Chapter 3: Vectors and Motion in Two Dimensions - TOO MUCH

• Vectors
• Projectile Motion
• Relative Motion
• Motion on a Ramp (next week)
• Circular Motion (with Ch 6)
Physics 20
Vectors!!!

Figure 3 — Sketch of reference frame $R_0$. Vectors $V_1$ and $V_2$ define the longitudinal axes of upper arm and forearm, respectively.

(b) Free-body diagram for the foot pulley
Cartesian Coordinate System

- Also called rectangular coordinate system
- $x$- and $y$- axes intersect at the origin
- Points are labeled $(x, y)$
Polar Coordinate System

- Origin and reference line are noted
- Point is distance $r$ from the origin in the direction of angle $\theta$, ccw from reference line
- Points are labeled $(r, \theta)$
Cartesian to Polar Coordinates

- \( r \) is the hypotenuse and \( \theta \) an angle

\[
\tan \theta = \frac{y}{x}
\]

\[
r = \sqrt{x^2 + y^2}
\]

\( \theta \) must be ccw from positive \( x \) axis for these equations to be valid
Example

• The Cartesian coordinates of a point in the $xy$ plane are $(x,y) = (-3.50, -2.50)$ m, as shown in the figure. Find the polar coordinates of this point.

Solution:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.50 \text{ m})^2 + (-2.50 \text{ m})^2} = 4.30 \text{ m}$$

$$\tan^{-1}\left(\frac{-2.50 \text{ m}}{-3.50 \text{ m}}\right) = 35.5^\circ$$

$$\theta = 180^\circ + 35.5^\circ = 216^\circ$$
Polar to Cartesian Coordinates

- Based on forming a right triangle from $r$ and $\theta$
- $x = r \cos \theta$
- $y = r \sin \theta$

\[
\sin \theta = \frac{y}{r}
\]
\[
\cos \theta = \frac{x}{r}
\]
\[
\tan \theta = \frac{y}{x}
\]
Vectors

Vector quantities have both magnitude and direction.

The length of the vector represents the *magnitude* of the vector. The orientation represents the *direction* or angle of the vector.

Example: velocity of 2 km/hr, 30 degrees north of east. 2 km/hr is the magnitude, 30 degrees north or east, the direction. Scalars only have magnitude: $T = 82$ degrees Celsius.
Writing Vectors

• Text books use BOLD. Hard to write bold!
• The vector is written with an arrow over head and includes both magnitude and direction.

• Magnitude (length of vector) is written with no arrow or as an absolute value:

$$|\vec{A}| = A = \text{magnitude} = 2\text{km}$$

$$\vec{A} = (2\text{km}, 30^\circ \text{NE})$$
Vector Example

• A particle travels from A to B along the path shown by the dotted red line
  – This is the *distance* traveled and is a scalar
• The *displacement* is the solid line from A to B
  – The displacement is independent of the path taken between the two points
  – Displacement is a vector
Equality of Two Vectors

- Two vectors are **equal** if they have the same magnitude and the same direction.
- \( \mathbf{A} = \mathbf{B} \) if \( A = B \) and they point along parallel lines.
- All of the vectors shown are equal.
WARNING!

Vector Algebra is Weird!!

Vectors have a different rules of operation for addition, subtraction, multiplication and division than ordinary real numbers!!!! Since vectors have magnitude and direction you can not always just simply add them!!!!

A & B co-linear

A & B NOT colinear
Adding Vectors
The Graphical Method

• When you have many vectors, just keep repeating the process until all are included.

• The resultant is still drawn from the origin of the first vector to the end of the last vector.

\[ R = A + B + C + D \]
Adding Vectors

Head to Tail

\[ R = A + B = B + A \]

When two vectors are added, the sum is independent of the order of the addition.

- This is the *commutative law of addition*
- \( A + B = B + A \)
- The sum forms the diagonal of a *Parallelogram*!
Adding Vectors
The Graphical Method

- Draw vectors to scale.
- Draw the vectors “Tip to Tail.”
- **IMPORTANT:** the angle of a vector is relative its own tail!
- The resultant, R, is drawn from the tail of the first to the head of the last vector.
- Use a ruler to *MEASURE* the resultant length.
- Use a protractor to *MEASURE* the resultant angle.
Each of the displacement vectors $\mathbf{A}$ and $\mathbf{B}$ shown has a magnitude of 3.00 m. Find graphically
(a) $\mathbf{A} + \mathbf{B}$,
(Report all angles counterclockwise from the positive $x$ axis.)

