Q14.1. **Reason:** There are many examples in daily life, such as a mass hanging from a spring, a tennis ball being volleyed back and forth, washboard road bumps, a beating heart, AC electric voltage, or a pendulum swinging.

**Assess:** The study of oscillations is important precisely because they occur so often in nature.

Q14.2. **Reason:** Frequency is the rate at which an event is occurring. Since the heartbeat is given as beats per minute, this is a frequency.

**Assess:** Any description of an event that tells you its rate of occurrence is a frequency.

Q14.3. **Reason:** We are given the graph of $x$ versus $t$. However, we want to think about the slope of this graph to answer velocity questions.

(a) When the $x$ versus $t$ graph is increasing, the particle is moving to the right. It has maximum speed when the positive slope of the $x$ versus $t$ graph is greatest. This occurs at 0 s and 4 s.

(b) When the $x$ versus $t$ graph is decreasing, the particle is moving to the left. It has maximum speed when the negative slope of the $x$ versus $t$ graph is greatest. This occurs at 2 s and 6 s.

(c) The particle is instantaneously at rest when the slope of the $x$ versus $t$ graph is zero. This occurs at 1 s, 3 s, 5 s, and 7 s.

**Assess:** This is reminiscent of material studied in Chapter 2; what is new is that the motion is oscillatory and the graph periodic.

Q14.4. **Reason:** Since the energy and amplitude of an oscillator are related by $E = kA^2/2$, we see that the amplitude is proportional to the square root of $E$ ($A = \sqrt{E/k}$). If $E$ is doubled, $A$ will be increased by a factor of $\sqrt{2}$. That is $A_{\text{new}} = \sqrt{2}A_{\text{old}}$. What we have done amounts to a qualitative analysis. We inspect the relationship and write down the answer. Some students find this difficult to do. If that is the case, you should attempt to solve the question in this manner and then use a more quantitative approach to convince yourself that your qualitative answer is correct. For example, write the relationship for the old and the new case as follows:

$$E_{\text{old}} = kA_{\text{old}}^2/2 \quad \text{and} \quad E_{\text{new}} = kA_{\text{new}}^2/2$$

Divide the new expression by the old one, solve for $A_{\text{new}}$, and insert $E_{\text{new}} = 2E_{\text{old}}$ to obtain

$$A_{\text{new}} = A_{\text{old}}\sqrt{E_{\text{new}}/E_{\text{old}}} = A_{\text{old}}\sqrt{2E_{\text{old}}/E_{\text{old}}} = \sqrt{2}A_{\text{old}}$$

The new amplitude is $\sqrt{2}(20 \text{ cm}) = 28 \text{ cm}$.

**Assess:** The qualitative method above requires more insight and the quantitative method requires more time. Both skills are needed and complement each other.

Q14.5. **Reason:** The maximum speed of the block is directly proportional to the amplitude. $v_{\text{max}} = 2\pi fA$.

So doubling the amplitude will double the maximum speed to 40 cm/s.

**Assess:** We assumed the frequency didn’t change.
Q14.6. **Reason:** The maximum kinetic energy is the same as the total mechanical energy. The total energy and amplitude of an oscillator are related by \( E = kA^2/2 \), we see that the energy is proportional to the square of the amplitude. If \( A \) is doubled, \( E \) will increase by a factor of four. That is, \( E_{\text{new}} = 4E_{\text{old}} = 4(2\ J) = 8\ J \).

**Assess:** This question may also be answered using a more quantitative approach as outlined in Question 14.4.

Q14.7. **Reason:** Telling us that the initial elongation is doubled is a way of telling us that the amplitude is doubled. Since energy is conserved, the maximum potential energy is equal to the maximum kinetic energy.

\[
kA^2/2 = \frac{mv_{\text{max}}^2}{2}
\]

Solving this for the maximum speed we obtain the following:

\[
v_{\text{max}} = A\sqrt{\frac{k}{m}}
\]

The above expression informs us that the maximum speed is proportional to the amplitude: If the amplitude doubles, the speed will double, hence

\[
(v_{\text{max}})_{\text{new}} = 2(v_{\text{max}})_{\text{old}} = 2(0.3\ \text{m/s}) = 0.6\ \text{m/s}
\]

**Assess:** Notice that we are solving the question by inspecting the relationship. This question may also be answered using a more quantitative approach as outlined in Question 14.4.

Q14.8. **Reason:** From the graph the strategy is to determine the period, then use \( f = 1/T \). As is done in Figure 14.4, one can measure the period between two crests; in this case it appears to be 2 s.

\[
f = \frac{1}{T} = \frac{1}{2\ \text{s}} = 0.5\ \text{Hz}
\]

The amplitude is the maximum distance from the equilibrium position. On this graph it appears that \( A = 10\ \text{cm} \).

**Assess:** The amplitude is not the distance from the maximum to the minimum—that would be \( 2A \). See Figure 14.6.

Q14.9. **Reason:** From the graph we read the period as 2 s and the maximum speed as 0.10 m/s. Knowing the period we can determine the frequency as follows:

\[
f = 1/T = 1/2\ \text{s} = 0.50\ \text{Hz}
\]

Knowing the maximum speed and the frequency we can determine the amplitude.

\[
v_{\text{max}} = \omega A = (2\pi/T)A
\]

or

\[
A = v_{\text{max}}T/2\pi = (0.10\ \text{m/s})(2.0\ \text{s})/2\pi = 0.032\ \text{m}
\]

**Assess:** These are fairly typical values for the frequency and amplitude of an oscillator.

Q14.10. **Reason:** The period of a block oscillating on a spring is given in Equation 14.26, \( T = 2\pi\sqrt{m/k} \). We are told that \( T_1 = 2.0\ \text{s} \).

(a) In this case the mass is doubled, \( m_2 = 2m_1 \).

\[
\frac{T_2}{T_1} = \frac{2\pi\sqrt{m_2/k_1}}{2\pi\sqrt{m_1/k_1}} = \sqrt{\frac{2m_1}{m_1}} = \sqrt{2}
\]

So \( T_2 = \sqrt{2}T_1 = \sqrt{2}(2.0\ \text{s}) = 2.8\ \text{s} \).

(b) In this case the spring constant is doubled, \( k_2 = 2k_1 \).

\[
\frac{T_2}{T_1} = \frac{2\pi\sqrt{m_1/k_2}}{2\pi\sqrt{m_1/k_1}} = \sqrt{\frac{k_1}{2k_1}} = \frac{1}{\sqrt{2}}
\]

So \( T_2 = T_1/\sqrt{2} = (2.0\ \text{s})/\sqrt{2} = 1.4\ \text{s} \).
(c) The formula for the period does not contain the amplitude; that is, the period is independent of the amplitude. Changing (in particular, doubling) the amplitude does not affect the period, so the new period is still 2.0 s.

**Assess:** It is equally important to understand what doesn’t appear in a formula. It is quite startling, really, the first time you realize it, that the amplitude doesn’t affect the period. But this is crucial to the idea of simple harmonic motion. Of course, if the spring is stretched too far, out of its linear region, then the amplitude would matter.

**Q14.11. Reason:** The period of a simple pendulum is given in Equation 14.31,  

\[ T = 2\pi\sqrt{\frac{L}{g}} \]  

We are told that \( T_1 = 2.0 \) s.

(a) In this case the mass is doubled, \( m_2 = 2m_1 \). However, the mass does not appear in the formula for the period of a pendulum; that is, the period does not depend on the mass. Therefore the period is still 2.0 s.

(b) In this case the length is doubled, \( L_2 = 2L_1 \).

\[
\frac{T_2}{T_1} = \frac{2\pi\sqrt{L_2/g}}{2\pi\sqrt{L_1/g}} = \sqrt{\frac{L_2}{L_1}} = \sqrt{2}
\]

So \( T_2 = \sqrt{2}T_1 = \sqrt{2}(2.0 \text{ s}) = 2.8 \text{ s} \).

(c) The formula for the period of a simple small-angle pendulum does not contain the amplitude; that is, the period is independent of the amplitude. Changing (in particular, doubling) the amplitude, as long as it is still small, does not affect the period, so the new period is still 2.0 s.

**Assess:** It is equally important to understand what doesn’t appear in a formula. It is quite startling, really, the first time you realize it, that the amplitude doesn’t affect the period. But this is crucial to the idea of simple harmonic motion. Of course, if the pendulum is swung too far, out of its linear region, then the amplitude would matter. The amplitude does appear in the formula for a pendulum not restricted to small angles because the small-angle approximation is not valid; but then the motion is not simple harmonic motion.

**Q14.12. Reason:** If it behaves like a mass on a spring, then trimming the wings will reduce the mass, and this will increase the frequency because \( f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \).

**Assess:** It would be easier to beat a wing quickly if it had less mass.

**Q14.13. Reason:** The relationship between the length of a pendulum, its period and the value of the acceleration due to gravity at that site is given by,  

\[ T = 2\pi\sqrt{\frac{L}{g}} \]  

Inspecting this relationship, we see that the period is inversely proportional to the square root of the acceleration due to gravity. Going from Miami to Denver results in a smaller value of \( g \) and hence a larger value of the period of the clock \( T \). If the period of the clock is increased, it takes longer for an oscillation and the clock will run slower. The clock will run slower in Denver compared to Miami.

**Assess:** Notice that we solved the question by inspecting the relationship between the period and the acceleration due to gravity. This question may also be answered using a more quantitative approach as outlined in Question 14.4.

**Q14.14. Reason:** Reducing the mass increases the frequency because \( f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \). So you would remove water.

**Assess:** Try it!

**Q14.15. Reason:** The leg acts somewhat like a pendulum as it swings forward. By bending their knees to bring their feet up closer to the body, sprinters are shortening the pendulums, which makes them swing faster.

**Assess:** See if you can notice this effect by first running as fast as you can, then again without bending your knees any higher than necessary to clear the ground.

**Q14.16. Reason:** As the gibbons move through the trees they are essentially swinging pendulums. By bending their knees and bringing their feet closer to their body, they are decreasing the length of the pendulum, which in turn decreases the period of the pendulum. Decreasing the period of the pendulum results in a shorter swing time and they can move through the trees faster.
Assess: You do the same thing when you run. As you run, you don’t keep your arms extended to their full length, instead you bend them at the elbow. If you didn’t the swing of your arms could not keep up with the swing of your legs and you would run in a very awkward manner—try it!

Q14.17. Reason: \( T \), the period, is the time for each cycle of the motion, the time required for the motion to repeat itself. \( \tau \), the damping time constant, is the time required for the amplitude of a damped oscillator to decrease to about 37\% of its original value.

Assess: Both are times, however, and the SI units would be seconds for each.

Q14.18. Reason: Natural frequency is the frequency that an oscillator will oscillate at on its own. You may drive an oscillator at a frequency other than its natural frequency.

Assess: If you are pushing a child in a swing, you build up the amplitude of the oscillation by driving the oscillator at its natural frequency. You can achieve resonance by driving an oscillator at its natural frequency.

Q14.19. Reason: Smaller auditory systems means less mass to move, so they would naturally oscillate at higher frequencies.

Assess: A Web search will show that the range of hearing for mice is about 1 kHz to 70 kHz.

