Chapter 13

- Hooke’s Law: $F = -kx$
- Periodic & Simple Harmonic Motion
- Springs & Pendula
- Waves
- Superposition

Next Week!
Review Physics 2A: Springs, Pendula & Circular Motion
Elastic Systems  \( F = -kx \)

Small Vibrations
All objects have a natural frequency of vibration or oscillation. Bells, tuning forks, bridges, swings and atoms all have a natural frequency that is related to their size, shape and composition. A system being driven at its natural frequency will resonate and produce maximum amplitude and energy.
Coupled Oscillators

Molecules, atoms and particles are modeled as coupled oscillators. Waves Transmit Energy through coupled oscillators. Forces are transmitted between the oscillators like springs. Coupled oscillators make the medium.
The Oscillation of Nothing

The Quantum Oscillator

- Classically, at $x=0$, the energy is zero.
- Therefore, the momentum is zero too.
- But this would violate the Heisenberg uncertainty principle!
- Therefore, the quantum oscillator cannot be completely at rest. Allowed energies, $E_n$, are:

\[ E_n = (n + \frac{1}{2})hf \quad (n=0,1,2,\ldots) \]

$f = \text{oscillation frequency}$

The minimum possible energy for the quantum oscillator is $E_0 = \frac{1}{2} hf$.

This is called the zero point energy.
Hooke’s Law

*Ut tensio, sic vis* - as the extension, so is the force

Hooke’s Law describes the *elastic* response to an applied force.

Elasticity is the property of an object or material which causes it to be restored to its original shape after distortion.

An elastic system displaced from equilibrium oscillates in a simple way about its equilibrium position with *Simple Harmonic Motion.*
Robert Hooke (1635-1703)

- Leading figure in Scientific Revolution
- Contemporary and arch enemy of Newton
- Hooke’s Law of elasticity
- Worked in Physics, Biology, Meteorology, Paleontology
- Devised compound microscope
- Coined the term “cell”
Hooke’s Law

It takes twice as much force to stretch a spring twice as far.

The linear dependence of displacement upon stretching force:

\[ F_{\text{applied}} = kx \]
Hooke’s Law

Stress is directly proportional to strain.

\[ F_{\text{applied}}(\text{stress}) = kx(\text{strain}) \]
Hooke’s Law

\[ F_{\text{Restoring}} = -kx \]

• The applied force displaces the system a distance \( x \).

• The reaction force of the spring is called the “Restoring Force” and it is in the opposite direction to the displacement.
An Ideal Spring

\[ F_{spring} = -kx \]

An ideal spring obeys Hooke’s Law and exhibits Simple Harmonic Motion.
Spring Constant $k$: *Stiffness*

- The larger $k$, the stiffer the spring
- Shorter springs are stiffer springs
- $k$ strength is inversely proportional to the number of coils
Spring Question

Each spring is identical with the same spring constant, $k$. Each box is displaced by the same amount and released. Which box, if either, experiences the greater net force?

Same!
Hooke’s Law: \[ F = -kx \]
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Hooke’s Law: $F = -kx$
Periodic Motion

Position vs Time: Sinusoidal Motion
SHM and Circular Motion

\[ x(t) = A \cos \theta(t) \]

\[ v(t) = -A\omega \sin \omega t \]

\[ a(t) = -A\omega^2 \cos \omega t \]

\[ v_t = R\omega = \frac{2\pi R}{T}, \quad a_t = R\alpha, \quad a_c = \frac{v^2}{R} = \omega^2 R \]

Terms:

Amplitude: \( [A] = m \)

Period: \( T = \text{time / cycle}, \ [T] = \text{sec} \)

Frequency: \( f = \frac{1}{T} (\# \text{cycles / sec}), \ [f] = \text{Hz} \)

Angular Frequency: \( \omega = 2\pi f = \frac{2\pi}{T} , \ [\omega] = \text{rad / s} \)

Displacement of Mass
Simple Harmonic Motion

\[ x(t) = A \cos \theta(t) \]

\[ \theta(t) = \omega t = \frac{2\pi}{T} t \]

Displacement of Mass

Period = T
Simple Harmonic Motion

\[ x(t) = A \cos \omega t \]

\[ v(t) = -A\omega \sin \omega t \]

\[ a(t) = -A\omega^2 \cos \omega t \]