Adding Vectors:
Graphical Method

5.2 m, 60°
Subtraction of Vectors

\[ A = C + (-B) \]
Parallelogram Method
Addition & Subtraction

\[ R = X + Y \]

\[ R = X - Y \]
Subtracting Vectors: Graphical Method

Each of the displacement vectors \( \mathbf{A} \) and \( \mathbf{B} \) shown has a magnitude of 3.00 m. Find graphically (a) \( \mathbf{A} + \mathbf{B} \), (b) \( \mathbf{A} - \mathbf{B} \), (c) \( \mathbf{B} - \mathbf{A} \)

Report all angles counterclockwise from the positive \( x \) axis.

\( 3 \text{m, } 330^\circ \)

\( 3 \text{m, } 150^\circ \)
Vector Components

\[ A_x = A \cos \theta_A \]

\[ A_y = A \sin \theta_A \]

\[ A = \sqrt{A_x^2 + A_y^2} \]

\[ \theta_A = \tan^{-1}\left(\frac{A_y}{A_x}\right) \]

\[ \vec{A} = (A_x, A_y) \]

\[ \vec{A} = (A, \theta_A) \]
Example

If the magnitude of $\mathbf{r}$ is 175m, find the magnitude of components $x$ and $y$.

\[ |x| = r_x = r \cos \theta_r \]
\[ = 175 \, m \cos 50^\circ \]
\[ |x| = 113 \, m \]

\[ |y| = r_y = r \sin \theta_r \]
\[ = 175 \, m \sin 50^\circ \]
\[ |y| = 134 \, m \]
Vector Addition
Components Method

\[ C = A + B \]
Vector Addition
Components Method

\[ C = A + B \]

\[ C_x = A_x + B_x \]

\[ C_y = A_y + B_y \]
Vector Addition
Components Method

\[ C = A + B \]

\[ |C| = \sqrt{C_x^2 + C_y^2} \]

\[ \theta_C = \tan^{-1}\left( \frac{C_y}{C_x} \right) \]

\[ C_x = A_x + B_x \]

\[ C_y = A_y + B_y \]
Adding Vectors: Component Method

Each of the displacement vectors $\mathbf{A}$ and $\mathbf{B}$ shown has a magnitude of 3.00 m. Find using components
(a) $\mathbf{A} + \mathbf{B}$,  (b) $\mathbf{A} - \mathbf{B}$,  (c) $\mathbf{B} - \mathbf{A}$,  (d) $\mathbf{A} - 2\mathbf{B}$.
Report all angles counterclockwise from the positive $x$ axis.

$$A_x = 3.00 \text{ m cos}(30.0^\circ) = 2.60 \text{ m}$$
$$A_y = 3.00 \text{ m sin}(30.0^\circ) = 1.50 \text{ m}$$
$$B_x = 0, \quad B_y = 3.00 \text{ m}$$

$$R_x = A_x + B_x = 2.60 \text{ m} \quad R_y = A_y + B_y = 4.50 \text{ m}$$

$$R = \sqrt{(2.60 \text{ m})^2 + (4.50 \text{ m})^2} = 5.20 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{4.5 \text{ m}}{2.6 \text{ m}}\right) = 60^\circ$$
Adding Vectors: Compents Method

Each of the displacement vectors $\mathbf{A}$ and $\mathbf{B}$ shown has a magnitude of 3.00 m. Find graphically (a) $\mathbf{A} + \mathbf{B}$, (b) $\mathbf{A} - \mathbf{B}$, (c) $\mathbf{B} - \mathbf{A}$ and Report all angles counterclockwise from the positive $x$ axis.