Q14.20. Reason: The truck is a “driven oscillator.” At the intermediate speed where the vertical motion is large and unpleasant, the driving frequency due to hitting the bumps of the washboard is close to the natural (resonance) frequency (which is determined by the suspension system and the mass of the truck); the resulting large-amplitude motion is resonance.

At speeds either significantly above or below that intermediate speed, the driving frequency of the bumps in the road is either smaller or greater than the resonance frequency and the response of the driven oscillator is small. See Figure 14.24.

Assess: It should be noted that a driven oscillator does oscillate at the driving frequency, whether that happens to be close to the resonance frequency or not. So the frequency of the up-and-down motion of the truck is the frequency with which it hits the regular bumps in the washboard road.

You could try an experiment by varying the natural frequency of your truck by loading it more or less and see if the speed that produces resonance on the same washboard road changes similarly.

Q14.21. Prepare: When the kangaroo increases its speed the tendons stretch more which stores more energy in them, so they spend more time in the air propelled by greater spring energy.

Assess: Kangaroo hopping can be quite efficient at high speeds.

Q14.22. Reason: (a) The unstretched equilibrium position is 20 cm. When we load the spring with 100 g we establish a new equilibrium position at 30 cm. When we pull the oscillator down to 40 cm (i.e., an additional 10 cm) and release it, it will oscillate with an amplitude of \( A = 10 \text{ cm} \). The correct choice is B.

(b) Knowing that 100 g stretched the spring 10 cm, we can determine the spring constant.

\[ k = \frac{mg}{x} = \frac{0.1 \text{ kg}}{0.1 \text{ m}} \frac{(0.8 \text{ m/s}^2)/(0.10 \text{ m})}{9.8 \text{ N/m}} = 9.8 \text{ N/m} \]

Knowing the spring constant and the mass on the spring, we can determine the oscillation frequency as follows:

\[ f = \frac{\sqrt{k/m}}{2\pi} = \sqrt{(9.8 \text{ N/m})/(0.10 \text{ kg})}/2\pi = 1.6 \text{ Hz} \]

The correct choice is C.

(c) The frequency of oscillation depends on \( k \), and \( k \) depends on \( g \); the smaller the \( g \) the smaller the \( k \) and the smaller the \( f \). The correct choice is A.

Assess: These are rather small but acceptable values for the spring constant and frequency.

Q14.23. Reason: This is simple harmonic motion where the equilibrium position is at \( x = 0 \).

(a) One can read the period off the graph by seeing how much time elapses from peak to peak or trough to trough. Reading peak to peak gives about 38 s – 14 s = 24 s. So the correct choice is B.

(b) The amplitude is the maximum displacement from the equilibrium position. This is easily read at a number of places on the graph, but the first positive peak occurs at about \( t = 14 \text{ s} \); the displacement there is 4.5 cm. The correct choice is C.
(c) At the time \( t = 30 \) s the graph crosses a grid line and allows us to read the answer, \( x = -2 \) cm. The correct choice is B.

(d) The velocity of the object is given by the slope of the \( x \) versus \( t \) graph. We are looking for the first time that slope is zero (the tangent line is horizontal); this occurs at \( t = 2 \) s. The correct choice is B.

(e) Kinetic energy is maximum when the speed is greatest; this occurs as the object moves through the equilibrium position, not at the end points of its motion where it is instantaneously at rest. Of the choices given, 8 s is the time where the graph crosses the \( t \)-axis and where \( x = 0 \) and the object is in equilibrium (but moving quickly). The correct choice is B.

Assess: It would be very instructive to construct a \( v \times t \) versus \( t \) graph for this same situation (it would be the slope of this \( x \) versus \( t \) graph) and think about the same questions in relation to the new graph.

Q14.24. Reason: Some of the question may be answered by comparing the expression given with the general equation for simple harmonic motion.

The expression given for the displacement is \( x = (0.350 \text{ m}) \cos(15.0 \pi t) \).

The genera expression for the displacement is \( x = A \cos \omega t \).

Comparing these two expressions for the displacement we see that the amplitude of oscillation is \( A = 0.350 \) m, choice B.

The frequency of oscillation may be determined by \( f = \omega / 2 \pi = (15.0 / 2) = 2.39 \) Hz, choice B.

The mass attached to the spring may be determined by \( m = k / \omega^2 = (200 \text{ N/m}) / (15 / 2)^2 = 0.89 \) kg, choice B.

The total mechanical energy of the oscillator is equal to its maximum potential energy, which may be determined by \( E_{Total} = U_{max} = kA^2 / 2 = (200 \text{ N/m})(0.350 \text{ m})^2 / 2 = 12.2 \) J, choice E.

The maximum speed may be determined by \( v_{max} = \omega A = (15.0 / 2)(0.350 \text{ m}) = 5.25 \) m/s, choice E.

Assess: Working this problem brings our attention to two things. First, there is a lot of information tucked away in the function given for the displacement of the oscillator. Second, there are a lot of details associated with an oscillation. It is important to know these details and their interconnection.

Q14.25. Prepare: Your arms act like simple pendulums and carrying the weights in your hands merely increases the mass of the bob, but that doesn’t affect the frequency of oscillation, \( f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \). The answer is B.

Assess: The mass of the bob doesn’t appear in the equation for the frequency of a simple pendulum.

Q14.26. Reason: The relationship between the period, the length of the pendulum, and the acceleration due to gravity at the site of the pendulum is \( T = 2\pi \sqrt{L / g} \). This expression may be solved for the length \( L \) to obtain:

\[
L = \left( \frac{t^2 g}{4 \pi^2} \right) \left( 9.80 \text{ m/s}^2 \right) / 4\pi = 7.5 \text{ m}
\]

The correct choice is C.

Assess: While this is a rather long pendulum, it is in the acceptable range.

Q14.27. Reason: Since the period of a simple pendulum depends on \( g \), it will not tick at the same rate on the moon as it did on the earth. The wristwatch that functions much like a mass on a spring will, however, keep good time on the moon because the period of the mass on a spring does not depend on \( g \).

The correct choice is B.

Assess: Because we can measure frequencies (and therefore periods) so accurately, we can use a pendulum to detect small differences in the local \( g \) due to varying subterranean features.

A further comment is that if \( g \) is reduced to zero (by moving your pendulum to intergalactic outer space) then the pendulum won’t swing at all because there is no restoring force.

Q14.28. Reason: We see in Figure 14.27 that the cells on the basilar membrane close to the stapes correspond to higher frequencies.

The correct choice is B.

Assess: People often lose hearing sensitivity in the higher frequencies with age.
Problems

P14.1. **Prepare:** The period of the vibration is the inverse of the frequency, so we will use Equation 14.1.

**Solve:** The frequency generated by a guitar string is 440 Hz, hence

\[ T = \frac{1}{f} = \frac{1}{440 \text{ Hz}} = 2.3 \times 10^{-3} \text{ s} = 2.3 \text{ ms} \]

**Assess:** The units of frequency are Hz, that is, cycles per second, or s\(^{-1}\), so the period is in seconds.

P14.2. **Prepare:** Knowing the relationship between period and frequency, we can determine the frequency from the period.

**Solve:** The period is determined by

\[ f = \frac{1}{T} = \frac{1}{(54 \text{ min})(1 \text{ min}/60 \text{ s})} = 3.1 \times 10^{-4} \text{ Hz} \]

**Assess:** We expect the frequency to be small for such a large period.

P14.3. **Prepare:** Your pulse or heartbeat is 75 beats per minute or 75 beats/60 s = 1.25 beats/s. The period is the inverse of the frequency, so we will use Equation 14.1.

**Solve:** The frequency of your heart’s oscillations is

\[ f = \frac{75 \text{ beats}}{60 \text{ s}} = 1.25 \text{ beats/s} = 1.3 \text{ Hz} \]

The period is the inverse of the frequency, hence

\[ T = \frac{1}{f} = \frac{1}{1.3 \text{ Hz}} = 0.80 \text{ s} \]

**Assess:** A heartbeat of 1.3 beats per second means that one beat takes a little less than 1 second, which is what we obtained above.

P14.4. **Prepare:** \(180^\circ = \pi\) radian. Small angle approximation holds for angles less than \(10^\circ\) or 0.17 rad.

**Solve:** (a) and (b)

<table>
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<th>(\sin\theta)</th>
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</tr>
<tr>
<td>12</td>
<td>0.2094</td>
<td>0.2079</td>
</tr>
</tbody>
</table>

(c) \(12^\circ\)

(d) The small-angle approximation is good to three significant figures for \(\theta\) up to \(10^\circ\).

P14.5. **Prepare:** For a small angle pendulum the restoring force is proportional to the displacement because \(\sin \theta \approx \theta\).

**Solve:** If we model this as a Hooke’s law situation, then doubling the distance will double the restoring force from 20 N to 40 N.

**Assess:** This would not work for angles much larger than \(10^\circ\).

P14.6. **Prepare:** The air-track glider oscillating on a spring is in simple harmonic motion. The glider completes 10 oscillations in 33 s, and it oscillates between the 10 cm mark and the 60 cm mark. We will use Equations 14.1 and 14.13.
Solve: (a) 

\[ T = \frac{33 \text{ s}}{10 \text{ oscillations}} = 3.3 \text{ s/oscillation} = 3.3 \text{ s} \]

(b) 

\[ f = \frac{1}{T} = \frac{1}{3.3 \text{ s}} = 0.30 \text{ Hz} \]

(c) The oscillation from one side to the other is equal to 60 cm – 10 cm = 50 cm = 0.50 m. Thus, the amplitude is \( A = \frac{1}{2}(0.50 \text{ m}) = 0.25 \text{ m} \).

(d) The maximum speed is 

\[ v_{\text{max}} = \left(\frac{2\pi}{T}\right)A = (2\pi/3.3 \text{ s})(0.25 \text{ m}) = 0.48 \text{ m/s} \]

Assess: The glider takes 3.3 seconds to complete one oscillation. Its maximum speed of 0.48 m/s, or 1 mph, is reasonable.

P14.7. Model: The air-track glider attached to a spring is in simple harmonic motion. The glider is pulled to the right and released from rest at \( t = 0 \text{ s} \). It then oscillates with a period \( T = 2.0 \text{ s} \) and a maximum speed \( 4v_{\text{max}} = 0 \text{ cm/s} = 0.40 \text{ m/s} \). While the amplitude of the oscillation can be obtained from Equation 14.13, the position of the glider can be obtained from Equation 14.10, \( x(t) = A \cos\left(\frac{2\pi}{T}t\right) \).

Solve: (a) 

\[ v_{\text{max}} = \frac{2\pi A}{T} \quad \Rightarrow \quad A = \frac{v_{\text{max}} T}{2\pi} = \frac{(0.40 \text{ m/s})(2.0 \text{ s})}{2\pi} = 0.127 \text{ m} = 0.13 \text{ m} \]

(b) The glider’s position at \( t = 0.25 \text{ s} \) is 

\[ x_{0.25} = (0.127 \text{ m})\cos\left[\frac{2\pi(0.25 \text{ s})}{2.0 \text{ s}}\right] = 0.090 \text{ m} = 9.0 \text{ cm} \]

Assess: At \( t = 0.25 \text{ s} \), which is less than one quarter of the time period, the object has not reached the equilibrium position and is still moving toward the left.

