Chapter 5

Dynamics of Uniform Circular Motion

Circular Motion, Orbits, and Gravity
Uniform Circular Motion

A particle moving in a circle at a constant speed undergoes uniform circular motion. In this chapter you’ll learn how to describe a particle’s motion in terms of its angular position and angular velocity.

We'll also review the important idea that a particle moving in a circle has an acceleration directed toward the center of the circle.

Dynamics of Uniform Circular Motion

Because a particle moving in a circle has an acceleration that points toward the center of the circle, there must be a net force toward the center to cause this acceleration.

Looking Back

2.2 Uniform motion
3.8 Circular motion

Looking Back

5.2 Using Newton's second law

The net force on the girl is directed toward the center of the circle.

The normal force of the track and the car's weight combine to provide a net force toward the circle’s center.
Newton's Law of Gravity
Newton discovered the law that governs gravity. You’ll learn how it applies to an apple falling to earth or a rock falling on the moon, and how this law governs the motions of the moon, the planets, and even distant galaxies.

Newton’s great insight was that the law of gravity described not only falling objects but also the orbits of the moon and planets.

Even the structure of distant galaxies is determined by Newton’s law of gravity.

Gravity and Orbits
If an object moves fast enough, it can orbit the earth, the sun, or another planet.

An orbit can be thought of as projectile motion where the ground curves away just as fast as the object falls.

The space station appears weightless, but gravity still acts strongly on it; only its apparent weight is zero.
Circular Motion
Milky Way Galaxy

We Are Here

Orbital Speed of Solar System: 220 km/s
Orbital Period: 225 Million Years
Mercury: 48 km/s
Venus: 35 km/s
Earth: 30 km/s
Mars: 24 km/s
Jupiter: 13 km/s
Neptune: 5 km/s
Differential Solar Rotation
Entangled Magnetic Field Lines make Sun Spots

Equatorial Rotational Period: 27.5 days
Precession causes the position of the North Pole to change over a period of 26,000 years.
Although the Moon is always lit from the Sun, we see different amounts of the lit portion from Earth depending on where the Moon is located in its month-long orbit.

Orbital Speed of Moon: \( \sim 1 \text{ km/s} \)
155mph~70m/s
Doppler Radar

RADAR: RAdio Detecting And Ranging

200mph~90m/s
Electrons in Bohr Orbit: 2 million m/s
(Speed of Light: 200 million m/s)
Maximal Kerr Black Hole

Rotational Speed = Speed of Light
Uniform Circular Motion

- The velocity vector is tangent to the path.
- The change in velocity vector is due to the change in direction.
- The centripetal acceleration changes the direction of motion:
  \[ \alpha = \frac{\Delta v}{\Delta t} \]
Period & Frequency of Rotation

In one period \( T \), the object travels around the circumference of the circle, a distance \( 2\pi R \).

PERIOD = Time for one revolution
\[
T = \frac{\text{TIME}}{\text{REVOLUTION}}
\]
\[
T = \underline{\text{sec}}
\]

FREQUENCY = Number of revolutions in one unit of time
\[
f = \frac{\text{REVOLUTION}}{\text{TIME}}
\]
\[
f = \underline{\text{Hz}} \quad \text{Hz} = \text{sec}^{-1}
\]

\[
T = \frac{1}{f}
\]
\[
f = \frac{1}{T}
\]

What is the period of the Earth? Moon?
Centripetal Acceleration

- The acceleration is always perpendicular to the path of the motion.
- The acceleration always points toward the center of the circle of motion.
- This acceleration is called the *centripetal acceleration*.
- Magnitude is given by: 
  \[ a_C = \frac{v^2}{r} \]
Circular Motion: centripetal acceleration

The velocity $\vec{v}$ is always tangent to the circle and perpendicular to $\vec{a}$ at all points.

The acceleration $\vec{a}$ always points toward the center of the circle.

There is an acceleration because the velocity is changing direction.

$$\vec{a} = \left( \frac{v^2}{r}, \text{toward center of circle} \right)$$

Centripetal acceleration of object moving in a circle of radius $r$ at speed $v$.
Centripetal and Centrifugal

Center SEEKING and Center FLEEING

\[ F_{\text{actual}} = \text{Centripetal Force} \]
\[ F_{\text{fictitious}} = \text{Centrifugal Force} \]
In Physics, we use **ONLY** Centripetal acceleration **NOT** Centrifugal acceleration!
5.3 Centripetal Force

Example 5: The Effect of Speed on Centripetal Force

The model airplane has a mass of 0.90 kg and moves at constant speed on a circle that is parallel to the ground. The path of the airplane and the guideline lie in the same horizontal plane because the weight of the plane is balanced by the lift generated by its wings. Find the tension in the 17 m guideline for a speed of 19 m/s.

\[ F_c = T = m \frac{v^2}{r} \]

\[ T = (0.90 \text{ kg}) \frac{(19 \text{ m/s})^2}{17 \text{ m}} = 19 \text{ N} \]
The Earth rotates once per day around its axis as shown. Assuming the Earth is a sphere, is the rotational speed at Santa Rosa greater or less than the speed at the equator?

