What is momentum?

*Inertia in Motion.*

\[ p = mv \]
From Newton’s 2\textsuperscript{nd} Law:

\[ \sum \vec{F} = m \vec{a} = m \frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} \]

\[ \sum \vec{F} = \frac{d\vec{p}}{dt} \]

The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle.
A 2000-kg truck traveling at a speed of 6.0 m/s makes a 90° turn in a time of 4.0 s and emerges from this turn with a speed of 4.0 m/s. What is the magnitude of the average resultant force on the truck during this turn?

a. 4.0 kN
b. 5.0 kN
c. 3.6 kN
d. 6.4 kN
e. 0.67 kN

\[ \Delta p = \overline{F} \Delta t \]
What is Impulse?

Impulse changes momentum.

\[ F = ma = m \frac{dv}{dt} \]

\[ \int F \, dt = \int m \, dv \]

\[ \vec{I} = \Delta \vec{p} = \int \vec{F} \, dt \]
- A collision is a short-duration interaction between two objects.
- The collision between a tennis ball and a racket is quick, but it is *not* instantaneous.
- Notice that the right side of the ball is flattened.
- It takes time to compress the ball, and more time for the ball to re-expand as it leaves the racket.
Atomic Model of a Collision

Object A approaches.

Before

Spring-like molecular bonds

A and B exert equal but opposite forces on each other.

During

The bonds compress.

After

Object A bounces back as the bonds re-expand.
A large force exerted for a small interval of time is called an **impulsive force**.

The figure shows a particle with initial velocity \( \vec{v}_i \).

The particle experiences an impulsive force of short duration \( \Delta t \).

The particle leaves with final velocity \( \vec{v}_f \).

\[
\vec{I} = \Delta \vec{p} = \int \vec{F} \, dt
\]
Impulse-Momentum Theorem

The impulse of the force $F$ acting on a particle equals the change in momentum of the particle.

$$\vec{I} = \Delta \vec{p} = \int \vec{F} \, dt$$

For a time-averaged force:

$$I = \Delta p = \bar{F} \Delta t$$
Same Impulse, different Force and Time.

\[ I = \Delta p = \bar{F} \Delta t \]
Same Impulse, different Force and Time.

\[ I = \Delta p = \bar{F} \Delta t \]

(a) \[ F \cdot t = \text{change in momentum} \]

(b) \[ F \cdot t = \text{change in momentum} \]
You awake in the night to find that your living room is on fire. Your one chance to save yourself is to throw something that will hit the back of your bedroom door and close it, giving you a few seconds to escape out the window. You happen to have both a sticky ball of clay and a super-bouncy Superball next to your bed, both the same size and same mass. You’ve only time to throw one. Which will it be? Your life depends on making the right choice!

A. Throw the Superball.
B. Throw the ball of clay.
C. It doesn’t matter. Throw either.
You awake in the night to find that your living room is on fire. Your one chance to save yourself is to throw something that will hit the back of your bedroom door and close it, giving you a few seconds to escape out the window. You happen to have both a sticky ball of clay and a super-bouncy Superball next to your bed, both the same size and same mass. You’ve only time to throw one. Which will it be? Your life depends on making the right choice!

A. **Throw the Superball.**  
   Larger $\Delta p \Rightarrow$ more impulse to door

B. Throw the ball of clay.

C. It doesn’t matter. Throw either.
Bounce/ No Bounce

No Bounce: \[ \Delta p = 0 - -mv_0 = mv_0 \]

Bounce: \[ \Delta p = p_f - p_0 = m(v_f - (-v_0)) \]
\[ = m(v_0 + v_0) = 2mv_0 \]

Bounce gives you twice as much WACK! Bounce has to stop the object AND give it p in the opposite direction!
- A rubber ball bounces off a wall.
- The ball is initially traveling toward the right, so $v_{ix}$ and $p_{ix}$ are positive.
- After the bounce, $v_{fx}$ and $p_{fx}$ are negative.
- The force on the ball is toward the left, so $F_x$ is negative.
- In this example, the impulse, or area under the force curve, has a negative value.
Bounce: Momentum is a Vector!!

A ball with mass .0047 kg. hits the ground as shown. What is the impulse applied to the ball by the floor?

\[ \vec{I} = \Delta \vec{p} = \vec{p}_f - \vec{p}_0 \]

\[ \begin{align*}
   &= (p_x \hat{i} + p_y \hat{j})_f - (p_x \hat{i} + p_y \hat{j})_i \\
   &= (p_y f - p_y 0) \hat{j} \\
   &= m(v \cos \theta - (-v \cos \theta)) \hat{j} \\
   &= 2mv \cos \theta \hat{j} = 2(.0047 \text{kg})(45 \text{m}/\text{s})(\cos 30) \hat{j} \\
   \end{align*} \]

\[ \vec{I} = (0.366 \text{kg} \cdot \text{m}/\text{s}) \hat{j} \]
Baseball Bat Wack!