P14.8. Prepare: Please refer to Figure P14.8. The oscillation is the result of simple harmonic motion. As the graph shows, the time to complete one cycle (or the period) is \( T = 2.0 \text{ s} \). We will use Equation 14.1 to find frequency.

Solve: (a) The amplitude \( A = 10 \text{ cm} \).

(b) 

\[ f = \frac{1}{T} = \frac{1}{2.0 \text{ s}} = 0.50 \text{ Hz} \]

Assess: It is important to know how to find information from graphs.

P14.9. Prepare: Please refer to Figure P14.9. The oscillation is the result of simple harmonic motion. As the graph shows, the time to complete one cycle (or the period) is \( T = 4.0 \text{ s} \). We will use Equation 14.1 to find frequency.

Solve: (a) The amplitude \( A = 20 \text{ cm} \).

(b) The period \( T = 4.0 \text{ s} \), thus 

\[ f = \frac{1}{T} = \frac{1}{4.0 \text{ s}} = 0.25 \text{ Hz} \]

Assess: It is important to know how to find information from a graph.
P14.10. **Prepare:** As the object is undergoing simple harmonic motion, we will use Equation 14.11 to describe its motion.

**Solve:** The position of the object is

\[ x(t) = A \cos(2\pi ft) = (6.0 \text{ cm}) \cos[2\pi(0.5 \text{ Hz})t] \]

A position graph (shown below) can now be generated by plugging in various values of \( t \) (in seconds).

![Position Graph](image)

**Assess:** It is a typical cosine function with an amplitude of 6.0 cm and a time period of \((1/0.5 \text{ Hz}) = 2.0 \text{ s}\).

P14.11. **Prepare:** Treating the building as an oscillator, the magnitude of the maximum displacement is the amplitude, the magnitude of the maximum velocity is determined by \( |v_{\text{max}}| = 2\pi fA \), and the magnitude of the maximum acceleration is determined by \( |a_{\text{max}}| = 2\pi^2 f^2 A \).

**Solve:** The magnitude of the maximum displacement is \( |x_{\text{max}}| = A = 0.30 \text{ m} \).

The magnitude of the maximum velocity is \( |v_{\text{max}}| = 2\pi fA = 2\pi(1.2 \text{ Hz})(0.30 \text{ m}) = 2.3 \text{ m/s} \).

The magnitude of the maximum acceleration is \( |a_{\text{max}}| = (2\pi f)^2 A = 4\pi^2(1.2 \text{ Hz})^2(0.30 \text{ m}) = 17 \text{ m/s}^2 \).

**Assess:** These are reasonable values for the magnitude of the maximum displacement, velocity, and acceleration.

P14.12. **Prepare:** We will assume that ship and passengers are approximately in simple harmonic motion. Equation 14.18 gives the maximum acceleration for an object in simple harmonic motion, \( a_{\text{max}} = (2\pi f)^2 A \).

\( A = 1 \text{ m} \) and \( f = 1/15 \text{ s} = 0.067 \text{ Hz} \).

**Solve:** (a) \[ a_{\text{max}} = (2\pi f)^2 A = (2\pi \times 0.067 \text{ Hz})^2(1 \text{ m}) = 0.2 \text{ m/s}^2 \]

(b) To one significant figure, \( g = 10 \text{ m/s}^2 \), so the passenger’s acceleration is about \( \frac{1}{15} g \).

**Assess:** This is not a large acceleration, but it can play havoc with some people’s stomachs.

P14.13. **Prepare:** Solve \( a_{\text{max}} = (2\pi f)^2 A \) for \( A \).

**Solve:**

(a) \[ A = \frac{a_{\text{max}}}{(2\pi f)^2} = \frac{0.20 \text{ m/s}^2}{(2\pi(1.3 \text{ Hz}))^2} = 3.0 \text{ mm} \]

(b) \[ v_{\text{max}} = 2\pi fA = 2\pi(1.3 \text{ Hz})(0.0030 \text{ m}) = 0.024 \text{ m/s} \]

**Assess:** These both seem like small but possible answers.

P14.14. **Prepare:** The total side-to-side motion is \( 2A \). Solve \( a_{\text{max}} = (2\pi f)^2 A \) for \( A \). \( a_{\text{max}} = (0.020)(9.8 \text{ m/s}^2) \).

**Solve:**

\[ 2A = 2 \frac{a_{\text{max}}}{(2\pi f)^2} = 2 \frac{(0.020)(9.8 \text{ m/s}^2)}{(2\pi(0.17 \text{ Hz}))^2} = 34 \text{ cm} \]

**Assess:** The height of the building is not needed.
P14.15. **Prepare:** The spring undergoes simple harmonic motion. The elastic potential energy in a spring stretched by a distance \( x \) from its equilibrium position is given by Equation 14.20, and the total mechanical energy of the object is the sum of kinetic and potential energies as in Equation 14.21. At maximum displacement, the total energy is simply \( E = \frac{1}{2} k A^2 \), Equation 14.22.

**Solve:** (a) When the displacement is \( x = \frac{1}{2} A \), the potential energy is

\[
U = \frac{1}{2} k x^2 = \frac{1}{2} k \left( \frac{1}{2} A \right)^2 = \frac{1}{4} \left( \frac{1}{2} k A^2 \right) = \frac{1}{4} E \Rightarrow K = E - U = \frac{3}{4} E
\]

Thus, one quarter of the energy is potential and three-quarters is kinetic.

(b) To have \( U = \frac{1}{4} E \) requires

\[
U = \frac{1}{2} k x^2 = \frac{1}{2} E = \frac{1}{2} \left( \frac{1}{2} k A^2 \right) \Rightarrow x = \frac{A}{\sqrt{2}}
\]

P14.16. **Prepare:** The block attached to the spring is in simple harmonic motion. At maximum displacement position, \( x = A \), and the kinetic energy is zero. We can use the energy conservation Equation 14.21 to find the amplitude.

**Solve:** (a) The conservation of mechanical energy equation \( K_f + U_f = K_i + U_i \) is

\[
\frac{1}{2} m v_1^2 + \frac{1}{2} k (\Delta x)^2 = \frac{1}{2} m v_0^2 + 0 \ J \Rightarrow 0 + \frac{1}{2} k A^2 = \frac{1}{2} m v_0^2 + 0 \ J
\]

\[
\Rightarrow A = \sqrt{\frac{m}{k}} v_0 = \sqrt{\frac{1.0 \ \text{kg}}{16 \ \text{N/m}}} (0.40 \ \text{m/s}) = 0.10 \ \text{m} = 10.0 \ \text{cm}
\]

(b) We have to find the velocity at a point where \( x = A/2 \). The conservation of mechanical energy equation \( K_f + U_f = K_i + U_i \) is

\[
\frac{1}{2} m v_2^2 + \frac{1}{2} k \left( \frac{A}{2} \right)^2 = \frac{1}{2} m v_0^2 = \frac{1}{2} m v_0^2 - \frac{1}{4} \left( \frac{1}{2} k A^2 \right) = \frac{1}{2} m v_0^2 - \frac{1}{4} \left( \frac{1}{2} m v_0^2 \right)
\]

\[
\Rightarrow v_2 = \sqrt{\frac{3}{4} v_0} = \sqrt{\frac{3}{4}} (0.40 \ \text{m/s}) = 0.346 \ \text{m/s} = 35 \ \text{cm/s}
\]

**Assess:** Speed decreases as an object moves away from the equilibrium position, so a decreased speed of 35 m/s compared to 40 m/s at the equilibrium position is reasonable.

P14.17. **Prepare:** The block attached to the spring is in simple harmonic motion. The period of an oscillating mass on a spring is given by Equation 14.27.
Solve: The period of an object attached to a spring is

\[ T = 2\pi \sqrt{\frac{m}{k}} = T_0 = 2.00 \text{ s} \]

where \( m \) is the mass and \( k \) is the spring constant.

(a) For mass \( 2m \),

\[ T = 2\pi \sqrt{\frac{2m}{k}} = (\sqrt{2})T_0 = 2.83 \text{ s} \]

(b) For mass \( \frac{1}{2}m \),

\[ T = 2\pi \sqrt{\frac{\frac{1}{2}m}{k}} = T_0/\sqrt{2} = 1.41 \text{ s} \]

(c) The period is independent of amplitude. Thus \( T = T_0 = 2.00 \text{ s} \).

(d) For a spring constant \( 2k \),

\[ T = 2\pi \sqrt{\frac{m}{2k}} = T_0/\sqrt{2} = 1.41 \text{ s} \]

Assess: As would have been expected, increase in mass leads to slower simple harmonic motion.

P14.18. Prepare: The air-track glider attached to a spring is in simple harmonic motion. Experimentally, the period of motion is \( T = (12.0 \text{ s})/(10 \text{ oscillations}) = 1.20 \text{ s} \). Equation 14.27 relates the period to the glider’s mass and the spring constant.

Solve: Using Equation 14.27 for the period,

\[ T = 2\pi \sqrt{\frac{m}{k}} \Rightarrow k = \left( \frac{2\pi}{T} \right)^2 \cdot \left( \frac{0.200 \text{ kg}}{1.20 \text{ s}} \right)^2 = 5.5 \text{ N/m} \]

Assess: \( k = 5.5 \text{ N/m} \) means that a force of 5.5 N or a mass of 0.56 kg (a little over a pound) will stretch/compress it by 100 cm. This is a soft spring.

P14.19. Prepare: The oscillating mass is in simple harmonic motion. The position of the oscillating mass is given by \( x(t) = (2.0 \text{ cm}) \cos(10t) \), where \( t \) is in seconds. We will compare this with Equation 14.10.

Solve: (a) The amplitude \( A = 2.0 \text{ cm} \).

(b) The period is calculated as follows:

\[ \frac{2\pi}{T} = 10 \text{ rad/s} \Rightarrow T = \frac{2\pi}{10 \text{ rad/s}} = 0.63 \text{ s} \]

(c) The spring constant is calculated from Equation 14.27 as follows:

\[ \frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow k = m\left( \frac{2\pi}{T} \right)^2 = (0.050 \text{ kg})(10 \text{ rad/s})^2 = 5.0 \text{ N/m} \]

(d) The maximum speed from Equation 14.26 is

\[ v_{max} = 2\pi fA = \left( \frac{2\pi}{T} \right) A = (10 \text{ rad/s})(2.0 \text{ cm}) = 20 \text{ cm/s} \]

(e) The total energy from Equation 14.22 is

\[ E = \frac{1}{2}kA^2 = \frac{1}{2}(5.0 \text{ N/m})(0.02 \text{ m})^2 = 1.0 \times 10^{-3} \text{ J} \]

(f) At \( t = 0.40 \text{ s} \), the velocity from Equation 14.12 is

\[ v_t = -(20.0 \text{ cm/s})\sin[(10 \text{ rad/s})(0.40 \text{ s})] = 15 \text{ cm/s} \]

Assess: Velocity at \( t = 0.40 \text{ s} \) is less than the maximum velocity, as would be expected.
P14.20. **Prepare:** The mass attached to the spring oscillates in simple harmonic motion. The mass oscillates at a frequency of 2.0 Hz. We will need the spring constant $k$ which we will determine using Equation 14.27.