- 366 m/s
- 464 m/s
The Earth rotates once per day around its axis. Assuming the Earth is a sphere with radius $6.38 \times 10^6$ m, find the tangential speed of a person at the equator and at 38 degrees latitude (Santa Rosa!) and their centripetal accelerations.

At the equator, $r = 6.38 \times 10^6$ m:

$$v = \frac{2\pi r}{\Delta t} = \frac{2\pi (6.38 \times 10^6 \text{ m})}{86,400 \text{ s}} = 464 \text{ m/s}$$

$$a_c = \frac{v^2}{r} = \frac{(464 \text{ m/s})^2}{6.38 \times 10^6 \text{ m}} = 0.034 \text{ m/s}^2$$

At Santa Rosa, $r = 6.38 \times 10^6$ m $\cos38^\circ$:

$$v = \frac{2\pi r}{\Delta t} = \frac{2\pi (6.38 \times 10^6 \text{ m}) \cos38^\circ}{86,400 \text{ s}} = 366 \text{ m/s}$$

$$a_c = \frac{v^2}{r \cos38} = \frac{(366 \text{ m/s})^2}{6.38 \times 10^6 \text{ m} \cos38} = 0.027 \text{ m/s}^2$$
Is your apparent weight as measured on a spring scale more at the Equator or at Santa Rosa?
Since you are standing on the Earth (and not in the can) the centrifugal force tends to throw you off the Earth. You weigh less where the centripetal force is greatest because that is also where the centrifugal force is greatest – the force that tends to throw you out of a rotating reference frame.
Centripetal & Centrifugal Force Depends on Your Reference Frame

Outside Observer (non-rotating frame) sees Centripetal Force pulling can in a circle.

Inside Observer (rotating reference frame) feels Centrifugal Force pushing them against the can.
Centrifugal Force is Fictitious?

The centrifugal force is a real effect. Objects in a rotating frame feel a centrifugal force acting on them, trying to push them out. This is due to your inertia – the fact that your mass does not want to go in a circle. The centrifugal force is called ‘fictitious’ because it isn’t due to any real force – it is only due to the fact that you are rotating. The centripetal force is ‘real’ because it is due to something acting on you like a string or a car.
Important: Inside vs Outside the Rotating Frame
Example 13: Artificial Gravity

At what speed must the surface of the space station move so that the astronaut experiences a push on his feet equal to his weight on earth? The radius is 1700 m.

\[
F_c = m \frac{v^2}{r} = mg
\]

\[
v = \sqrt{rg}
\]

\[
= \sqrt{(1700 \text{ m})(9.80 \text{ m/s}^2)}
\]

\[
= 130 \text{ m/s}
\]
Example 13: Artificial Gravity

At what speed must the surface of the space station move so that the astronaut experiences a push on his feet equal to his weight on earth? The radius is 1700 m.

\[ v = 130 \text{ m/s} \]

What is the period of rotation?
Space Station Rotation

A space station of diameter 80 m is turning about its axis at a constant rate. If the acceleration of the outer rim of the station is 2.5 m/s², what is the period of revolution of the space station?

a. 22 s  
b. 19 s  
c. 25 s  
d. 28 s  
e. 40 s
Horizontal (Flat) Curve

- The force of static friction supplies the centripetal force
  \[ \sum F_c = f = \frac{mv^2}{r} \]

- The maximum speed at which the car can negotiate the curve is
  \[ v = \sqrt{\mu gr} \]

- Note, this does not depend on the mass of the car

\[ \sum F_y = N - mg = 0 \]

\[ f = \mu mg \]
A highway curve has a radius of 0.14 km and is unbanked. A car weighing 12 kN goes around the curve at a speed of 24 m/s without slipping. What is the magnitude of the horizontal force of the road on the car? What is $\mu$? Draw FBD.

a. 12 kN  
b. 17 kN  
c. 13 kN  
d. 5.0 kN  
e. 49 kN
Example Problem

A level curve on a country road has a radius of 150 m. What is the maximum speed at which this curve can be safely negotiated on a rainy day when the coefficient of friction between the tires on a car and the road is 0.40?
5.4 Banked Curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car’s weight.
5.4 Banked Curves

\[ F_c = F_N \sin \theta = m \frac{v^2}{r} \]

\[ F_N \cos \theta = mg \]
5.4 Banked Curves

\[ F_N \sin \theta = m \frac{v^2}{r} \]

\[ F_N \cos \theta = mg \]

\[ \tan \theta = \frac{v^2}{rg} \]
5.4 Banked Curves

Example 8: The Daytona 500

The turns at the Daytona International Speedway have a maximum radius of 316 m and are steeply banked at 31 degrees. Suppose these turns were frictionless. At what speed would the cars have to travel around them in order to remain on the track?