What is the impulse and average force exerted on a 0.140 kg baseball by a bat given that the ball’s initial speed is 45.0 m/s and its final speed, after 1.3 ms impact, is 65.0 m/s in the opposite direction?

\[ I = \Delta p = p_f - p_0 = m(v_f - v_0) \]

\[ = 0.14 \text{ kg} (65 \text{ m/s} - (-45 \text{ m/s})) = 15.4 \text{ kg m/s} \]

\[ \bar{F} = \frac{\Delta p}{\Delta t} = \frac{(15.4 \text{ kg} \cdot \text{m/s})}{1.3 \times 10^{-3} \text{ s}} = 1.18 \times 10^4 \text{ N} \]

NOTE: Direction Matters! Vectors!!
In a slow-pitch softball game, a 0.200-kg softball crosses the plate at 15.0 m/s at an angle of 45.0° below the horizontal. The batter hits the ball toward center field, giving it a velocity of 40.0 m/s at 30.0° above the horizontal.

(a) Determine the impulse delivered to the ball.
(b) If the force on the ball increases linearly for 4.00 ms, holds constant for 20.0 ms, and then decreases to zero linearly in another 4.00 ms, what is the maximum force on the ball?
EXAMPLE 9.1 Hitting a baseball

A 150 g baseball is thrown with a speed of 20 m/s. It is hit straight back toward the pitcher at a speed of 40 m/s. The interaction force between the ball and the bat is shown in the figure below. What maximum force $F_{\text{max}}$ does the bat exert on the ball? What is the average force of the bat on the ball?

MODEL Model the baseball as a particle and the interaction as a collision.

1. Draw the before-and-after pictures.
   Before:
   $v_{ix} = -20 \text{ m/s}$
   $m = 0.15 \text{ kg}$

   Establish a coordinate system.

2. After:
   $v_{fx} = 40 \text{ m/s}$

3. Define symbols.

4. List known information.

5. Identify desired unknowns.

6. Draw a momentum bar chart.

3. The ball moves to the right with a higher speed.

2. It's hit to the right.

1. The ball was initially moving to the left.

$p_{ix} + J_{x} = p_{fx}$
EXAMPLE 9.1 Hitting a baseball

SOLVE Until now we’ve consistently started the mathematical representation with Newton’s second law. Now we want to use the impulse-momentum theorem:

\[ \Delta p_x = J_x = \text{area under the force curve} \]

We know the velocities before and after the collision, so we can calculate the ball’s momenta:

\[ p_{tx} = mv_{tx} = (0.15 \text{ kg})(-20 \text{ m/s}) = -3.0 \text{ kg m/s} \]
\[ p_{tx} = mv_{tx} = (0.15 \text{ kg})(40 \text{ m/s}) = 6.0 \text{ kg m/s} \]

Thus the change in momentum is

\[ \Delta p_x = p_{tx} - p_{tx} = 9.0 \text{ kg m/s} \]

The force curve is a triangle with height \( F_{\text{max}} \) and width 6.0 ms. The area under the curve is

\[ J_x = \text{area} = \frac{1}{2} \times F_{\text{max}} \times (0.0060 \text{ s}) = (F_{\text{max}})(0.0030 \text{ s}) \]

According to the impulse-momentum theorem,

\[ 9.0 \text{ kg m/s} = (F_{\text{max}})(0.0030 \text{ s}) \]

Thus the maximum force is

\[ F_{\text{max}} = \frac{9.0 \text{ kg m/s}}{0.0030 \text{ s}} = 3000 \text{ N} \]

The average force, which depends on the collision duration \( \Delta t = 0.0060 \text{ s} \), has the smaller value:

\[ F_{\text{avg}} = \frac{J_x}{\Delta t} = \frac{\Delta p_x}{\Delta t} = \frac{9.0 \text{ kg m/s}}{0.0060 \text{ s}} = 1500 \text{ N} \]
The cart’s change of momentum $\Delta p_x$ is

A. $-20$ kg m/s.
B. $-10$ kg m/s.
C. $0$ kg m/s.
D. $10$ kg m/s.
E. $30$ kg m/s.
The cart’s change of momentum $\Delta p_x$ is

A. $-20$ kg m/s.
B. $-10$ kg m/s.
C. 0 kg m/s.
D. 10 kg m/s.
E. 30 kg m/s.

$\Delta p_x = 10$ kg m/s $- (-20$ kg m/s $) = 30$ kg m/s

Negative initial momentum because motion is to the left and $v_x < 0$. 
A 2.0 kg object moving to the right with speed 0.50 m/s experiences the force shown. What are the object’s speed and direction after the force ends?