**Solve:**

(a) The period using Equation 14.1 is $T = 1/f = 1/2.0 \text{ Hz} = 0.50 \text{ s}$.

(b) Using energy conservation $\frac{1}{2}kA^2 = \frac{1}{2}kx_0^2 + \frac{1}{2}m(v_0)^2$.

Using $x_0 = 5.0 \text{ cm}$, $(v_0)_0 = -30 \text{ cm/s}$, and $k = m(2\pi f)^2 = 0.2 \text{ kg}(2\pi(2.0 \text{ Hz}))^2 = 31.58 \text{ N/m}$, we get $A = 5.54 \text{ cm}$, which is to be reported as 5.5 cm.

(c) The maximum speed from Equation 14.26 is $v_{\text{max}} = 2\pi fA = 2\pi(2.0 \text{ Hz})(5.54 \text{ cm}) = 69.6 \text{ cm/s}$, which will be reported as 70 cm/s.

(d) The total energy is $E = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}(0.200 \text{ kg})(0.696 \text{ m/s})^2 = 0.049 \text{ J}$.

P14.21. **Prepare:** The mass attached to the spring is in simple harmonic motion.

**Solve:**

(a) The period using Equation 14.27 is

$$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{(0.507 \text{ kg})}{(20 \text{ N/m})}} = 1.00 \text{ s}$$

(b) Using Equation 14.26, the maximum speed $v_{\text{max}} = 2\pi fA = (2\pi/1.00 \text{ s})(0.10 \text{ m}) = 0.628 \text{ m/s}$.

(c) The total energy from Equation 14.23 is $E = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}(0.507 \text{ kg})(0.628 \text{ m/s})^2 = 0.100 \text{ J}$.

P14.22. **Prepare:** The oscillator is in simple harmonic motion and its mechanical energy is conserved.

**Solve:** The energy conservation equation $E_1 = E_2$ is

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

$$\frac{1}{2}(0.30 \text{ kg})(0.954 \text{ m/s})^2 + \frac{1}{2}k(0.030 \text{ m})^2 = \frac{1}{2}(0.30 \text{ kg})(0.714 \text{ m/s})^2 + \frac{1}{2}k(0.060 \text{ m})^2$$

$$\Rightarrow k = 44.48 \text{ N/m}$$

The total energy of the oscillator is

$$E_{\text{total}} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}(0.30 \text{ kg})(0.954 \text{ m/s})^2 + \frac{1}{2}(44.48 \text{ N/m})(0.030 \text{ m})^2 = 0.1565 \text{ J}$$

Because $E_{\text{total}} = \frac{1}{2}mv_{\text{max}}^2$,

$$0.1565 \text{ J} = \frac{1}{2}(0.300 \text{ kg})v_{\text{max}}^2 \Rightarrow v_{\text{max}} = 1.02 \text{ m/s}$$

**Assess:** A maximum speed of 1.02 m/s is reasonable.

P14.23. **Prepare:** Assume a small angle of oscillation so there is simple harmonic motion. We will use Equation 14.31 for the pendulum’s time period.

**Solve:** The period of the pendulum is

$$T_0 = 2\pi \sqrt{\frac{L_0}{g}} = 4.00 \text{ s}$$

(a) The period is independent of the mass and depends only on the length. Thus $T = T_0 = 4.00 \text{ s}$.

(b) For a new length $L = 2L_0$,

$$T = 2\pi \sqrt{\frac{2L_0}{g}} = \sqrt{2}T_0 = 5.66 \text{ s}$$
(c) For a new length \( L = L_0/2 \),

\[ T = 2\pi \sqrt{\frac{L_0/2}{g}} = \frac{1}{\sqrt{2}} T_0 = 2.83 \text{ s} \]

(d) The period is independent of the amplitude as long as there is simple harmonic motion. Thus \( T = 4.00 \text{ s} \).

P14.24. **Prepare:** Because the angle of displacement is less than 10\(^\circ\), the small-angle approximation holds and the pendulum exhibits simple harmonic motion. We will use Equation 14.31 and \( g = 9.80 \text{ m/s}^2 \).

**Solve:** The period is \( T = 12.0 \text{ s/10 oscillations} = 1.20 \text{ s} \) and is given by the formula

\[ T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow L = \left( \frac{T}{2\pi} \right)^2 g = \left( \frac{1.20 \text{ s}}{2\pi} \right)^2 (9.80 \text{ m/s}) = 35.7 \text{ cm} \]

**Assess:** A length of 35.7 cm for the simple pendulum is reasonable.

P14.25. **Prepare:** The pendulum undergoes simple harmonic motion according to \( \theta(t) = (0.10 \text{ rad}) \cos(5t) \).

We are dealing with angular quantities here, so the amplitude will be in radians.

**Solve:** (a) The amplitude is 0.10 rad.

(b) The frequency of oscillations is

\[ 2\pi f = 5 \text{ rad/s} \Rightarrow f = \frac{5 \text{ rad/s}}{2\pi} = 0.796 \text{ Hz} \]

which we will report as 0.80 Hz.

(c) The length can be obtained from the period, Equation 14.31

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \Rightarrow L = \left( \frac{1}{2\pi f} \right)^2 g = \left( \frac{1}{2\pi(0.796 \text{ Hz})} \right)^2 (9.8 \text{ m/s}^2) = 0.39 \text{ m} \]

(d) At \( t = 2.0 \text{ s} \), \( \theta_{2.0} = (0.10 \text{ rad}) \cos(5(2.0 \text{ s})) = -0.084 \text{ rad.} \)

**Assess:** The angle is smaller than the amplitude, as would be expected. The negative sign with the angle indicates that the pendulum is on the left side of the equilibrium position.

P14.26. **Model:** Assume that the swinging lamp makes a small angle with the vertical so that there is simple harmonic motion. We will use Equation 14.31.

**Solve:** (a) Using the formula for the period of a pendulum,

\[ T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow L = g \left( \frac{T}{2\pi} \right)^2 = (9.80 \text{ m/s}^2) \left( \frac{5.5 \text{ s}}{2\pi} \right)^2 = 7.5 \text{ m} \]

**Assess:** Lamp chains in cathedrals are pretty long, so a length of 7.5 m is reasonable.

P14.27. **Prepare:** Assume the pendulum to have small-angle oscillations. In this case, the pendulum undergoes simple harmonic motion. We will use Equation 14.31.

**Solve:** The periods of the pendulums on the moon and on the earth are

\[ T_{\text{earth}} = 2\pi \sqrt{\frac{L_{\text{earth}}}{g_{\text{earth}}}} \quad \text{and} \quad T_{\text{moon}} = 2\pi \sqrt{\frac{L_{\text{moon}}}{g_{\text{moon}}}} \]

Because \( T_{\text{earth}} = T_{\text{moon}} \),

\[ 2\pi \sqrt{\frac{L_{\text{earth}}}{g_{\text{earth}}}} = 2\pi \sqrt{\frac{L_{\text{moon}}}{g_{\text{moon}}}} \Rightarrow L_{\text{moon}} = \left( \frac{g_{\text{moon}}}{g_{\text{earth}}} \right) L_{\text{earth}} = \left( \frac{1.62 \text{ m/s}^2}{9.8 \text{ m/s}^2} \right)(2.00 \text{ m}) = 33.1 \text{ cm} \]

**Assess:** Because of smaller \( g \) on the moon, a smaller pendulum length to have the same period as on the earth was indeed expected.

P14.28. **Prepare:** Assume a small angle of oscillation so that the pendulum has simple harmonic motion. We will use Equation 14.31.
Oscillations

Solve: The time periods of the pendulums on the earth and on Mars are

\[ T_{\text{earth}} = 2\pi \sqrt{\frac{L}{g_{\text{earth}}}} \quad \text{and} \quad T_{\text{Mars}} = 2\pi \sqrt{\frac{L}{g_{\text{Mars}}}} \]

Dividing these two equations,

\[ \frac{T_{\text{earth}}}{T_{\text{Mars}}} = \sqrt{\frac{g_{\text{Mars}}}{g_{\text{earth}}}} \Rightarrow g_{\text{Mars}} = g_{\text{earth}} \left( \frac{T_{\text{earth}}}{T_{\text{Mars}}} \right)^2 = (9.80 \text{ m/s}^2) \left( \frac{1.50 \text{ s}}{2.45 \text{ s}} \right)^2 = 3.67 \text{ m/s}^2 \]

Assess: Because \( T_{\text{Mars}} > T_{\text{earth}} \), the same length of the pendulum would imply smaller \( g \) on Mars, as obtained above.

P14.29. Prepare: To complete a whole period, the wrecking ball will have to swing down, up to the other side, back down, and up again to the original position. So the time it takes to swing from maximum height down to lowest height once is one-quarter of a period. We will assume that the wrecking ball is a simple small-angle pendulum and so it’s period is given by \( T = 2\pi \sqrt{\frac{L}{g}} \). Solve:

\[ \frac{1}{4} T = \frac{1}{4} \pi \sqrt{\frac{10 \text{ m}}{9.8 \text{ m/s}^2}} = 1.6 \text{ s} \]

Assess: This is enough time to dive out of the way, but it is still wiser to not stand in the way of wrecking balls.

P14.30. Prepare: Treating the lower leg as a physical pendulum we can determine the moment of inertia by combining \( T = 2\pi \sqrt{\frac{I}{mgL}} \) and \( T = 1/f \).

Solve: Combining the above expressions and solving for the moment of inertia we obtain

\[ I = mgL/(2\pi f)^2 = (5.0 \text{ kg})(9.80 \text{ m/s}^2)(0.18 \text{ m})/(2\pi(1.6 \text{ Hz}))^2 = 8.7 \times 10^{-2} \text{ kg \cdot m}^2 \]

Assess: NASA determines the moment of inertia of the shuttle in a similar manner. It is suspended from a heavy cable, allowed to oscillate about its vertical axis of symmetry with a very small amplitude, and from the period of oscillation one may determine the moment of inertia. This arrangement is called a torsion pendulum.

P14.31. Prepare: For part (a) just plug the value of \( t \) in to the equation for \( \theta(t); \theta(t) = (0.175 \text{ rad}) \sin(\pi t) \).

For part (b) we refer to the angular version of Equation 14.19, \( \theta(t) = A \cos(2\pi f t) \).

For part (c) we realize that a uniform solid rod is not a simple pendulum, but a physical pendulum whose frequency is given by Equation 14.32. The moment of inertia of a solid rod pivoted about its end is \( I = \frac{1}{3} mL^2 \).

Solve: (a)

\[ \theta(0.250 \text{ s}) = (0.175 \text{ rad}) \sin(\pi(0.250 \text{ s})) = 0.124 \text{ rad} \]

(b) Comparing \( \theta(t) = (0.175 \text{ rad}) \sin(\pi t) \) to \( \theta(t) = A \cos(2\pi f t) \) shows that \( \pi = 2\pi f \). (This is true even though one function is cos and the other sin; they have the same frequency.) Therefore \( f = (1/2) \text{ Hz} \) and \( T = 1/f = 2.00 \text{ s} \).