\[
\tan \theta = \frac{v^2}{rg} \quad \Rightarrow \quad v = \sqrt{rg \tan \theta}
\]

\[
v = \sqrt{(316 \text{ m})(9.8 \text{ m/s}^2) \tan 31^\circ} = 43 \text{ m/s} \ (96 \text{ mph})
\]
Vertical Circle has Non-Uniform Speed

Where is the speed Max? Min?
Where is the Tension Max? Min?
Example Problem: Loop-the-Loop

A roller coaster car goes through a vertical loop at a constant speed. For positions A to E, rank order the:

- centripetal acceleration
- normal force
- apparent weight
What is the maximum speed the vehicle can have at B and still remain on the track?
Maximum Speed for Vertical Circular Motion

What is the maximum speed the car can have as it passes this highest point without losing contact with the road?

Max speed without losing contact MEANS:

\[ \text{Take: } n = 0 \]

Therefore:

\[ mg = \frac{m v^2}{r} \]

\[ v = \sqrt{gr} \]

Maximum Speed to not lose contact with road only depends on R! ROOT GRRRRRRRR
What is the maximum speed the vehicle can have at B and still remain on the track?

\[ v = \sqrt{gr} \]
Humps in the Road: Outside the Vertical Loop

A car that’s out of gas coasts over the top of a hill at a steady 20 m/s. Assume air resistance is negligible. Which free-body diagram describes the car at this instant?

- [Image of free-body diagrams A, B, C, D, E]
Humps in the Road: Outside the Vertical Loop

A car that’s out of gas coasts over the top of a hill at a steady 20 m/s. Assume air resistance is negligible. Which free-body diagram describes the car at this instant?

Now the centripetal acceleration points down.
A roller coaster car does a loop-the-loop. Which of the free-body diagrams shows the forces on the car at the top of the loop? Rolling friction can be neglected.

A. 
B. 
C. 
D. 
E.
A roller coaster car does a loop-the-loop. Which of the free-body diagrams shows the forces on the car at the top of the loop? Rolling friction can be neglected.

The track is above the car, so the normal force of the track pushes down.

Options:

A.  
B.  
C.  
D.  
E.  

Correct Answer: E.
A roller-coaster car has a mass of 500 kg when fully loaded with passengers. At the bottom of a circular dip of radius 40 m (as shown in the figure) the car has a speed of 16 m/s. What is the magnitude of the force of the track on the car at the bottom of the dip?

a. 3.2 kN  
b. 8.1 kN  
c. 4.9 kN  
d. 1.7 kN  
e. 5.3 kN
Projectile Motion/Orbital Motion

Projectile Motion is Orbital motion that hits the Earth!
Curvature of Earth

Curvature of the Earth: Every 8000 m, the Earth curves by 5 meters!

If you threw the ball at 8000 m/s off the surface of the Earth (and there were no buildings or mountains in the way) how far would it travel in the vertical direction in 1 second?

$$\Delta y = \frac{1}{2} gt^2 \sim 5 m/s^2 \cdot (1 s^2) = 5 m$$

The ball will achieve orbit.
Orbital Velocity

If you can throw a ball at 8000m/s, the Earth curves away from it so that the ball continually falls in free fall around the Earth – it is in orbit around the Earth!

Above the atmosphere

Ignoring air resistance.
Orbital Motion & Escape Velocity

8km/s: Circular orbit
Between 8 & 11.2 km/s: Elliptical orbit
11.2 km/s: Escape Earth
42.5 km/s: Escape Solar System!
There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.

\[ F = \frac{m_s M_E G}{r^2} \]

\[ a = \frac{v^2}{r} \]

\[ F = m_s a \]
Orbit Question

Find the orbital speed of a satellite 200 km above the Earth. Assume a circular orbit. \( M_E = 5.97 \times 10^{24} \text{ kg}, R_E = 6.38 \times 10^6 \text{ m} \)

\[
F = \frac{m_s M_E G}{r^2} = m_s \frac{v^2}{r} \]

\[
F = m_s a \quad a = \frac{v^2}{r}
\]

\[
v = \sqrt{\frac{M_E G}{R_E + h}}
\]

\[
v = \sqrt{\frac{(5.97 \times 10^{24} \text{ kg})(6.67 \times 10^{-11} \text{ Nm}^2 / \text{ kg}^2)}{6.58 \times 10^6 \text{ m}}}
\]

\[
v = 7.78 \times 10^3 \text{ m/s}
\]