A. 0.50 m/s left.
B. At rest.
C. 0.50 m/s right.
D. 1.0 m/s right.
E. 2.0 m/s right.
A 2.0 kg object moving to the right with speed 0.50 m/s experiences the force shown. What are the object’s speed and direction after the force ends?

A. 0.50 m/s left.
B. At rest.
C. 0.50 m/s right.
D. 1.0 m/s right. $\Delta p_x = J_x$ or $p_{fx} = p_{ix} + J_x$
E. 2.0 m/s right.
Newton’s Third Law

\[ F_{\text{action}} = -F_{\text{reaction}} \]

\[ \frac{\Delta p_{\text{ball}}}{\Delta t} = -\frac{\Delta p_{\text{cannon}}}{\Delta t} \]

\[ \Delta p_{\text{ball}} = -\Delta p_{\text{cannon}} \]

\[ \frac{dp_{\text{total}}}{dt} = 0 \]

\[ (m_b v_{bf} - m_b v_{b0}) = -(m_c v_{cf} - m_c v_{c0}) \]

\[ m_b v_{bf} + m_c v_{cf} = m_b v_{b0} + m_c v_{c0} \]

Conservation of Momentum!
Conservation of Momentum

If there are no external forces acting on a system of objects then the total momentum before an event is equal to the total momentum after an event.

\[ \sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}} \]

System: You define the system so that momentum is conserved.
- No external forces acting on the system
- Internal forces cancel out and don’t change momentum
- Include all the objects that collide or ‘go bump’
- Always apply conservation laws to a system

When you have defined the system so that the net external forces acting is zero, THEN & ONLY THEN can you invoke Conservation of momentum!
ONLY AFTER you have defined the system so that the net external forces acting on it is zero, THEN & ONLY THEN can you invoke Conservation of Momentum and apply:

\[ \sum \vec{P}_{\text{before}} = \sum \vec{P}_{\text{after}} \]
Conservation of Momentum

Momentum is conserved for which system?

(a) 8-ball system  
(b) cue-ball system  
(c) cue-ball + 8 ball system

INCLUDE EVERYTHING THAT GOES BUMP!

NOTICE HOW THE INTERNAL FORCES CANCEL!
Conservation of Momentum

If there are no external forces acting on a system of objects then the total momentum before an event is equal to the total momentum after an event.

\[ \sum p_{before} = mv + 0 = mv \]

\[ \sum p_{after} = 2m \frac{v}{2} = mv \]
Conservation Depends on The System

\[ \sum p_{\text{before}} = mv \]

\[ \sum p_{\text{after}} = m \frac{v}{2} \]

Momentum is NOT conserved for this system!
Warning!

MOMENTUM IS CONSERVED!
NOT VELOCITY!

\[ \sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}} \]

\[ \sum \vec{v}_{\text{before}} = \sum \vec{v}_{\text{after}} \]

This is true
ONLY for
The CM!
Perfectly Inelastic Collisions:  
*Stick*  

Velocity Kinetic Energy is NOT Conserved!
Elastic Collisions: *Bounce*

Momentum is transferred!

Kinetic Energy is Conserved!
<table>
<thead>
<tr>
<th></th>
<th>Truck</th>
<th>Car</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (kg)</td>
<td>3000</td>
<td>1000</td>
</tr>
<tr>
<td>vel. (m/s)</td>
<td>20.0</td>
<td>0.0</td>
</tr>
<tr>
<td>mom. (kg m/s)</td>
<td>60 000</td>
<td>0</td>
</tr>
</tbody>
</table>

**MOMENTUM TRANSFER**
<table>
<thead>
<tr>
<th>Car</th>
<th>Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (kg)</td>
<td>1000</td>
</tr>
<tr>
<td>vel. (m/s)</td>
<td>20.0</td>
</tr>
<tr>
<td>mom. (kg m/s)</td>
<td>20 000</td>
</tr>
</tbody>
</table>
Collision Decision