(c) Following the derivation in Example 14.11 with \( d = L/2 \)

\[ f = \frac{1}{2\pi} \sqrt{\frac{mgd}{I}} = \frac{1}{2\pi} \sqrt{\frac{mg(L/2)}{\frac{1}{3} mL^2}} = \frac{1}{2\pi} \sqrt{\frac{3g}{2L}} \]

\[ T = 1/f \text{ so,} \]

\[ T = 2\pi \sqrt{\frac{2L}{3g}} \]
Now solve this for $L$.

$$L = \frac{3}{2} g \left( \frac{T}{2\pi} \right)^2 = \frac{3}{2} (9.8 \text{ m/s}^2) \left( \frac{2.00 \text{ s}}{2\pi} \right)^2 = 1.49 \text{ m}$$

Assess: 1.49 m is a realistic length for a pendulum in a clock. Notice that part (b) helped us with part (c).

**P14.32. Prepare:** We will model the rope as a simple small-angle pendulum. We want to hang on to the rope for a half a period to get back to land.

**Solve:**

$$t_{\text{hang}} = \frac{T}{2} = \pi \sqrt{\frac{L}{g}} = \pi \sqrt{\frac{15 \text{ m}}{9.8 \text{ m/s}^2}} = 3.9 \text{ s}$$

Assess: This seems reasonable for such a long pendulum.

**P14.33. Prepare:** Treating the hoop suspended from the nail as a physical pendulum we can determine the period of oscillation using $T = 2\pi \sqrt{I/mgL}$.

**Solve:** The period of oscillation for the hoop suspended from its rim on a nail may be determined by

$$T = 2\pi \sqrt{I/mgL} = 2\pi \sqrt{2mR^2/mgR} = 2\pi \sqrt{2(0.22 \text{ m})/(9.80 \text{ m/s}^2)} = 1.3 \text{ s}$$

Assess: This is a reasonable period for this situation.

**P14.34. Prepare:** Model the elephant legs as physical pendulums with $I = \frac{1}{2} mL^2$. The distance from the pivot point to the center of gravity is $d = \frac{1}{2} L$. For a leg to swing forward requires half a period.

**Solve:**

(a) $T = \pi \sqrt{\frac{1}{mgd}} = \pi \sqrt{\frac{\frac{1}{2}mL^2}{mg(\frac{1}{2}L)}} = \pi \sqrt{\frac{2L}{3g}} = \pi \sqrt{\frac{2(2.3 \text{ m})}{3(9.8 \text{ m/s}^2)}} = 1.24 \text{ s} \approx 1.2 \text{ s}$

(b) The right leg takes 1.24 s to swing forward, then stay planted while the left leg takes 1.24 s to swing forward, so it takes 2.48 s for a whole period of the process. That is, each leg hits the ground $\frac{1}{2.48}$ times per second. There are 4 legs.

$$\frac{1}{2.48 \text{ s}} (4 \text{ legs}) \left( \frac{60 \text{ s}}{1 \text{ min}} \right) = 97 \text{ steps/min}$$

Assess: This seems to jibe with the nature shows we’ve seen.

**P14.35. Prepare:** The motion is a damped oscillation. The maximum displacement or amplitude of the oscillation at time $t$ is given by Equation 14.33, $x_{\text{max}}(t) = Ae^{-\frac{t}{\tau}}$, where $\tau$ is the time constant. Using $x_{\text{max}} = 0.368 A$ and $t = 10.0 \text{ s}$, we can calculate the time constant.

**Solve:** From Equation 14.33

$$0.368A = Ae^{-10.0 \frac{s}{\tau}} \Rightarrow \ln (0.368) = -\frac{10.0 \text{ s}}{\tau} \Rightarrow \tau = -\frac{10.0 \text{ s}}{\ln (0.368)} = 10.0 \text{ s}$$

Assess: The above result says that the oscillation decreases to about 37% of its initial value after one time constant.
P14.36. **Prepare:** Assume $A=1$ in arbitrary units. The object continues to oscillate but $x_{\text{max}}$ decreases due to the damping.

**Solve:** The equation for the graph is $x(t) = e^{-t/\tau} \cos(2\pi(1.0\,\text{Hz})t)$.

![Graph of oscillations](image)

**Assess:** The oscillations damp out at about the rate we expect for $\tau = 4.0\,\text{s}$.

P14.37. **Prepare:** The initial amplitude is $A = 6.5\,\text{cm}$.

**Solve:**

The equation is $x_{\text{max}}(t) = Ae^{-t/\tau}$ with $x_{\text{max}}(8.0\,\text{s}) = 1.8\,\text{cm}$.

(a) $1.8\,\text{cm} = (6.5\,\text{cm})e^{-8.0/\tau} \quad \Rightarrow \quad \tau = \frac{8.0\,\text{s}}{\ln \frac{1.8\,\text{cm}}{6.5\,\text{cm}}} = 6.23\,\text{s} \approx 6.2\,\text{s}$

(b) Use $x_{\text{max}}(t) = Ae^{-t/\tau}$ with $t = 4.05$ and $\tau = 6.23\,\text{s}$

$$x_{\text{max}}(4.0\,\text{s}) = (6.5\,\text{s})e^{-4.05/(6.23\,\text{s})} = 3.4\,\text{cm}$$

**Assess:** We expect the amplitude at 4.0 s to be between 6.5 cm and 1.8 cm.

P14.38. **Prepare:** Assume damped oscillations. We examine the graph carefully.

**Solve:** It looks from the graph that the period is 0.5 s so $f = 2\,\text{Hz}$. Looking at the peaks, it appears the amplitude has decreased to about 37% of its value after only 0.25 s, so that is our guess for $\tau$.

**Assess:** You want the time constant for damping on your car to be short.

P14.39. **Prepare:** We will model the child on the swing as a simple small-angle pendulum. To make the amplitude grow large quickly we want to drive (push) the oscillator (child) at the natural resonance frequency. In other words, we want to wait the natural period between pushes.

**Solve:**

$$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{2.0\,\text{m}}{9.8\,\text{m/s}^2}} = 2.8\,\text{s}$$

**Assess:** You could also increase the amplitude by pushing every other time (every $2T$), but that would not make the amplitude grow as quickly as pushing every period. The mass of the child was not needed; the answer is independent of the mass.

P14.40. **Prepare:** We first determine the frequency of the oscillation, then look at the graph to determine whether we need to increase or decrease the frequency to decrease the amplitude.

**Solve:** The frequency is

$$\frac{20\,\text{mi/h}}{10\,\text{ft}} \left(\frac{5280\,\text{ft}}{1\,\text{mi}}\right) \left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = 2.93\,\text{Hz}$$

This is to the right of the peak of the graph, so we need to increase the frequency to decrease the amplitude; you should speed up.

**Assess:** Experience teaches that sometimes it helps to speed up on a washboard road.
P14.41. **Prepare:** Given the mass and the resonant frequency, we can determine the effective spring constant using the relationship \( \omega = 2\pi f = \sqrt{k/m} \).

**Solve:** Solving the above expression for the spring constant, obtain

\[
k = (2\pi f)^2 m = (2\pi (29 \text{ Hz}))^2 (7.5 \times 10^{-3} \text{ kg}) = 249 \text{ N/m}
\]

**Assess:** As spring constants go, this is a fairly large value, however the musculature holding the eyeball in the socket is strong and hence will have a large effective spring constant.

P14.42. **Prepare:** For part (a) start from Hooke’s law \( F_y = -k\Delta y \) and use the derivation in Equation 14.4.

**Solve:** (a) Solve Equation 14.5 for \( k \).

\[
k = \frac{mg}{\Delta L} = \frac{(0.080 \text{ kg})(9.8 \text{ m/s}^2)}{0.040 \text{ m}} = 20 \text{ N/m}
\]

(b) The period of a massive object (such as a ball) on a spring is

\[
T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{0.080 \text{ kg}}{20 \text{ N/m}}} = 0.40 \text{ s}
\]

(c) 

**Assess:** These are typical values for springs like those used in college physics labs.

P14.43. **Prepare:** Since all forces are conservative, conservation of energy may be used to solve the problem. The following sketch will be helpful in the process of thinking about this problem.

In the figure,

(a) shows the unstretched spring—this is the point of zero potential energy for the spring.

(b) shows the spring with the ball hanging at rest. The spring has been stretched an amount \( d \) and it is now at rest at the new equilibrium position. At this position the spring is stretched an amount \( d \) until the restoring force the spring exerts is equal to the weight of the ball. That is \( kd = mg \) or \( d = mg/k \). As a matter of convenience let’s choose this new equilibrium position as the point for zero potential energy. Since we are free to choose zero gravitational energy at any point we wish, this seems like a convenient choice.

(c) shows the ball pulled down an additional amount \( A \).
Next let’s write a statement of total energy at the stretched position and at the equilibrium position. Since energy is conserved, we will equate these expressions for total energy and solve for the speed of the ball at the equilibrium position, which is the maximum speed (because the total potential energy is zero at this point).

**Solve:** The initial total energy when the ball is at the stretched position is \( E_i = K + U_i + U_s = 0 - mgA + k(d + A)^2/2 \).

The final total energy when the ball is at the new spring equilibrium position is \( E_f = K + U_i' + U_s' = mv^2/2 + 0 + kd^2/2 \).

Equate the initial and final total energy we have \( mv^2/2 + kd^2/2 = -mgA + k(d + A)^2/2 \).

Expanding this obtains \( mv^2/2 + kd^2/2 = -mgA + k(d^2 + 2dA + A^2)/2 \).

Multiplying by 2 and doing a little rearranging, obtain \( mv^2 + kd^2 = -2mgA + kd^2 + 2kdA + kA^2 \).

Cancel the \( kd^2 \) term and insert \( d = mg/k \) to obtain \( mv^2 = -2mgA + 2k(mg/k)A + kA^2 \).

After canceling the \( 2mgA \) terms and solving for \( v \), obtain

\[
v = \sqrt{\frac{k}{m}A} = \sqrt{\frac{(12 \text{ N/m})/(0.40 \text{ kg})(0.20 \text{ m})}{2}} = 1.1 \text{ m/s}
\]

**Assess:** It is interesting to note that we ended up with the same result as the horizontal case.

**P14.44. Prepare:** The vertical oscillations constitute simple harmonic motion given by Equation 14.27. A pictorial representation of the spring and the book is given.

**Solve:** (a) At equilibrium, Newton’s first law applied to the physics book is

\[(F_y)_e - mg = 0 \Rightarrow -k\Delta y - mg = 0 \Rightarrow k = -mg/\Delta y = -(0.500 \text{ kg})(9.8 \text{ m/s}^2)/(-0.20 \text{ m}) = 24.5 \text{ N/m}\]

which we will report as 25 N/m to two significant figures.

(b) To calculate the period from Equation 14.27

\[
\frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow \frac{2\pi}{m} = 7.0 \text{ rad/s} \Rightarrow T = \frac{2\pi \text{ rad}}{7.0 \text{ rad/s}} = 0.898 \text{ s} = 0.90 \text{ s}
\]

(c) The maximum speed from Equation 14.26 is

\[

v_{max} = A(2\pi f) = A\frac{2\pi}{T} = (0.10 \text{ m})(7.0 \text{ rad/s}) = 0.70 \text{ m/s}
\]

Maximum speed occurs as the book passes through the equilibrium position.