(a) 5.2 m, 60°  
(b) 3 m, 330°  
(c) 3 m, 150°
Problem

Three displacement vectors of a croquet ball are shown, where $|A| = 20.0 \text{ m}$, $|B| = 40.0 \text{ m}$, and $|C| = 30.0 \text{ m}$. Find (a) the resultant components (b) the magnitude and direction of the resultant displacement.

\[ R_x = A_x + B_x + C_x \quad \text{and} \quad R_y = A_y + B_y + C_y \]

\[ R_x = 40.0 \cos 45.0^\circ m + 30.0 \cos 45.0^\circ m = 49.5 m \]

\[ R_y = 40.0 \sin 45.0^\circ m - 30.0 \sin 45.0^\circ m + 20.0 m = 27.1 m \]

(a) \( R = (49.5 m, 27.1 m) \)

(b) \[ |R| = \sqrt{(49.5 m)^2 + (27.1 m)^2} = 56.4 m \quad \theta = \tan^{-1}\left(\frac{27.1}{49.5}\right) = 28.7^\circ \]
Problem

A person going for a walk follows the path shown. The total trip consists of four straight-line paths. At the end of the walk, what is the person's resultant displacement measured from the starting point?

\[
\begin{align*}
\mathbf{d}_1 &= (100m, 0) \\
\mathbf{d}_2 &= (0, -300m) \\
\mathbf{d}_3 &= (-150\cos(30.0^\circ)m, -150\sin(30.0^\circ)m) = (-130m, -75.0m) \\
\mathbf{d}_4 &= (-200\cos(60.0^\circ)m, 200\sin(60.0^\circ)m) = (-100m, 173m)
\end{align*}
\]

\[
\mathbf{R} = \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 + \mathbf{d}_4 = (-130m, -202m)
\]

\[
|\mathbf{R}| = \sqrt{(-130)^2 + (-202)^2} = 240 \text{ m}
\]

\[
\phi = \tan^{-1}\left(\frac{202}{130}\right) = 57.2^\circ
\]

\[
\theta = 180 + \phi = 237^\circ
\]
Subtracting Vectors

- If $A - B$, then use $A + (-B)$
- The negative of the vector will have the same magnitude, but point in the opposite direction
- Two ways to draw subtraction:
The distance between two vectors is equal to the magnitude of the difference between them!

Two points in the $xy$ plane have Cartesian coordinates $(2.00, -4.00)$ m and $(-3.00, 3.00)$ m. Determine (a) the distance between these points and (b) their polar coordinates. Draw the vectors!!!
1. Draw 1st vector tail \( A \) at origin
2. Draw 2nd vector \( B \) with tail at the head of the 1st vector, \( A \). The angle of \( B \) is measured relative to an imaginary axis attached to the tail of \( B \).
3. The resultant is drawn from the tail of the first vector to the head of the last vector.
Subtracting Vectors: Very Important for 2-D Kinematics

\[ \vec{v} = \frac{\Delta \vec{r}}{\Delta t} \]

\[ \vec{a} = \frac{\Delta \vec{v}}{\Delta t} \]
Projectile Motion

1-D vs 2-D
A ball is tossed up in the air. Taking up as the positive direction, at its very highest point, the ball’s instantaneous acceleration $a_y$ is

A. Positive.
B. Negative.
C. Zero.
A ball is tossed up in the air. Taking up as the positive direction, at its very highest point, the ball’s instantaneous acceleration $a_y$ is

A. Positive.

✓ B. Negative.

C. Zero.
Zero at the Top!

Y component of velocity is zero at the top of the path in both cases!

You know they have the y component of velocity is the same in both cases because they reached the same height!
Throwing up is Also Free Fall!
Symmetry of G Field.

\[ a = g \sim 10 \text{ m/s}^2 \]

\[ v_f = v_0 + gt \]
\[ \Delta y = v_0 t + \frac{1}{2} gt^2 \]
What Goes Up Must Come Down

Someone standing at the edge of a cliff throws one ball straight up and one straight down at the same speed. Ignoring air resistance, which ball strikes the ground with the greatest speed?
Free Fall: Throwing Up

What is the speed at the top of the path? 
ZERO!

What is the acceleration at the top? 
a = -9.80 m/s²

What is the velocity at the same height on the way down? 
-30 m/s

With what velocity will the rock hit the ground? 
-59.4 m/s
SAME as if you threw it straight down at 30m/s!
Projectile Motion
 Ignore Air Resistance!

Most Important:
X and Y components are INDEPENDENT of each other!
First the SIMPLE Case: Horizontal Launch

(Ignore Air Resistance)

The x-component doesn’t change (no acceleration in x-direction.)
The y-component changes (a = -g.)
Projectile Motion

Same cannons, Same height. One dropped, One shot. Which hits the ground first? SAME!

Both falling the same height! Horizontal speed doesn’t affect vertical speed or the time to hit the ground!