Both trucks have the same speed and mass. In our simple physics world, what is NOT allowed:

a) Bounce off each other Elastic
b) Stick and move to the left Impossible
c) Stick and move to the right Impossible
d) Stick and stop dead in the center Inelastic
e) Vaporize Matter-Antimatter trucks!
Easiest: Perfectly Inelastic Collisions

Granny (m=50kg) whizzes around the rink at 3m/s and snatches up Andy (m=25kg). What is their final velocity? (Ignore friction)
Granny: m = 50kg, v₀ = 3m/s
Andy: m = 25kg, v₀ = 0

\[ \sum p_{\text{initial}} = \sum p_{\text{final}} \]

\[ m_1 v_{1i} = (m_1 + m_2) v_f \]

\[ v_f = \frac{m_1 v_{1i}}{(m_1 + m_2)} \]

\[ = \frac{50\text{kg} \cdot 3\text{m/s}}{(50\text{kg} + 25\text{kg})} \]

\[ = 2\text{m/s} \]
The mass of the big fish is 4X the mass of the little fish.
Speed of Small Fish = 5 km/hr
Perfectly Inelastic Collision
What’s the Final velocity?
2-D Collisions

<table>
<thead>
<tr>
<th></th>
<th>Blue Car</th>
<th></th>
<th>Red Car</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (kg)</td>
<td>1000</td>
<td>mass (kg)</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>vel. (m/s)</td>
<td>20.0, East</td>
<td>vel. (m/s)</td>
<td>10.0, North</td>
<td></td>
</tr>
<tr>
<td>mom. (kg m/s)</td>
<td>20 000, East</td>
<td>mom. (kg m/s)</td>
<td>10 000, North</td>
<td></td>
</tr>
</tbody>
</table>
2-D Momentum

Collision Intuition

Think Symmetry

Equal Masses
Equal Speeds
2-D Momentum

Collision Intuition

Think Symmetry

Equal Masses
Equal Speeds
Unequal Masses

Equal Speeds

Collision Intuition

2-D Momentum
2-D Momentum

Collision Intuition

Unequal Masses
Equal Speeds
2-D Momentum

Collision Intuition

Equal Masses
Unequal Speeds
2-D Momentum

Collision Intuition

Equal Masses
Unequal Speeds
2-D Momentum

Collision Intuition

Unequal Masses
Unequal Speeds
A 3.0-kg mass moving in the positive $x$ direction with a speed of 10 m/s collides with a 6.0-kg mass initially at rest. After the collision, the speed of the 3.0-kg mass is 8.0 m/s, and its velocity vector makes an angle of 35° with the positive $x$ axis. What is the magnitude of the velocity of the 6.0-kg mass after the collision?

a. 2.2 m/s
b. 2.9 m/s
c. 4.2 m/s
d. 3.5 m/s
e. 4.7 m/s
Exploding Systems

\[ \sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}} \]
Charlie and his gun, $m = 80\text{kg}$, shoots off a bullet, $m = .01\text{kg}$, straight ahead at $300 \text{ m/s}$. What is Charlie’s (+gun) recoil velocity?

1. Define the System so net external forces is zero.

2. THEN momentum is conserved

\[
\sum p_{\text{initial}} = \sum p_{\text{final}}
\]

\[
p_{\text{charlie},i} + p_{\text{bullet},i} = p_{\text{charlie},f} + p_{\text{bullet},f}
\]

\[
0 = p_{\text{charlie},f} + p_{\text{gun},f}
\]

\[
p_{\text{charlie},f} = -p_{\text{bullet},f}
\]

\[
m_c v_{cf} = -m_b v_{bf}
\]

\[
v_{cf} = -\frac{m_b}{m_c} v_{bf}
\]

\[
= -\frac{0.01\text{kg}}{80\text{kg}} \times 300\text{m/s} = -0.04\text{m/s}
\]

Frictionless surface
Conservation of Momentum

The toy rocket in deep space breaks into equal masses. Find the final velocities of the masses and show that the velocity of the CM is equal to the initial velocity. Ignore gravity and air resistance.

\[ \sum \vec{p}_i = \sum \vec{p}_f \]

\[ \sum p_{xi} = \sum p_{xf} \quad \sum p_{yi} = \sum p_{yf} \]

\[
2mv = mv_1 \cos 30 + mv_2 \cos 60 \quad 0 = mv_1 \sin 30 - mv_2 \sin 60 \quad v_1 = v_2 \frac{\sin 60}{\sin 30}
\]

Substitute:

\[
2mv = m\left(v_2 \frac{\sin 60}{\sin 30}\right) \cos 30 + mv_2 \cos 60
\]

\[
v_2 = \frac{2(50m/s)}{\sin 60 / \tan 30 + \cos 60} = 50m/s
\]

\[
v_1 = v_2 \frac{\sin 60}{\sin 30} = (50m/s) \frac{\sin 60}{\sin 30} = 86.6m/s
\]