**P14.45. Prepare:** The vertical oscillations constitute simple harmonic motion. A pictorial representation of the spring and the ball is shown in the following figure. The period and frequency of oscillations are
$T = \frac{20 \text{ s}}{30 \text{ oscillations}} = 0.6667 \text{ s}$ and $f = \frac{1}{T} = \frac{1}{0.6667 \text{ s}} = 1.50 \text{ Hz}$

Since $k$ is known, we can obtain the mass $m$ using Equation 14.27.

**Solve:**
(a) The mass can be found as follows:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \Rightarrow m = \frac{k}{(2\pi f)^2} = \frac{15.0 \text{ N/m}}{[2\pi(1.50 \text{ Hz})]^2} = 0.169 \text{ kg}$$

(b) The maximum speed is given by Equation 14.26, $v_{\text{max}} = 2\pi fA = 2\pi(1.50 \text{ Hz})(0.0600 \text{ m}) = 0.565 \text{ m/s}.$

**Assess:** Both the mass of the ball and its maximum speed are reasonable.

**P14.46. Prepare:** The vertical oscillations constitute simple harmonic motion. To find the oscillation frequency using Equation 14.27, $2\pi f = \sqrt{k/m},$ we first need to find the spring constant $k$. In equilibrium, the weight $mg$ of the block and the spring force $k\Delta L$ are equal and opposite. That is, $mg = k\Delta L \Rightarrow k = mg/\Delta L.$ As shown in the next figure, the mass of the mug drops out of the calculations, and the stretch length $\Delta L$ is 2.0 cm.

**Solve:** The frequency of oscillation $f$ is

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{mg/\Delta L}{m}} = \frac{1}{2\pi} \sqrt{\frac{9.8 \text{ m/s}^2}{0.020 \text{ m}}} = 3.5 \text{ Hz}$$

**P14.47. Prepare:** The vertical oscillations constitute simple harmonic motion. We will use Equation 14.27.
Solve: At the equilibrium position, the net force on mass $m$ on Planet X is the following:

$$F_{net} = k\Delta L - mg_x = 0 \Rightarrow \frac{k}{m} = \frac{g_x}{\Delta L}$$

For simple harmonic motion, Equation 14.27 yields $k/m = (2\pi f)^2$, thus

$$(2\pi f)^2 = \frac{g_x}{\Delta L} \Rightarrow g_x = \left(\frac{2\pi}{T}\right)^2 \Delta L = \left(\frac{2\pi}{14.5 \text{ s}/10}\right)^2 (0.312 \text{ m}) = 5.86 \text{ m/s}^2$$

Assess: This value of $g$ is of the same order of magnitude as the one for the earth, and would thus seem to be reasonable.

P14.48. Prepare: The object is undergoing simple harmonic motion. We will use Equation 14.19 for velocity of an object undergoing simple harmonic motion.

Solve: The velocity of an object oscillating on a spring is

$$v_x(t) = -(2\pi f)A\sin(2\pi ft)$$

(a) For $A' = 2A$ and $f' = f/2$, we have

$$v'_x(t) = -(2\pi f/2)(2A)\sin[(2\pi f/2)t] = -(2\pi f)A\sin(\pi ft)$$

That is, the maximum velocity $A'(2\pi f')$ remains the same, but the frequency of oscillation is halved.

(b) For $m' = 4m$,

$$f' = \frac{1}{2\pi} \sqrt{\frac{k}{m'}} = \frac{1}{2\pi} \sqrt{\frac{k}{4m}} = \frac{f}{2} \quad \text{and} \quad A' = A \Rightarrow v'_{max} = A'(2\pi f') = A\pi f = v_{max}/2$$

Quadrupling of the mass halves the frequency or doubles the time period, and halves the maximum velocity.

Assess: It is important to know how to present information on a graph.

P14.49. Prepare: The vertical mass/spring systems are in simple harmonic motion. Please refer to Figure P14.49.

Solve: (a) For system A, $y$ is positive for one second as the mass moves downward and reaches maximum negative $y$ after two seconds. It then moves upward and reaches the equilibrium position, $y = 0$, at $t = 3$ seconds. The maximum speed while traveling in the upward direction thus occurs at $t = 3.0 \text{ s}$. The frequency of oscillation is 0.25 Hz.

(b) For system B, all the mechanical energy is potential energy when the position is at maximum amplitude, which for the first time is at $t = 1.5 \text{ s}$. The time period of system B is thus 6.0 s.
(c) Spring/mass A undergoes three oscillations in 12 s, giving it a period \( T_A = 4.0 \) s. Spring/mass B undergoes two oscillations in 12 s, giving it a period \( T_B = 6.0 \) s. From Equation 14.27, we have

\[
T_A = 2\pi \sqrt{\frac{m_A}{k_A}} \quad \text{and} \quad T_B = 2\pi \sqrt{\frac{m_B}{k_B}} \Rightarrow \frac{T_A}{T_B} = \sqrt{\frac{m_A}{m_B}} \left( \frac{k_B}{k_A} \right) = \frac{4.0 \text{ s}}{6.0 \text{ s}} = \frac{2}{3}
\]

If \( m_A = m_B \), then

\[
\frac{k_B}{k_A} = \frac{4}{9} \Rightarrow \frac{k_A}{k_B} = \frac{9}{4} = 2.25
\]

Assess: It is important to learn how to read a graph.

**P14.50. Prepare:** First we figure the mass of the chair alone, then the mass of the chair plus astronaut, then subtract.

**Solve:**

\[
m_{ch} = \frac{T^2}{4\pi^2}k = \frac{(0.901 \text{ s})^2}{4\pi^2}(606 \text{ N/m}) = 12.46 \text{ kg}
\]

\[
m_{tot} = \frac{T^2}{4\pi^2}k = \frac{(2.09 \text{ s})^2}{4\pi^2}(606 \text{ N/m}) = 67.05 \text{ kg}
\]

\[
m_a = m_{tot} - m_{ch} = 67.05 \text{ kg} - 12.46 \text{ kg} = 54.59 \text{ kg} \quad \text{which we report as 54.6 kg.}
\]

Assess: This is a reasonable weight for a light astronaut.

**P14.51. Prepare:** The ball attached to the spring is in simple harmonic motion. The position and velocity at time \( t \) are \( x_0 = -5 \) cm and \( v_0 = 20 \) cm/s. An examination of Equations 14.19 shows that \( x(t) = A \cos(2\pi ft) \) and \( \frac{v(t)}{2\pi f} = -A \sin(2\pi ft) \). Adding the squares of these equations and using the trigonometric relationship \( \cos^2 \theta + \sin^2 \theta = 1 \), we have

\[
A = \sqrt{x(t)^2 + \left(\frac{v(t)}{2\pi f}\right)^2}
\]

**Solve:**

(a) The oscillation frequency is

\[
f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{2.5 \text{ N/m}}{0.10 \text{ kg}}} = 0.796 \text{ Hz}
\]

Using Equation 14.27, the amplitude of the oscillation is

\[
A = \sqrt{x(t)^2 + \left(\frac{v(t)}{2\pi f}\right)^2} = \sqrt{(-5.00 \text{ cm})^2 + \left(\frac{20 \text{ cm/s}}{2\pi(0.796 \text{ Hz})}\right)^2} = 6.40 \text{ cm}
\]

(b) We can use the conservation of energy between \( x_i = -5 \) cm and \( x_f = 3 \) cm as follows:

\[
\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 \Rightarrow v_f = \sqrt{v_i^2 + \frac{k}{m}(x_f^2 - x_i^2)} = 0.283 \text{ m/s} = 28.3 \text{ cm/s}
\]

Assess: Because \( k \) is known in SI units of N/m, the energy calculation must be done using SI units of m, m/s, and kg. Both the amplitude and speed are reasonable.

**P14.52. Prepare:** The transducer undergoes simple harmonic motion. The maximum restoring force that the disk can withstand is 40,000 N. By applying Newton’s second law to the disk of mass 0.10 g, we will first find its maximum acceleration and then use Equation 14.18 to find the maximum oscillation amplitude. Once we find the amplitude, we will use Equation 14.26 to find the disk’s maximum speed.
Solve: Newton’s second law for the transducer is

\[ F_{\text{restoring}} = ma_{\max} \Rightarrow 40,000 \text{ N} = (0.10 \times 10^{-3} \text{ kg})a_{\max} \Rightarrow \]  

\[ a_{\max} = 4.0 \times 10^8 \text{ m/s}^2 \]

Because from Equation 14.18 \( a_{\max} = (2\pi f)^2A \),

\[ A = \frac{a_{\max}}{(2\pi f)^2} = \frac{4.0 \times 10^8 \text{ m/s}^2}{[2\pi(1.0 \times 10^6 \text{ Hz})]^2} = 1.01 \times 10^{-3} \text{ m} = 10 \mu\text{m} \]

(b) The maximum speed is

\[ v_{\max} = \omega A = 2\pi(1.0 \times 10^6 \text{ Hz})(1.01 \times 10^{-5} \text{ m}) = 64 \text{ m/s} \]

Assess: Apparently, the amplitude is very small and the maximum oscillation speed is large. However, to oscillate at high frequencies such as 1.0 MHz, you would expect a small amplitude and a large speed.

P14.53. Prepare: The compact car is in simple harmonic motion. The mass on each spring for the empty car is \((1200 \text{ kg})/4 = 300 \text{ kg}\). However, the car carrying four persons means that each spring has, on the average, an additional mass of 70 kg. For both parts we will use Equation 14.27.

Solve: (a) The spring constant can be calculated as follows:

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \Rightarrow k = m(2\pi f)^2 = (300 \text{ kg})[2\pi(2.0 \text{ Hz})]^2 = 4.74 \times 10^4 \text{ N/m} = 4.7 \times 10^4 \text{ N/m} \]

to two significant figures.

(b) Here \( m = 370 \text{ kg} \), so

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{4.74 \times 10^4 \text{ N/m}}{370 \text{ kg}}} = 1.8 \text{ Hz} \]

Assess: A small frequency change from the additional mass is reasonable because frequency is inversely proportional to the square root of the mass.

P14.54. Prepare: The frequency is \( f = (6.0 \text{ m/s})/(5.0 \text{ m}) = 1.2 \text{ Hz} \). We solve Hooke’s law for the distance moved, \( F = -k\Delta x \Rightarrow \Delta x = F/k \) where we drop the minus sign because we already know the car will rise up. \( F \) is the weight of the people and is 300 kg.

Solve:

\[ \Delta x = \frac{mg}{k} = \frac{m_{\text{people}}g}{(2\pi f)^2m_{\text{tot}}} = \frac{300 \text{ kg})(9.8 \text{ m/s}^2)}{(2\pi(1.2\text{ Hz}))^2}(1400 \text{ kg})} = 3.7 \text{ cm} \]

Assess: This seems to be a reasonable amount for the car to rise when four people get out.
**P14.55. Prepare:** A completely inelastic collision between the two gliders results in simple harmonic motion. Let us denote the 250 g and 500 g masses as \( m_1 \) and \( m_2 \), which have initial velocities \((v_1)_i\) and \((v_2)_i\). After \( m_1 \) collides with and sticks to \( m_2 \), the two masses move together with velocity \( v_f \). We will first find the final velocity using momentum conservation and then use the mechanical energy conservation equation for the two stuck gliders to determine the amount of compression or the amplitude.