Only $\Delta y$ determines time!
Projectile Motion
Gravity acts in the vertical direction but not in the horizontal direction!!

Speed in vertical direction speeds up!
Speed in horizontal direction stays the same!
Projectile Motion

No Change

\[ a_x = 0 \]

Actual path is a vector sum of horizontal and vertical motions.

change

\[ a_y = g \]
An airplane traveling at a constant speed and height drops a care package. Ignoring air resistance, at the moment the package hits the ground, where is it relative to the plane?

a) Behind the plane.  

b) Under the plane.  

c) In front of the plane.

Any object dropped from a plane has the same initial velocity as the plane!
Care Package

An airplane moves horizontally with constant velocity of 115 m/s at an altitude of 1050m and drops a care package as shown. How far from the release point does the package land?
Care Package

Strategy: Find the time the package drops to get the horizontal distance. The time to drop is just the free fall time!!! The horizontal displacement takes the same time as it takes the package to drop.

\[ \Delta x = v_x t \]
**Care Package**

**Knowns:**

\[ v_{y0} = 0, \quad v_x = 115 \text{ m/s} \]

\[ \Delta y = -1050 \text{ m}, \quad \Delta x = ? \]

\[ a_y = -9.8 \text{ m/s}^2, \quad a_x = 0 \]

**Strategy:** Find time from \( y \) info to solve for \( \Delta x = v_x t \).

\[ \Delta y = v_0 t + \frac{1}{2} a_y t^2 \rightarrow t = \sqrt{\frac{2\Delta y}{a_y}} \]

\[ t = \sqrt{\frac{2(-1050 \text{ m})}{(-9.8 \text{ m/s}^2)}} \quad t = 14.6 \text{s} \]

\[ \Delta x = v_x t \]

\[ = 115 \text{ m/s}(14.6 \text{s}) \]

\[ \Delta x = 1680 \text{ m} \]

With what velocity does it hit the ground?
Care Package

**Knowns:**
- \( v_{y0} = 0, \quad v_x = 115 \text{ m/s} \)
- \( \Delta y = -1050 \text{ m}, \quad \Delta x = 1680 \text{ m} \)
- \( a_y = -9.8 \text{ m/s}^2, \quad a_x = 0 \)
- \( t = 14.6 \text{ s} \)

**Strategy:** Find final velocity in y direction and use it in:

\[
v = \sqrt{v_x^2 + v_y^2}, \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)
\]

\[
v_{yf} = v_{y0} + a_y t
\]

\[
= 0 + (-9.8 \text{ m/s}^2)(14.6 \text{ s})
\]

\[
= -143 \text{ m/s}
\]

\[
v = \sqrt{(115 \text{ m/s})^2 + (-143 \text{ m/s})^2}
\]

\[
= 184 \text{ m/s}
\]

\[
\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-143 \text{ m/s}}{115 \text{ m/s}}\right)
\]

\[
\theta = -51.3^\circ
\]

\[\bar{v} = (184 \text{ m/s}, -51.3^\circ)\]
You Try

The ball is thrown horizontally at 20 m/s. About how long does it take to hit the ground?

\[ \Delta y = v_{yi} t + \frac{1}{2} gt^2 \]

Solving for time, \( t = \sqrt{\frac{2\Delta y}{g}} = 1\text{s} \)

How far does it travel in the horizontal direction?

\[ \Delta x = v_{xi} t = 20 \frac{m}{s} \times 1\text{s} = 20\text{m} \]
The ball is thrown horizontally at 30 m/s. About how long does it take to hit the ground?

\[ g = 10 \text{ m/s}^2 \]

\[ \Delta y = v_{yi}t + \frac{1}{2}gt^2 \]

\[ t = \sqrt{\frac{2\Delta y}{g}} = 1 \text{s} \]

How far does it travel in the horizontal direction?

\[ \Delta x = v_{xi}t = 30 \frac{m}{s} \cdot 1 \text{s} = 30 \text{m} \]
The ball is thrown horizontally at 100 m/s. About how long does it take to hit the ground?

\[
\Delta y = v_{yi}t + \frac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{\frac{2\Delta y}{g}} = 1\text{s}
\]

Only \( \Delta y \) determines time!

