Energy in an Oscillating Spring-Mass System

A simple way to test conservation of mechanical energy, is by using a spring system with a vertically hanging mass $m$ and a spring constant $k$. The restoring force applied by an ideal spring is proportional to how much it is stretched or compressed, given by Hooke’s law:

$$ F = -kx $$

In this experiment, you will determine the spring constant of an ideal spring, and test if the mechanical energy of the oscillating mass-spring system is conserved.

Energy is present in three forms for the mass and spring system. The mass $m$, with velocity $v$, can have kinetic energy $KE$

$$ K = \frac{1}{2}mv^2 $$

The spring can hold elastic potential energy, or $PE_{elastic}$. We calculate $U_{elastic}$ by using

$$ U_{elastic} = \frac{1}{2}kx^2 $$

where $k$ is the spring constant and $x$ is the extension or compression of the spring measured from the equilibrium position.

The mass and spring system also has gravitational potential energy ($U_{gravitational} = mgy$), but we do not have to include the gravitational potential energy term if we measure the spring length from the hanging equilibrium position. We can then concentrate on the exchange of energy between kinetic energy and elastic potential energy.

If there are no other forces experienced by the system, then the principle of conservation of energy tells us that the sum

$$ \Delta K + \Delta U_{elastic} = 0 $$

which we can test experimentally.

OBJECTIVES

- Determine the spring constant of an ideal spring
- Examine the energies involved in simple harmonic motion.
- Test the principle of conservation of energy.

MATERIALS

- computer
- Vernier computer interface
- Logger Pro
- Vernier Motion Detector
- wire basket
- slotted mass set, 50 g to 300 g in 50 g steps
- slotted mass hanger
- spring
- Table clamp and rods (see setup below)
- Pendulum brass ball & string (for plumb bob)
- Damper (cd, records, etc)
Experimental Procedure

Open a Word document and the Excel data sheet for your group lab report. Use the Logger Pro files in the Physics 40 Lab folder. Include sample data plots from logger pro, the excel data table and plot, all labeled, and answer all the questions in the Analysis section.

Experimental Procedure

1. Weigh the mass of the spring and the mass hanger and put this data into your lab report. Attach the spring to a horizontal rod connected to the lab stand with the denser part of the spring at the top. Hang the mass hanger from the spring on it as shown above.

2. Connect the Motion Detector to DIG/SONIC 1 of the LabPro

3. Place the Motion Detector at least 75 cm below the mass. Make sure there are no objects near the path between the detector and mass, such as a table edge. Place the wire basket over the Motion Detector to protect it. Open the Logger Pro and excel files from Physics 40 Energy of a Spring Folder.

4. Make a preliminary run to make sure things are set up correctly. Lift the mass upward a few centimeters and release. The mass should oscillate along a vertical line only. Click to begin data collection. The position graph should show a clean sinusoidal curve. If it has flat regions or spikes, reposition the Motion Detector and try again.

Part 1: Finding the Spring Constant

1. Set the equilibrium position of the spring – hanger system by zeroing the motion detector. Do this by allowing the mass to hang free and at rest. The zero is under the experiment menu in Logger Pro. Collect data to confirm that the motion detector is reading the bottom of the hanger as the zero point.

2. Add 100 grams to mass hanger and let the spring system come to rest. Measure the distance to the hanger by collecting data. THE MASS IN NOT OSCILLATING!!!! Use the statistics analysis to find the mean value of the distance. Copy and paste a sample logger pro graph into your report. Repeat for 100 g increments for a total of 5 data points. Calculate the value of k for each weight and displacement. USE EXCEL FORMULAS to do the calculations. Copy and paste the formulas as text to show that you did it. Average the values of k and find the average deviation between them. This will be the uncertainty in your k. Plot the applied force (the added weights) vs the displacement and do a linear fit to find the slope of the line. This will give you the force constant, k, for the spring. Copy and paste the F vs x graph into your lab along with the linear fit information. Write the value of k in standard form: $k \pm \Delta k$

Part 2: Testing Conservation of Mechanical Energy

1. Open Logger pro file: Energy in a Spring 2. Adjust the mass and spring constant for the system on the right side of the window, using a total of 200 g and the k value found in part 1.

2. Set the equilibrium position of the spring – hanger system by zeroing the motion detector. Test to make sure it is properly zeroed as above. It can’t be moving at all!

3. Then set the system in oscillation by displacing the mass about 5.0 cm from equilibrium so that you produce very clean data graphs for both position vs. time and the velocity vs.
From the x vs t and v vs t graphs, find the maximum displacement (m) and the maximum velocity. Enter these into the data sheet. Put a copy in your word document.

4. Use Excel (for goodness sakes!) to calculate the maximum kinetic energy and elastic spring potential energy from the x and v values found in step 3 above. These are considered the ‘theoretical’ values for energies.

5. In the x vs t graph, click on the label on the vertical axis and click on ‘more’ and then click on ‘potential energy’ so that all both graphs are shown simultaneously. Do the same in the v vs t graph to show both the velocity and kinetic energy graphs simultaneously. (cool eh?) From the graphs, find the experimental values for the potential and kinetic energies and enter these values into the data sheet. Copy and paste each into your lab report.

6. On the top graph click more and select Kinetic Energy, Potential Energy, Total Energy and force so that they are all showing together. On the bottom graph show the energy only. Using the stats tool on the bottom energy graph, find the mean value of the energy during the entire data set. Enter the value into the data sheet for Eex, the experimental value of the total energy of the system. Study the relationships between the plots and discuss with your lab partners if they agree with your expectations. You are free to show different plots together if that works for you. Copy and paste into your lab report.

7. Repeat steps 1-6 above for two more mass combinations. (e.g. 300g, 400g)

8. If a non-conservative force such as air resistance becomes important, the data graphs will change. Predict how the graphs would look, then tape an index card to the bottom of your hanging mass. Collect data again for one mass and compare to your prediction. (You do not need to enter data into a table nor do calculations for this part – it is qualitative. But do include the logger pro plot in your lab report and discuss the results.) Try some fits for the envelope to try to get a function that describes the motion.

9. Find the period of oscillation for your mass spring system. Does it depend on mass? How far you stretch it? Figure it out.

ANALYSIS

1. How did the k value from the average of the 10 data points vary from the k from the slope of the linear fit? How good was the fit (R-squared value)? Which value do you trust more? Why? Can you think of any other way to find the spring constant, k, to test this value? Explain.

2. How do the maximum kinetic energy and potential energies compare? At what values of x do they occur? How does the force compare? That is, when F is maximum or a minimum, what are the values K and U?

3. What is the theoretical value for the total energy of the system? Calculate it and put it into the table for each mass. Calculate the percent difference for the total Energy.

4. Is the mechanical energy conserved for the mass spring system? That is, is the total energy of the system constant? How do the potential energy vs. time and the kinetic energy vs. time relate to each other? How do they show that the total mechanical energy of the system is conserved or not? Explain.

5. Discuss your prediction and results for part 8 above.

6. Discuss your results for 9 above.