![Graph](image)

**Solve:** The momentum conservation equation \( p_i = p_f \) for the completely inelastic collision is \((m_1 + m_2)v_f = m_1(v_1)_i + m_2(v_2)_i\). Substituting the given values,

\[
(0.750 \text{ kg})(0.40 \text{ m/s}) = (0.25 \text{ kg})(1.20 \text{ m/s}) + (0.50 \text{ kg})(0 \text{ m/s}) \Rightarrow v_f = 0.40 \text{ m/s}
\]

We now use the conservation of mechanical energy equation,

\[
(K + U)_{\text{compressed}} = (K + U)_{\text{equilibrium}} \Rightarrow 0 + \frac{1}{2}kA^2 = \frac{1}{2}(m_1 + m_2)v_f^2 + 0 \text{ J}
\]

\[
\Rightarrow A = \sqrt{\frac{m_1 + m_2}{k}}v_f = \sqrt{\frac{0.750 \text{ kg}}{10 \text{ N/m}}} \cdot (0.40 \text{ m/s}) = 0.11 \text{ m}
\]

The period is

\[
T = 2\pi \sqrt{\frac{m_1 + m_2}{k}} = 2\pi \sqrt{\frac{0.750 \text{ kg}}{10 \text{ N/m}}} = 1.7 \text{ s}
\]

**Assess:** The magnitudes of both the amplitude and the time period are physically reasonable.

**P14.56. Prepare:** A completely inelastic collision between the bullet and the block results in simple harmonic motion. Let us denote the bullet’s and block’s masses as \( m_b \) and \( m_b \), which have initial velocities \( v_b \) and \( v_B \). After \( m_b \) collides with and sticks to \( m_B \), the two move together with velocity \( v_f \) and exhibit simple harmonic motion. Since the amplitude of the harmonic motion is given, we will first find the final velocity of the bullet + block system using the mechanical energy conservation equation. The momentum conservation will then give us the bullet’s speed.
Solve: (a) The equation for conservation of energy after the collision is

$$\frac{1}{2} kA^2 = \frac{1}{2} (m_b + m_n) v_f^2 \Rightarrow v_f = \sqrt{\frac{k}{m_b + m_n}} A = \sqrt{\frac{2500 \text{ N/m}}{1.010 \text{ kg}}} (0.10 \text{ m}) = 4.975 \text{ m/s}$$

The momentum conservation equation for the perfectly inelastic collision $p_{after} = p_{before}$ is

$$(m_b + m_n) v_f = m_b v_{b} + m_n v_{n}$$

$$(1.010 \text{ kg})(4.975 \text{ m/s}) = (0.010 \text{ kg}) v_{b} + (1.00 \text{ kg})(0 \text{ m/s}) \Rightarrow v_{b} = 502 \text{ m/s}$$

(b) No. The oscillation frequency $\frac{1}{2\pi} \sqrt{k/(m_b + m_n)}$ depends on the masses but not on the speeds.

Assess: The bullet’s speed is high but not unimaginable.

**P14.57. Prepare:** When the block is displaced from the equilibrium position one spring is compressed and exerts a restoring force on the block while the other spring is stretched and also exerts a restoring force on the block. These two forces have the same magnitude since the springs are identical and a stretch or compression of the same amount produces the same restoring force.

The restoring force would be the same if the springs were on the same side of the block. For springs in parallel like that, the total $k_{tot}$ is the sum of the two $k$’s. The restoring force would even be the same if there were only one spring with a spring constant twice as big (since the two springs each have the same $k$).

**Solve:** $f_{two}$ is the frequency for the system with two springs.

$$f_{two} = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}} = \frac{1}{2\pi} \sqrt{\frac{20 \text{ N/m} + 20 \text{ N/m}}{2.5 \text{ kg}}} = 0.64 \text{ Hz}$$

Assess: The answer seems reasonable.

If the two $k$’s are the same $k_1 = k_2 = k$ (as in this case), you can see the general formula would be

$$f_{two} = \frac{1}{2\pi} \sqrt{\frac{k_1 + k_2}{m}} = \frac{1}{2\pi} \sqrt{\frac{2k}{m}} = \sqrt{\frac{1}{2\pi} \sqrt{\frac{k}{m}}} = f_{one}$$

The frequency with two identical springs is $\sqrt{2}$ times the frequency with one spring.

An even more general result can be obtained with similar reasoning even where the two $k$’s differ, $f_{two}^2 = f_1^2 + f_2^2$. 
P14.58. **Prepare:** The Super Bungee will stretch until all of the bus’s kinetic energy is stored as elastic potential energy in the cord. The bus will stop at one-fourth the period of oscillation.

**Solve:** The bus stops when all its kinetic energy is stored as elastic potential energy in the cord. This information may be used to determine the spring constant of the Super Bungee.

\[
\frac{mv^2}{2} = k\Delta x^2 \quad \text{or} \quad k = \frac{mv^2}{\Delta x^2} = \frac{(12 \times 10^3 \text{ kg})(21.2 \text{ m/s})^2}{(50 \text{ m})^2} = 2.16 \times 10^3 \text{ N/m}
\]

The bus stops in a time equal to one-fourth the period of oscillation

\[
t = \frac{T}{4} = \frac{(1/4)(2\pi/v)}{(1/4)(2\pi)} = \frac{\pi/2}{\pi A/2v} = \frac{\pi}{2}(50 \text{ m})/2(21.2 \text{ m/s}) = 3.70 \text{ s}
\]

**Assess:** Many students will be tempted to use the kinematic equations from the early chapters of the text. However these equations will not work because they are applicable only for the case of constant acceleration. In this case the acceleration is not a constant.

P14.59. **Prepare:** The strategy will be to find how long the block on the left will take to fall to the table in free fall and use that as one-half of a period for the oscillator on the right. The spring is 30 cm long, so before release the spring is neither stretched nor compressed and the block is at its maximum height (not in equilibrium because the force of gravity is still acting on it). To go from maximum height to minimum height takes one-half of a period.

Once we know the period and the mass of the block on the right we can solve for \( k \).

**Solve:**

\[
\Delta y = \frac{1}{2} a_y (\Delta t)^2
\]

\[
\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-0.30 \text{ m})}{-9.8 \text{ m/s}^2}} = 0.247 \text{ s} = \frac{1}{2} T
\]

\[
T = 2(0.247 \text{ s}) = 0.495 \text{ s}
\]

Solve Equation 14.27 for \( k \).

\[
k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 (0.050 \text{ kg})}{(0.495 \text{ s})^2} = 8.1 \text{ N/m}
\]

**Assess:** This is a reasonable spring, neither extremely stiff nor extremely loose. The mass of the block on the left is irrelevant since all objects have the same acceleration in free fall. The block on the right will pass through its equilibrium position when it falls far enough that \( k\Delta y = mg \); that position is halfway down its total descent.

P14.60. **Prepare:** Assume a small angle oscillation of the pendulum so that it has simple harmonic motion. We will use Equation 14.31.

**Solve:** (a) At the equator, the period of the pendulum is

\[
T_{\text{equator}} = 2\pi \sqrt{\frac{1.000 \text{ m}}{9.78 \text{ m/s}^2}} = 2.009 \text{ s}
\]

The time for 100 oscillations is 200.9 s.

(b) At the North Pole, the period is

\[
T_{\text{pole}} = 2\pi \sqrt{\frac{1.000 \text{ m}}{9.83 \text{ m/s}^2}} = 2.004 \text{ s}
\]

The time for 100 oscillations is 200.4 s.

(c) The period on the top of the mountain is 2.010 s. The acceleration due to gravity can be calculated by rearranging the formula for the period:

\[
g_{\text{mountain}} = L \left( \frac{2\pi}{T_{\text{mountain}}} \right)^2 = (1.000 \text{ m}) \left( \frac{2\pi}{2.010 \text{ s}} \right)^2 = 9.77 \text{ m/s}^2
\]

**Assess:** The difference between the answers at the equator and the North Pole is 0.5 s, and this difference is quite measurable with a hand-operated stopwatch. The acceleration due to gravity at the top of a high mountain at the equator is reasonable because \( g \) decreases with altitude and it has decreased by 0.01 m/s^2.
P14.61. **Prepare:** Because this is a “heavy bob supported on a very thin rod” we can model it as a simple pendulum; we will also assume the amplitude of oscillation is small.

As the temperature rises the thin steel rod will lengthen according to Equation 12.19, \( \Delta L = \alpha L \Delta T \) (where this \( T \) is the temperature). Use Table 12.3 to look up \( \alpha_{\text{steel}} = 12 \times 10^{-6} \text{K}^{-1} \).

**Solve:**

(a) The period of the simple pendulum is given by

\[
T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{1.00000 \text{ m}}{9.80 \text{ m/s}^2}} = 2.00709 \text{ s}
\]

(b) First compute the new length of the pendulum as follows:

\[
\Delta L = \alpha L \Delta T = (12 \times 10^{-6} \text{K}^{-1})(1.00000 \text{ m})(10 \text{ K}) = 0.00012 \text{ m}
\]

\[L_i = L_i + \Delta L = 1.00012 \text{ m}\]

Now put the new length in the formula for the period.

\[
T = 2\pi \sqrt{\frac{L_i}{g}} = 2\pi \sqrt{\frac{1.00012 \text{ m}}{9.80 \text{ m/s}^2}} = 2.00721 \text{ s}
\]

(c) Every period the warm clock is off by the difference of the two periods, 0.00012 s; so the warm clock will be off by 1.0 s in 1/0.00012 s periods. That’s 8304 periods, or 8304(2.00709 s) = 16667 s = 4.6 h.

**Assess:** When you do integrated problems like this you are even more likely than usual to run into conflicting uses of the same symbol. Please keep careful track of when \( T \) is the temperature and when it is the period. Because of these timekeeping inaccuracies due to temperature variations, which are really not acceptable, clockmakers such as John Harrison in the 18th century made pendulums by combining strips of two different metals that counteracted each other’s changes in length.

P14.62. **Prepare:** The pendulum falls, then undergoes small-amplitude oscillations in simple harmonic motion. We have placed the origin of the coordinate system at the bottom of the arc. To find the oscillation frequency using Equation 14.31, we need to first find the length of the pendulum, which we will do using the mechanical conservation equation.

[Diagram of pendulum with origin at bottom of arc, initial velocity \( v_i = 5.0 \text{ m/s} \), final position \( y_f = 2L \), and length \( L \).]