How far does it travel in the horizontal direction?

\[
\Delta x = v_{xi}t = 100 \frac{m}{s} \times 1\text{s} = 100\text{m}
\]
NO MATTER HOW FAST YOU THROW THE BALL IN THE HORIZONTAL DIRECTION the time it takes to drop only depends on the height from which it was thrown!!!!!! The x and y components of motion are independent of each other!! But a strange thing happens on the Earth because it is not FLAT....
Curvature of Earth

Curvature of the Earth: Every 8000 m, the Earth curves by 5 meters!
Curvature of Earth

If you threw the ball at 8000 m/s off the surface of the Earth (and there were no buildings or mountains in the way) how far would it travel in the vertical and horizontal directions in 1 second?

Does the ball ever hit the Earth??
Curvature of Earth

If you threw the ball at 8000 m/s off the surface of the Earth (and there were no buildings or mountains in the way) how far would it travel in the vertical and horizontal directions in 1 second?

\[
\text{horizontal} : \quad \Delta x = v_x t = (8000 \text{ m/s})(1 \text{ s}) = 8000 \text{ m}
\]

\[
\text{vertical} : \quad \Delta y = \frac{1}{2} gt^2 = 5t^2 = 5(1 \text{ s})^2 = 5 \text{ m}
\]
Orbital Velocity

If you can throw a ball at 8000m/s, the Earth curves away from it so that the ball continually falls in free fall around the Earth – it is in orbit around the Earth!

Above the atmosphere

Ignoring air resistance.
Orbital Motion & Escape Velocity

8 km/s: Circular orbit
Between 8 & 11.2 km/s: Elliptical orbit
11.2 km/s: Escape Earth
42.5 km/s: Escape Solar System!
Projectile Motion IS Orbital Motion

The Earth is in the way!
Projectiles Launched at an Angle: The simple case: $\Delta y=0$
Projectile Motion

A place kicker kicks a football at an angle of 40 degrees above the horizontal with an initial speed of 22 m/s. Ignore air resistance and find the total time of flight, the maximum height and the range the ball attains.

$H = \text{Maximum height}$

$R = \text{Range}$
Projectile Motion
Launched at an Angle

(Ignore Air Resistance)

The x-component doesn’t change (no acceleration in x-direction.)
The y-component changes (a = -g.)
Sample Problem!

What is the Range of motion?

\[ x_f = u_{xi} t \]

GET t from y info!!!

\[ \Delta y = u_{yi} t + \frac{1}{2} a_y t^2 \]

solve \((-45 = 20 \sin 30^\circ t - 4.9 t^2)\)

\[ t = 4.22 \text{ s} \]

\[ x_f = 20 \text{ m/s} \cos 30^\circ (4.22 \text{ s}) \]

\[ x_f = 73.1 \text{ m} \]
Relative Velocity

- Two observers moving relative to each other generally do not agree on the outcome of an experiment.
- For example, observers A and B below see different paths for the ball and measure different velocities:

\[ \vec{v}_{bB} = \vec{v}_{bA} + \vec{v}_{AB} \]

Velocity of ball relative to observer B

Velocity of A relative to observer B

Velocity of ball relative to observer A
Co-Linear Motion

Just add or subtract the magnitudes of vectors!

\[ \mathbf{v}_{PG} = \mathbf{v}_{PT} + \mathbf{v}_{TG} \]

Notice how the inner subscripts cancel!

\[ \mathbf{v}_{PT} = +2.0 \text{ m/s} \]

\[ \mathbf{v}_{TG} = +9.0 \text{ m/s} \]

\[ \mathbf{v}_{PG} = +11.0 \text{ m/s} \]
2D Relative Velocity

The boat can travel 2.50 m/s relative to the river. The river current flows at 1.00 m/s relative to the Earth. What is the total velocity of the boat relative to the Earth (shore = Earth)?

\[ \vec{v}_{bE} = \vec{v}_{br} + \vec{v}_{rE} \]

\[ v_{bE} = \sqrt{(2.5 \text{m/s})^2 + (1 \text{m/s})^2} = 2.69 \text{m/s} \]

\[ \theta = \tan^{-1}\left(\frac{1 \text{m/s}}{2.5 \text{m/s}}\right) = 21.8^\circ \]
Relative Velocity Again

The boat can travel 10 m/s relative to the river. The river current flows at 5.00 m/s relative to the shore. If the boat wants to travel straight across, what must be his heading? What is its total speed?

\[ \vec{v}_{bE} = \vec{v}_{br} + \vec{v}_{rE} \]

