy in a Mass-Spring

\[ K = \frac{1}{2} mv^2 \quad U = \frac{1}{2} kx^2 \quad E = \frac{1}{2} kA^2 \]

\[ v = \pm \sqrt{\frac{k}{m} (A^2 - x^2)} = \pm \omega^2 \sqrt{A^2 - x^2} \]

<table>
<thead>
<tr>
<th>t</th>
<th>x</th>
<th>v</th>
<th>a</th>
<th>K</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>0</td>
<td>-\omega^2 A</td>
<td>0</td>
<td>\frac{1}{2} kA^2</td>
</tr>
<tr>
<td>T/4</td>
<td>0</td>
<td>-\omega A</td>
<td>0</td>
<td>\frac{1}{2} kA^2</td>
<td>0</td>
</tr>
<tr>
<td>T/2</td>
<td>-A</td>
<td>0</td>
<td>\omega^2 A</td>
<td>0</td>
<td>\frac{1}{2} kA^2</td>
</tr>
<tr>
<td>3T/4</td>
<td>0</td>
<td>\omega A</td>
<td>0</td>
<td>\frac{1}{2} kA^2</td>
<td>0</td>
</tr>
<tr>
<td>T</td>
<td>A</td>
<td>0</td>
<td>-\omega^2 A</td>
<td>0</td>
<td>\frac{1}{2} kA^2</td>
</tr>
</tbody>
</table>

© 2007 Thomson Higher Education
Energy of Mass-Spring

There is only ONE energy state possible for a Mass-spring system and the total mechanical energy is constant in the absence of nonconservative forces. Energy is continuously being transferred between potential energy stored in the spring and the kinetic energy of the block. Notice that the energy is proportional to the amplitude square. This is VERY important.

\[ E = \frac{1}{2} kA^2 \]
A block oscillates on a very long horizontal spring. The graph shows the block’s kinetic energy as a function of position. What is the spring constant?

A. 1 N/m.
B. 2 N/m.
C. 4 N/m.
D. 8 N/m.
E. I have no idea.
A block oscillates on a very long horizontal spring. The graph shows the block’s kinetic energy as a function of position. What is the spring constant?

A. 1 N/m  
B. 2 N/m.  
C. 4 N/m.  
D. 8 N/m.  
E. I have no idea.

\[ E = K_{\text{max}} = 8 \text{ J} = \frac{1}{2} kA^2 \]

\[ k = \frac{16 \text{ J}}{(2 \text{ m})^2} = 4 \text{ N/m} \]
Simple Pendulum

For small angles, simple pendulums exhibit SHM. Ignore mass of string and air resistance and treat the mass as a point particle.
Simple Pendulum

For small angles, simple pendulums exhibit SHM because for small angles \( \theta = s / L = x / L \)

Two ways to find \( \omega \).

**Rectilinear Coordinates:**

\[
x(t) = A \cos \left( \omega t + \phi \right)
\]

\[
a = -\omega^2 x
\]

**Angular Coordinates:**

\[
\theta(t) = \theta_0 \cos \left( \omega t + \phi \right)
\]

\[
\alpha = -\omega^2 \theta
\]

They are equivalent since \( a = \alpha r, x = \theta r \)
Simple Pendulum: Rectilinear

The displacement from equilibrium, \( x \) is the arclength \( s = L\theta \).

\[
\theta = s / L = x / L
\]

Accelerating & Restoring Force in the tangential direction, taking cw as positive initial displacement direction:

\[
\sum F = -mg \sin \theta \approx -mg\theta
\]

\[
F \approx -mg \frac{x}{L} = ma
\]

\[
a = \frac{g}{L} x = -\omega^2 x
\]

\[
\omega = \sqrt{\frac{g}{L}}
\]

\[
T = 2\pi \sqrt{\frac{L}{g}}
\]

\[
x(t) = A \cos (\omega t + \phi)
\]

\[
a = -\omega^2 x
\]
Simple Pendulum: Angular

θ is the displacement from equilibrium, x.

Accelerating & Restoring Torque in the angular direction: \( \tau = \vec{r} \times \vec{F} = I \alpha \)

\[ \tau = -Lmg \sin \theta \approx -Lmg \theta \]

\[ I = mL^2 \]

\[ mL^2 \alpha \approx -mgL \theta \]

\[ \alpha = \frac{g}{L} \theta = -\omega^2 \theta \]

\[ \theta(t) = \theta_0 \cos(\omega t + \phi) \]

\[ \alpha = -\omega^2 \theta \]

\[ \omega = \sqrt{\frac{g}{L}} \quad T = 2\pi \sqrt{\frac{L}{g}} \]
A ball on a massless, rigid rod oscillates as a simple pendulum with a period of 2.0 s. If the ball is replaced with another ball having twice the mass, the period will be

A. 1.0 s.
B. 1.4 s.
C. 2.0 s.
D. 2.8 s.
E. 4.0 s.
A ball on a massless, rigid rod oscillates as a simple pendulum with a period of 2.0 s. If the ball is replaced with another ball having twice the mass, the period will be

A. 1.0 s.
B. 1.4 s.
C. 2.0 s.  
D. 2.8 s.
E. 4.0 s.

C. 2.0 s.
On Planet $X$, a ball on a massless, rigid rod oscillates as a simple pendulum with a period of 2.0 s. If the pendulum is taken to the moon of Planet $X$, where the free-fall acceleration $g$ is half as big, the period will be

A. 1.0 s.
B. 1.4 s.
C. 2.0 s.
D. 2.8 s.
E. 4.0 s.
On Planet X, a ball on a massless, rigid rod oscillates as a simple pendulum with a period of 2.0 s. If the pendulum is taken to the moon of Planet X, where the free-fall acceleration $g$ is half as big, the period will be

A. 1.0 s.
B. 1.4 s.
C. 2.0 s.
D. 2.8 s.  $T \propto \frac{1}{\sqrt{g}}$
E. 4.0 s.