Notice:

\[ a = -\omega^2 x \]

Displacement of Mass

\[ \omega = \frac{2\pi}{T} \]
Newton’s 2nd Law?

\[ F = ma \]
\[ = m(-\omega^2 x) \]
\[ = -kx \quad k = m\omega^2 \]

\[ \omega = \sqrt{\frac{k}{m}} \]

\{ angular frequency \}
\[ \omega = \frac{2\pi}{T} \]
Simple Harmonic Motion

$$\omega = \sqrt{\frac{k}{m}} \quad T = \frac{2\pi}{\omega}$$

Does the period depend on the displacement, x?

The period depends only on how stiff the spring is and how much inertia there is.

Despite the image, the diagram does not directly show the motion of the mass, but it illustrates the concept of simple harmonic motion with a spring and a mass attached to it, moving along a plane.
Finding The Spring Constant

A 2.00 kg block is at rest at the end of a horizontal spring on a frictionless surface. The block is pulled out 20.0 cm and released. The block is measured to have a frequency of 4.00 hertz.
a) What is the spring constant?

\[ f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

\[ k = 4\pi^2 f^2 m \]

\[ k = 4\pi^2 (4 / s)^2 (2kg) \frac{m^2}{m^2} = 1263 N / m \]
Harmonic Motion Described

\[ F_{\text{restoring}} = -kx \]

\[ T = 2\pi \sqrt{\frac{m}{k}}, \quad \omega = \sqrt{\frac{k}{m}} \]

\[ x(t) = A \cos \omega t \]

\[ v(t) = -A\omega \sin \omega t \]

\[ a(t) = -A\omega^2 \cos \omega t \]
Maximum Velocity and Acceleration

\[ v = -A\omega \sin \omega t \]

\[ v_{\text{max}} = A\omega \]

\[ @\omega t = n_{\text{odd}} \frac{\pi}{2} \]

\[ a = -A\omega^2 \cos \omega t \]

\[ a_{\text{max}} = A\omega^2 \]

\[ @\omega t = n\pi \]
Finding Maximum Velocity

A 2.00 kg block is at rest at the end of a horizontal spring on a frictionless surface. The block is pulled out 20.0 cm and released. The block is measured to have a frequency of 4.00 hertz. 

a) What is the spring constant? b) What is the maximum velocity?

\[ v(t) = -A\omega \sin \omega t \quad \Rightarrow \quad v_{\text{max}} = A\omega = A\sqrt{\frac{k}{m}} \]

\[ v_{\text{max}} = (.2m)\sqrt{\frac{1263 \text{ N/m}}{2\text{ kg}}} = 5.0 \text{ m/s} \]
Simple Harmonic Motion

Consider a \( m = 2.00 \) kg object on a spring in a horizontal frictionless plane. The sinusoidal graph represents displacement from equilibrium position as a function of time.

A) What is the amplitude of motion?
B) What is the period, \( T \)? frequency, \( f \)?
C) What is the equation representing the displacement?

A) \( A = 0.08 \) m

B) \( T = 4 \) s \( f = 1/T = 0.25 \) Hz

C) \( x(t) = A \cos \frac{2\pi}{T} t = 0.08m \cos \frac{2\pi}{4s} t = 0.08m \cos \frac{\pi}{2} t \)
Simple Harmonic Motion

Consider a $m = 2.00$ kg object on a spring in a horizontal plane. The sinusoidal graph represents displacement from equilibrium position as a function of time. $x(t) = 0.08m \cos \frac{\pi}{2} t$

D) What is the speed of the object at $t = 1s$?

$v = -A\omega \sin \omega t \quad \rightarrow \quad v_{\text{max}} = -A\omega = -A \frac{2\pi}{T} = -0.08m \frac{2\pi}{4s}$

$v_{\text{max}} = -0.13 m/s$

E) What is the acceleration of the object at $t = 1s$? zero!
Simple Harmonic Motion

Consider a \( m = 2.00 \) kg object on a spring in a horizontal plane. The sinusoidal graph represents displacement from equilibrium position as a function of time. \( x(t) = 0.08m \cos \frac{\pi}{2} t \)

F) What is the spring constant?