You can also use vector addition!!!
A 4.0-kg mass has a velocity of 4.0 m/s, east when it explodes into two 2.0-kg masses. After the explosion one of the masses has a velocity of 3.0 m/s at an angle of 60° north of east. What is the magnitude of the velocity of the other mass after the explosion?

a. 7.9 m/s  

b. 8.9 m/s  

c. 7.0 m/s  

d. 6.1 m/s  

e. 6.7 m/s
A 2.0 kg object moving to the right with speed 0.50 m/s experiences the force shown. What are the object’s speed and direction after the force ends?

A. 0.50 m/s left.
B. At rest.
C. 0.50 m/s right.
D. 1.0 m/s right.
E. 2.0 m/s right.
A 2.0 kg object moving to the right with speed 0.50 m/s experiences the force shown. What are the object’s speed and direction after the force ends?

A. 0.50 m/s left.
B. At rest.
C. 0.50 m/s right.
D. 1.0 m/s right.
E. 2.0 m/s right.

\[ \Delta p_x = J_x \text{ or } p_{fx} = p_{ix} + J_x \]
A 2.0 kg object moving to the right with speed 0.50 m/s experiences the force shown. What are the object’s speed and direction after the force ends?

A. 0.50 m/s left.
B. At rest.
C. 0.50 m/s right.
D. 1.0 m/s right.
E. 2.0 m/s right.
A 2.0 kg object moving to the right with speed 0.50 m/s experiences the force shown. What are the object’s speed and direction after the force ends?

A. 0.50 m/s left.  
B. **At rest.**  
C. 0.50 m/s right  
D. 1.0 m/s right.  
E. 2.0 m/s right.
A force pushes the cart for 1 s, starting from rest. To achieve the same speed with a force half as big, the force would need to push for

A. \( \frac{1}{4} \) s.
B. \( \frac{1}{2} \) s.
C. 1 s.
D. 2 s.
E. 4 s.
A force pushes the cart for 1 s, starting from rest. To achieve the same speed with a force half as big, the force would need to push for

A. \( \frac{1}{4} \) s.
B. \( \frac{1}{2} \) s.
C. 1 s.
D. 2 s. **(Correct Answer)**
E. 4 s.
A light plastic cart and a heavy steel cart are both pushed with the same force for 1.0 s, starting from rest. After the force is removed, the momentum of the light plastic cart is ________ that of the heavy steel cart.

A. greater than
B. equal to
C. less than
D. Can’t say. It depends on how big the force is.
A light plastic cart and a heavy steel cart are both pushed with the same force for 1.0 s, starting from rest. After the force is removed, the momentum of the light plastic cart is \_______\ that of the heavy steel cart.

A. greater than

B. equal to

C. less than

D. Can’t say. It depends on how big the force is.

**Solution:***

A. greater than

B. equal to

C. less than

D. Can’t say. It depends on how big the force is.

**Reasoning:**

Same force, same time \(\Rightarrow\) same impulse

Same impulse \(\Rightarrow\) same change of momentum
Tactics: Drawing a Before-and-After Pictorial Representation

**TACTICS BOX 9.1 Drawing a before-and-after pictorial representation**

1. **Sketch the situation.** Use two drawings, labeled “Before” and “After,” to show the objects *before* they interact and again *after* they interact.
2. **Establish a coordinate system.** Select your axes to match the motion.
3. **Define symbols.** Define symbols for the masses and for the velocities before and after the interaction. Position and time are not needed.

Exercises 17–19
**TACTICS BOX 9.1 Drawing a before-and-after pictorial representation**

4. **List known information.** Give the values of quantities that are known from the problem statement or that can be found quickly with simple geometry or unit conversions. Before-and-after pictures are simpler than the pictures for dynamics problems, so listing known information on the sketch is adequate.

5. **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined in step 3.

6. **If appropriate, draw a momentum bar chart** to clarify the situation and establish appropriate signs.

Exercises 17–19
Two 1.0 kg stationary cue balls are struck by cue sticks. The cues exert the forces shown. Which ball has the greater final speed?

A. Ball 1.
B. Ball 2.
C. Both balls have the same final speed.
Two 1.0 kg stationary cue balls are struck by cue sticks. The cues exert the forces shown. Which ball has the greater final speed?

A. Ball 1.
B. Ball 2.
C. Both balls have the same final speed.