Waves on a String

Introduction

Standing (stationary) waves can be produced on a string tied at both ends. Both the wavelength and the frequency, \( f \), are related to the length of the string, \( L \), by

\[
f_n = \frac{v}{\lambda_n}, \quad \lambda_n = \frac{2L}{n}, \quad \text{where } n = 1, 2, 3, \ldots (1)
\]

and \( v \) is the velocity of the waves. If the tension is constant, then the velocity and the length are constant so each frequency is an integer multiple of the fundamental:

\[
f_n = nf_1, \quad (2)
\]

The velocity of a wave is given by the wave equation

\[
v = \lambda f. \quad (3)
\]

The wave velocity on a string is also related to the tension in the string, \( T \), and the mass per unit length, \( \mu \), by

\[
v = \sqrt{\frac{T}{\mu}} \quad (4)
\]

When one end of a string is held in place and the other end is vibrated, a wave travels down the stretched string and is reflected from the fixed end. Superposition of the incident and reflected waves produces standing waves of different wavelengths and frequencies. If the frequency of the vibration and the tension in the string are of appropriate values such that the length of the string is some integral number of half wavelengths, a standing wave will be produced:

\[
\lambda_n = \frac{2L}{n} \quad (5)
\]

In this experiment, standing waves are produced in a string using a vibrator of variable frequency. The vibrator is placed under one end of the string, which is tied to a rod, and the other end of the string passes over a pulley held at the end of the lab bench as shown in the sketch below. The tension in the string can be changed by changing the suspended mass, and the frequency can be adjusted to produce a standing wave with a wavelength equal to the length of the string. The velocity of the wave in the string can be calculated and compared using equations (3) and (4).
**Apparatus:** Computer, string vibrator, frequency generator, string, short rod, table clamp, pulley clamp, pulley, mass hanger and masses (50-1, 100-2, 200-1, 500-1), meter stick, meter stick calipers, electronic balance.

**Objective:** To investigate the physics of waves on a string under tension; to find the speed of waves using two different methods and compare them; to produce standing waves and determine if harmonic frequencies are integer multiples of the fundamental and how the velocity, frequency and wavelength of a wave on a string depends on the tension of the string.

**Procedure:** (Please include uncertainties in your measurements!)

A. Before tying any knots in the string, measure the total length and the mass of the string. Record the values in data table and do a spreadsheet calculation to calculate the mass per unit length, $\mu$, in kg/m.

B. Set up the system as shown by your instructor and the diagram above. The vibrating part of the string should be about 100-150 cm long. Tie the string to the rod and suspend a mass hanger with slotted masses from the other end so the total suspended mass is around 200 g. Weigh the hanger and all the masses to get the actual mass. Don’t believe the stamped value! Have excel calculate the tension in the string and the velocity of the wave on the string using equation 4.

C. Place the string vibrator under the string with the vibrator about 2-3 cm from the rod and adjust the height of the pulley and string so the string is perfectly horizontal and not quite touching the bottom of the slot in the string vibrator.

D. Measure the length of the vibrating string which is equal to one half the wavelength. You should be able to place the calipers on the nodes without changing the wavelength. Calculate the wavelength from that and enter it into the data sheet.

E. Using the speed you calculated in step B, calculate the fundamental frequency. Then dial in that using the frequency generator. Do you see the fundamental mode? The frequency should be fine-tuned with small adjustments up and down until a stable wave of **maximum amplitude** is produced. Record the frequency of the wave on the data sheet along with an uncertainty that you should estimate by the range of frequencies that seem to give you the desired mode.

F. Increase the frequency of the vibrator to find the next harmonic frequency. Can you predict what frequency it will be? Enter it into the data sheet. This is your 2nd harmonic. Measure the wavelength with a meter stick. Repeat to find the third harmonic.

G. How many harmonics can you find? Keep increasing the frequency and find the highest frequency that you can see. You should find data for at least the 5th harmonic.

H. Now increase the tension in the string by adding 100 g to the mass hanger and repeat procedures D through F. Repeat for a total of five different masses for 3 runs. Measure the mass to be as accurate as possible in each run!

I. Calculate the % difference between the methods of finding the velocity for each weight and then average the % difference. The % difference is different than the % error! It is the difference between two values divided by the average of the two values.
For each harmonic you need to measure the wavelength, frequency and wave speed from equation (3) and a % difference between the speed found from equation (4) for each tension.

**Analysis and Summary Questions**

1. For the tension that gave you the most harmonics, plot data for equation (2) – that is, harmonic frequency vs harmonic integer \((f_n \text{ vs } n)\). Do a linear fit. Discuss your results.

2. How did the two methods for finding the wave speed compare? Which method for finding the velocity do you trust the most? Which method has the most experimental uncertainty? How did your measurement of \(\lambda\) effect your ability to determine the velocity using equation (3)?

3. Did the harmonic frequencies and wavelengths follow the pattern predicted by equations (1) and (2)? Did the wavelengths you measured depend on \(L\) as predicted by equation (1)?

4. How does the wavelength, speed and frequency change as the tension increases? Will the fundamental frequency be increased or decreased when the tension is increased?