\[
T = 2\pi \sqrt{\frac{m}{k}} \quad \Rightarrow \quad k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 (2\text{kg})}{(4\text{s})^2}
\]

\[ k = 4.94 \text{N/m} \]
The position of a simple harmonic oscillator is given by

\[ x(t) = (0.5 \text{ m}) \cos \left( \frac{\pi}{3} t \right) \]

where \( t \) is in seconds.

a) What is the period of this oscillator?
b) What is the maximum velocity of this oscillator?

a) \[ x(t) = A \cos \omega t \quad \Rightarrow \quad \frac{\pi}{3} = \omega = \frac{2\pi}{T} \quad \Rightarrow \quad T = 6s \]

b) \[ v(t) = -A\omega \sin \omega t \quad \Rightarrow \quad v_{\text{max}} = A\omega = A \frac{2\pi}{T} \]

\[ v_{\text{max}} = 0.5m \frac{2\pi}{6s} = 0.52m/s \]
Energy in a Spring

Mechanical Energy is Conserved!

\[ P E_{spring} = \frac{1}{2} k A^2 \]

\[ K E_{spring} = \frac{1}{2} m v^2 \]
Energy in a Spring: Quiz Question

What speed will a 25g ball be shot out of a toy gun if the spring (spring constant = 50.0N/m) compressed 0.15m? Ignore friction.

Use Energy!

\[ PE_{spring} = KE_{ball} \]

\[ \frac{1}{2} kx^2 = \frac{1}{2} mv^2 \]

\[ v = \sqrt{\frac{k}{m}} x \]

\[ v = \sqrt{\frac{50.0 \, N/m}{.025 \, kg}} (0.15m) = 6.7 \, m/s \]

Note: this is also

\[ = \omega A \]

\[ = v_{max} \]
Spring Question

A 1.0-kg object is suspended from a spring with $k = 16 \text{ N/m}$. The mass is pulled 0.25 m downward from its equilibrium position and allowed to oscillate. What is the maximum kinetic energy of the object?

Maximum kinetic energy happens at the equilibrium position.

\[ KE_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 \]

\[ v(t) = -A \omega \sin \omega t \Rightarrow v_{\text{max}} = A \omega = A \sqrt{\frac{k}{m}} \]

\[ KE_{\text{max}} = \frac{1}{2} m v_{\text{max}}^2 = \frac{1}{2} m \left( A \sqrt{\frac{k}{m}} \right)^2 = \frac{1}{2} A^2 k = 0.5J = PE_{\text{max}} \]
Simple Pendulum

For small angles, simple pendulums exhibit Simple Harmonic Motion:
Simple Pendulum

For small angles, simple pendulums exhibit Simple Harmonic Motion:

Restoring Force:

\[ F = -mg \sin \theta \approx -mg\theta \]

\[ \theta = s / L \]

\[ F \approx -mgs / L = -ks \]

\[ k = mg / L \]

\[ \omega = \sqrt{k / m} = \sqrt{mg / L / m} = \sqrt{g / L} \]

Amplitude: \( x = s \)

For angles less than 15 degrees:

\[ \omega = \sqrt{g / L} \]

\[ T = 2\pi \sqrt{L / g} \]
Moon Clock

At what rate will a pendulum clock run on the moon where $g_M = 1.6\text{m/s}^2$, in hours?

$$T_{Earth} = 2\pi \sqrt{\frac{L}{g_E}} \quad T_{Moon} = 2\pi \sqrt{\frac{L}{g_M}}$$

Divide:

$$\frac{T_{Moon}}{T_{Earth}} = \frac{2\pi \sqrt{\frac{L}{g_M}}}{2\pi \sqrt{\frac{L}{g_E}}} \quad \Rightarrow \quad T_{Moon} = T_{Earth} \sqrt{\frac{g_E}{g_M}} = 2.45\text{h}$$
Waves
Waves

- result from periodic disturbance
- same period (frequency) as source
- Longitudinal or Transverse Waves
- Characterized by
  - amplitude (how far do the “bits” move from their equilibrium positions? Amplitude of MEDIUM)
  - period or frequency (how long does it take for each “bit” to go through one cycle?)
  - wavelength (over what distance does the cycle repeat in a freeze frame?)
  - wave speed (how fast is the energy transferred?)