Notice that this is less 8 km/s!
Orbit Question

What is the period of a satellite orbiting 200 km above the Earth? Assume a circular orbit.

\[ M_E = 5.97 \times 10^{24} \text{ kg}, \quad R_E = 6.38 \times 10^6 \text{ m} \]

\[ v = \frac{2\pi r}{T} \]

\[ T = \frac{2\pi r}{v} \]

\[ = \frac{2\pi (6.58 \times 10^6 \text{ m})}{7.78 \times 10^3 \text{ m/s}} \]

\[ T = 5314 \text{ s} = 88 \text{ min} \]

If you don’t know the velocity:

\[ F = \frac{GMm}{r^2} = m \frac{v^2}{r} \]

\[ = m \frac{(2\pi r / T)^2}{r} \]

\[ T^2 = \frac{4\pi^2}{GM} r^3 \]

Kepler’s 3\textsuperscript{rd}!

Period increases with \( r \)!
If an object is some distance \( h \) above the Earth’s surface, \( r \) becomes \( R_E + h \).

The tangential speed of an object is its orbital speed and is given by the centripetal acceleration, \( g \):

\[
\frac{v^2}{r} = g
\]

Orbital speed decreases with increasing altitude!

\[
v = \sqrt{\frac{GM_E}{(R_E + h)}}
\]
5.5 Satellites in Circular Orbits

What altitude for a Geosynchronous orbit?

\[ T = 24 \text{ hours} \quad \quad T = \frac{2\pi r^{3/2}}{\sqrt{GM_E}} \]
Additional Questions

A satellite orbits the earth. A Space Shuttle crew is sent to boost the satellite into a higher orbit. Which of these quantities increases?

A. Speed  
B. Angular speed  
C. Period  
D. Centripetal acceleration  
E. Gravitational force of the earth
A satellite orbits the earth. A Space Shuttle crew is sent to boost the satellite into a higher orbit. Which of these quantities increases?

A. Speed
B. Angular speed
C. **Period**
D. Centripetal acceleration
E. Gravitational force of the earth
Types of Orbits

- Low-Earth Orbit
- Polar orbit
- Eccentric orbit
- Geostationary orbit
Speed in Circular Earth Orbit

Altitude (Log scale)

100,000 km
10,000 km
1,000 km
100 km
10 km
0 km

The Moon
GEO
GPS
Lageos
HST
ISS

Put mouse pointer on a feature for info

Jet plane going east at the equator

Earth's surface at the equator
1071 operational satellites in orbit around the Earth. 50 percent of which were launched by the United States. Half of that 1071 are in Low-Earth Orbit, just a few hundred kilometers above the surface.
https://www.youtube.com/watch?v=3AzhmNQfVR8
THE GROWING PROBLEM OF SPACE DEBRIS

Debris in Low Earth Orbit
- 10 cm and Larger = 20,000+ objects
- 1-10 cm = 500,000 objects
- Under 1 cm = over 10 million untrackable objects

Low Earth Orbit
approx. 160 to 2,000 km asl

International Space Station
approx. 300 to 400 km asl

Tracking and Speed
- Most debris is tracked using radar by the US Department of Defense.
- Orbiting debris moving over 28,000 kph. (23 times the speed of sound)

Potential Damage
At high speeds, even paint flecks can cause serious damage to satellites, spacecraft and the ISS. Every collision creates more debris, producing a cascading problem.

GEO and the larger problem
The world depends on satellites in geosynchronous Earth orbit for communications and GPS. But debris is largely untrackable at this orbit, meaning that cascading collisions could make it unusable.
Satellites and debris in low Earth orbit, 1960-2010. Courtesy NASA.
Dealing with space debris

https://www.youtube.com/watch?v=eYVsVRgiS0w
War in Space
SPACE ECOLOGY
THE FINAL FRONTIER
OF ENVIRONMENTALISM

By Lynda Williams

First there was the “Big Ocean Theory”, which basically meant that the ocean is so big that humans could dump any amount of waste into it without environmental consequence. Of course, that theory has proven to be false as ocean ecosystems today suffer from dying coral reefs and fish populations poisoned with mercury and other pollutants. Next came the “Big Atmosphere Theory”, which assumed that we could belch out billions of tons of air pollution and carbon dioxide from our smoke stacks and tail pipes without environmental repercussions. We all know how that idea has impacted the planet: air pollution, acid rain, ozone depletion and global warming.

Now we have a “Big Space Theory”; namely, that space is so big that the waste we create in it will cause no harm. That’s right folks, fifty years after Sputnik launched the space age, humans have turned space into yet another junk yard, with millions of pieces of manmade debris orbiting the Earth. The space debris problem is becoming so critical that space may become too trashed to use at all. What the world needs now, before it becomes too late, is an environmental movement in heaven: Space Ecology.