From the triangle:

\[ \theta = \sin^{-1}\left(\frac{5 \text{ m/s}}{10 \text{ m/s}}\right) = 30^\circ \]

\[ v_{br}^2 = v_{bE}^2 + v_{rE}^2 \]

\[ v_{bE} = \sqrt{(10 \text{ m/s})^2 - (5 \text{ m/s})^2} = 8.66 \text{ m/s} \]
Train Rain

A person looking out the window of a stationary train notices that raindrops are falling vertically down at a speed of 5.00 m/s relative to the ground. When the train moves at a constant velocity, the rain drops make an angle of 25 degrees when the move past the window, as the drawing shows. How fast is the train moving?

We know:

\[ v_{RT} = v_{RG} + v_{GT} \]

\[ v_{RG} - v_{TG} \]

We want \( v_{TG} \):

\[ v_{TG} = v_{RG} \tan 25^\circ \]

\[ = \frac{5 \text{m/s}}{\tan 25^\circ} = 2.33 \text{m/s} \]
You Try Rain

A car travels in a due northerly direction at a speed of 55 km/h. The traces of rain on the side windows of the car make an angle of 60 degrees with respect to the horizontal. If the rain is falling vertically with respect to the earth, what is the speed of the rain with respect to the earth?

a. 48 km/h
b. 95 km/h
c. 58 km/h
d. 32 km/h
e. 80 km/h
Previous problems all involved right triangles….how do you solve if you don’t have right triangle relationship between relative velocities?
Using vector component addition

A ferry boat is traveling in a direction 35.1 degrees north of east with a speed of 5.12 m/s relative to the water. A passenger is walking with a velocity of 2.71m/s due east relative to the boat. What is the velocity of the passenger with respect to the water? Determine the angle relative to due east.

\[ v_{PW} = v_{PB} + v_{BW} \]

\[ v_{PW} = \sqrt{(6.9)^2 + (2.94)^2} \text{ m/s} = 7.50 \text{ m/s} \]

\[ \theta_{PW} = \tan^{-1}\left(\frac{2.94}{6.9}\right) = 23.1^\circ \]
Relative Velocity

A plane is moving at 45 m/s due north relative to the air, while its velocity relative to the ground is 38.0 m/s, 20 degrees west of north. What is the velocity of the wind relative to due west?

\[
\vec{v}_{\text{plane/ground}} = \vec{v}_{\text{plane/wind}} + \vec{v}_{\text{wind/ground}}
\]

One Method: Use Law of Cosines:

\[
c^2 = a^2 + b^2 - 2ab \cos 20^\circ
\]

\[
c = \sqrt{38^2 + 45^2 - 2(38)(45) \cos 20^\circ}
\]

\[
\vec{v}_w = 16 \text{ m} / \text{s}
\]

\[
\theta = \cos^{-1} \left( \frac{38 \sin(20) \text{ m} / \text{s}}{16 \text{ m} / \text{s}} \right) = 35.7^\circ
\]
While driving a car in the rain falling straight down relative to the ground, the rear window can remain dry! Why?

We know the velocity of the raindrop and the car relative to the ground. To determine whether the raindrop hits the window we need to consider the velocity of the raindrop relative to the car:

\[ v_{RC} = v_{RG} + v_{GC} \]
\[ = v_{RG} - v_{CG} \]

From the vector diagram of the relative velocities:

\[ \theta_R = \tan^{-1}\left( \frac{v_{CG}}{v_{RG}} \right) \]

If the direction of the rain relative to the car, \( \theta_R \), is greater than the angle of the rear window, \( \theta_w \), the rain will not hit the rear window! The faster the car, the greater the angle of the rain and the rear window can remain dry!
Maximum range is achieved at a launch angle of 45°!
Symmetry in the Projectile Range

$\sin 2\theta$ is symmetric about 45°

Range Equation:

$$R = \frac{v_i^2 \sin 2\theta_i}{g}$$
Same rock, same speed, same angle.

Which rock hits the water first?
a) Rock 1           b) Rock 2           c) same

Which rock hits the water with the greatest speed?
 a) Rock 1           b) Rock 2           c) same
Same rock, same speed, same angle.

Which rock hits the water first?
(a) Rock 1  (b) Rock 2  (c) same

Which rock hits the water with the greatest speed?
a) Rock 1  b) Rock 2  (c) same

Spatial Symmetry In G Field!