\[ f = \frac{1}{T} \]

\[ v = \lambda f \]
Types of Waves

Sound

Wavelength = \lambda

Transverse

String

Direction of wave travel

WATER particle moves on circular path

Transverse component

Longitudinal component
Sound is a Longitudinal Wave

Pulse

Tuning Fork

Guitar String
Spherical Waves

Wavelength
Wavelength and Frequency are *Inversely* related: \[ f = \frac{v}{\lambda} \]

The *shorter* the wavelength, the *higher* the frequency. The *longer* the wavelength, the *lower* the frequency.
Wave speed: Depends on Properties of the Medium: Temperature, Density, Elasticity, Tension, Relative Motion

\[ v = \lambda f \]
Wave Speed

Speed of wave depends on properties of the MEDIUM

\[ v = \lambda f \]

Particle Speed

Speed of particle in the Medium depends on SOURCE: SHM

\[ v(t) = -A\omega \sin \omega t \]
Waves on Strings

\[ v = \lambda f \]

\[ v = \sqrt{\frac{F}{\mu}} \quad (1D \text{ string}) \]

\[ \mu = m / L \quad (\text{linear mass density}) \]
Problem:  

The displacement of a vibrating string vs position along the string is shown. The wave speed is 10cm/s.

A) What is the amplitude of the wave?  
4 cm

B) What is the wavelength of the wave?  
6 cm

C) What is the frequency of the wave?  

\[ f = \frac{v}{\lambda} = \frac{10 \text{ cm/s}}{6 \text{ cm}} = 1.67 \text{ Hz} \]
The displacement of a vibrating string vs position along the string is shown. The wave speed is 10cm/s.

D) If the linear density of the string is .01kg/m, what is the tension of the string?
The displacement of a vibrating string vs position along the string is shown. The wave speed is 10 cm/s.

D) If the linear density of the string is 0.01 kg/m, what is the tension of the string?

\[ F = \frac{F}{m/L} \]

\[ F = (0.1 m)^2 (0.01 \text{ kg/m}) = 10^{-5} \text{ N} \]
The displacement of a vibrating string vs position along the string is shown. The wave speed is 10 cm/s.

e) If the tension doubles, how does the wave speed change? Frequency? Wavelength?

\[ v = \sqrt{\frac{F}{mL}} \]

Wave speed increases by a factor of \( \sqrt{2} \)
Wave PULSE:

- traveling disturbance
- transfers energy and momentum
- no bulk motion of the medium
- comes in two flavors
  - LONGitudinal
  - TRANSverse
Reflected PULSE:

If the end is bound, the pulse undergoes an inversion upon reflection: “a 180 degree phase shift”

If it is unbound, it is not shifted upon reflection.
Reflection of a Wave Pulse
Reflection of a Traveling Wave
Superposition

Waves interfere temporarily.

(a) Overlap begins

(b) Total overlap; the slinky has twice the height of either pulse

(c) The receding pulses
Pulsed Interference
Standing Waves: Boundary Conditions
Transverse Standing Wave

Produced by the superposition of two identical waves moving in opposite directions.
Standing Waves on a String

Harmonics
Standing Waves on a String

\[ \lambda_1 = 2L \]

\[ \lambda_2 = L \]

\[ \lambda_3 = \frac{2L}{3} \]
Standing Waves on a String

$$\lambda_n = \frac{2L}{n}$$

$$f_n = \frac{\nu}{\lambda_n}$$

$$f_n = n \frac{\nu}{2L}$$
Standing Waves on a String

Harmonics

The possible frequency and energy states of a wave on a string are quantized.

\[ f_1 = \frac{v}{\lambda} = \frac{v}{2l} \]

\[ f_n = n \frac{v}{2l} \]

\[ f_n = nf_1 \]
Strings & Atoms are Quantized

The possible frequency and energy states of an electron in an atomic orbit or of a wave on a string are quantized.

\[ f = n \frac{\nu}{2l} \]

\[ E_n = nhf, \quad n = 0, 1, 2, 3, \ldots \]

\( h = 6.626 \times 10^{-34} \text{ Js} \)
Multiple Harmonics can be present at the same time.
Which harmonics (modes) are present on the string?

The Fundamental and third harmonic.
Standing Waves

Standing waves form in certain *MODES* based on the length of the string or tube or the shape of drum or wire. Not all frequencies are permitted!