**Solve:** We need to find the length of the pendulum. The conservation of mechanical energy equation for the pendulum’s fall is \((K + U_y)_{\text{top}} = (K + U_y)_{\text{bottom}}\).

\[
\frac{1}{2}mv_i^2 + mg y_i = \frac{1}{2}mv_f^2 + mg y_f \Rightarrow 0 + mg(2L) = \frac{1}{2}m(5.0 \text{ m/s})^2 + 0 J
\]

\[
\Rightarrow L = \frac{1}{4} \frac{(5.0 \text{ m/s})^2}{g} = 0.6377 \text{ m}
\]

Using \( L = 0.6377 \text{ m} \), we can find the frequency \( f \) as

\[
f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} = \frac{1}{2\pi} \sqrt{\frac{9.8 \text{ m/s}^2}{0.6377 \text{ m}}} = 0.62 \text{ Hz}
\]

**Assess:** A length of 0.64 m for the pendulum and an oscillation frequencies of 0.62 Hz are reasonable.
P14.63. **Prepare:** When we have two oscillators at slightly different frequencies we observe a regular pattern when they are in step and out of step. This is called beats.

**Solve:** (a) For the 24.8-cm pendulum

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} = \frac{1}{2\pi} \sqrt{\frac{9.8 \text{ m/s}^2}{0.248 \text{ m}}} = 1.00 \text{ Hz} \]

\[ T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{0.248 \text{ m}}{9.8 \text{ m/s}^2}} = 1.00 \text{ s} \]

For the 38.8-cm pendulum

\[ f = \frac{1}{2\pi} \sqrt{\frac{g}{L}} = \frac{1}{2\pi} \sqrt{\frac{9.8 \text{ m/s}^2}{0.388 \text{ m}}} = 0.800 \text{ Hz} \]

\[ T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{0.388 \text{ m}}{9.8 \text{ m/s}^2}} = 1.25 \text{ s} \]

(b) Say they both tick at \( t = 0.0 \text{ s} \). The slower one will tick at \( t = 1.25 \text{ s}, t = 2.50 \text{ s}, t = 3.75 \text{ s}, \) and \( t = 5.0 \text{ s} \), at which time it will be back on an integer second and coincide with the faster one, which ticks every second. It took 5.00 s for this pattern to happen and it will repeat each 5.00 s.

(c) The frequency of this phenomenon is \( 1/T = 1/5.00 \text{ s} = 0.200 \text{ Hz} \). What we notice is that this frequency is the difference of the original frequencies.

**Assess:** The beat frequency is the difference between the frequencies of the two oscillators. This phenomenon is most often observed in daily life with sound. Musicians tune their instruments by playing together, listening for the beats (which indicate the frequencies aren’t identical), and adjusting their instruments or embouchures until the beats go away, or the beat frequency is zero.

P14.64. **Prepare:** Assume the orangutan is a simple small-angle pendulum. One swing of the arm would be half a period of the oscillatory motion. The horizontal distance traveled in that time would be

\[ d = 2(0.90 \text{ m}) \sin(20^\circ) = 0.616 \text{ m} \]

from analysis of a right triangle.

**Solve:**

\[ T = \frac{2\pi}{2} \sqrt{\frac{L}{g}} = \frac{2\pi}{2} \sqrt{\frac{0.90 \text{ m}}{9.8 \text{ m/s}^2}} = 0.952 \text{ s} \]

\[ \text{speed} = \frac{\text{dist}}{\text{time}} = \frac{0.616 \text{ m}}{0.952 \text{ s}} = 0.65 \text{ m/s} \]

**Assess:** This isn’t very fast, but isn’t out of the reasonable range.

P14.65. **Prepare:** The doll’s head is in simple harmonic motion and is damped, so we will use Equations 14.27 and 14.33. The maximum amplitude of the oscillation is 2 cm.

**Solve:** (a) The oscillation frequency is

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \Rightarrow k = m(2\pi f)^2 = (0.015 \text{ kg})(2\pi)^2(4.0 \text{ Hz})^2 = 9.475 \text{ N/m} = 9.5 \text{ N/m} \]

to two significant figures.

(b) The maximum speed is

\[ v_{\text{max}} = (2\pi f)A = \sqrt{\frac{k}{m}} A = \sqrt{\frac{9.475 \text{ N/m}}{0.015 \text{ kg}}} (0.020 \text{ m}) = 0.503 \text{ m/s} = 0.50 \text{ m/s} \]

(c) Using \( x(t) = Ae^{-\tau t} \), we get

\[ (0.5 \text{ cm}) = (2.0 \text{ cm})e^{-\left(\frac{4.0 \text{ s}}{\tau}\right)} \Rightarrow 0.25 = e^{-4/\tau} \Rightarrow -4/\tau = \ln 0.25 \Rightarrow \tau = 2.9 \text{ s} \]

P14.66. **Prepare:** The oscillator is in simple harmonic motion and is damped, so we will use Equation 14.33. The time period of the oscillation is 0.50 s and its initial amplitude is 10 cm.
Solve: \( (a) \) The maximum displacement at time \( t \) of a damped oscillator is

\[
x_{\text{max}}(t) = Ae^{-\frac{t}{\tau}}
\]

Using \( x_{\text{max}} = 0.98A \) at \( t = 0.50 \text{ s} \), we can find the time constant \( \tau \) to be

\[
\tau = \frac{0.5 \text{ s}}{\ln(0.98)} = 24.75 \text{ s}
\]

25 oscillations will be completed at \( t = 25\tau = 12.5 \text{ s} \). At that time, the amplitude will be

\[
x_{\text{max, 12.5 s}} = (10.0 \text{ cm})e^{-12.5 \text{ s}/24.75 \text{ s}} = 6.0 \text{ cm}
\]

Assess: The oscillation decreases to 37\% of its initial value after \( \tau \). But, since \( t < \tau \), we have not reached the amplitude \( 0.37 \times 10 \text{ cm} = 3.7 \text{ cm} \) yet. So, a value of 6.0 cm for the amplitude at \( t = 12.5 \text{ s} \) is reasonable.

P14.67. Prepare: The maximum speed occurs at the equilibrium position.

Solve:

\[
v_{\text{max}} = \frac{2\pi}{T} A = \frac{2\pi}{0.50 \text{ s}} (0.30 \text{ m}) = 0.377 \text{ m/s}
\]

After it detaches it is in free fall.

\[
H = \frac{v_0^2}{2g} = \frac{(3.77 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 0.7251 \text{ m} \approx 73 \text{ cm}
\]

Assess: This is a safe and reasonable number. The spring will not put the baby into orbit.

P14.68. Prepare: We won’t have good accuracy from reading off the graph.

Solve:

(a) Looking at the graph, the period appears to be about 0.75 s.

(b) \[m = \left(\frac{T}{2\pi}\right)^2 k = \left(\frac{0.75 \text{ s}}{2\pi}\right)^2 (1.2 \text{ N/m}) = 17 \text{ g}\]

(c) From peak to peak the amplitude appears to decrease by a factor of four in one period.

\[
\tau = \frac{t}{\ln(\frac{1}{4})} = \frac{0.75 \text{ s}}{\ln(\frac{1}{4})} = 0.54 \text{ s}
\]

Assess: The answers seem to fit with the graph.

P14.69. Prepare: The oscillator is in simple harmonic motion and is damped, so we will use Equation 14.33.

Solve: The maximum displacement, or amplitude, of a damped oscillator decreases as \( x_{\text{max}}(t) = Ae^{-\frac{t}{\tau}} \), where \( \tau \) is the time constant. We know \( x_{\text{max}}/A = 0.60 \) at \( t = 50 \text{ s} \), so we can find \( \tau \) as follows:

\[
-\frac{t}{\tau} = \ln\left(\frac{x_{\text{max}}(t)}{A}\right) \Rightarrow \tau = -\frac{50 \text{ s}}{\ln(0.60)} = 97.88 \text{ s}
\]

Now we can find the time \( t_{50} \) at which \( x_{\text{max}}/A = 0.30 \)

\[
t_{50} = -\tau \ln\left(\frac{x_{\text{max}}(t)}{A}\right) = -(97.88 \text{ s})\ln(0.30) = 118 \text{ s}
\]

The undamped oscillator has a frequency \( f = 2 \text{ Hz} = 2 \text{ oscillations per second} \). Then the number of oscillations before the amplitude decays to 30\% of its initial amplitude is \( N = f \cdot t_{50} = (2 \text{ oscillations/s}) \cdot (118 \text{ s}) = 236 \text{ oscillations} \) or 240 oscillations to two significant figures.
P14.70. **Prepare:** With no numbers given we must get all our information from the graph.

**Solve:** From the graph count three full periods in about 3.4 s, so it appears that $T = 1.1$ s.
From the graph after a full 4.0 s it looks like the amplitude is about 0.4 of the original amplitude, this is close to 37%, so we guess that $\tau = 4.0$ s.

**Assess:** Part b can also be done by reading off the time it takes for the amplitude to decrease by half. This appears to be about 2.8 s, so $\tau = \frac{\ln(0.5)}{\ln(0.13)} = 4.0$ s.

P14.71. **Prepare:** In one time constant the displacement is decreased to 37% of its initial value, $y = A(1/e) = .37A$; in two time constants it will be $A(1/e)^2 = 0.13A$.

The initial total energy is $E = \frac{1}{2} kA^2 = \frac{1}{2} (220 \text{ N/} \text{ms})(0.15 \text{ m})^2 = 2.475 \text{ J}$.

**Solve:**

$$E_{\text{new}} = \frac{1}{2} ky^2 = \frac{1}{2} k(0.13A)^2 = (0.13)^2 \frac{1}{2} kA^2 = (0.13)^2 (2.475 \text{ J}) = 0.0453 \text{ J}$$

The energy dissipated is the difference between the original total energy and what is left. $E_{\text{diss}} = 2.475 \text{ J} - 0.0453 \text{ J} = 2.43 \text{ J}$, which should be rounded to 2.4 J.

**Assess:** There is really very little energy left after $2\tau$.

P14.72. **Prepare:** We are given $m = 60 \text{ kg}$ and $\Delta y = 2.5 \text{ mm}$.

**Solve:**

(a) $$k = \frac{F}{\Delta y} = \frac{mg}{\Delta y} = \frac{(60 \text{ kg})(9.80 \text{ m/s}^2)}{2.5 \text{ mm}} = 235200 \text{ N/m} \approx 2.4 \times 10^5 \text{ N/m}$$

(b) $$T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{60 \text{ kg}}{235200 \text{ N/m}}} = 0.10 \text{ s}$$

(c) The running stride period is not the same as the oscillation period. The conversion from kinetic to potential and back to kinetic energy should take a whole oscillation period, so that is the time between landing and liftoff, or 0.10 s.

(d) Yes, it corresponds nicely.

**Assess:** It’s nice how the parts fit together.

P14.73. **Prepare:** Since the web is horizontal, it will sag until the effective spring force of the web is equal to the weight of the fly. That is $kx = mg$.

**Solve:** Solving the above expression for $k$, obtain

$$k = \frac{mg}{x} = \frac{(12 \times 10^{-6} \text{ kg})(9.80 \text{ m/s}^2)}{3 \times 10^{-3} \text{ m}} = 0.039 \text{ N/m}$$

The correct choice is A.

**Assess:** Since the web is very delicate, we expect a small effective spring constant.

P14.74. **Prepare:** Example 14.7 shows how this is done, $k = mg/\Delta L$. And $f = (1/2\pi)\sqrt{k/m}$.

**Solve:** Combine the two equations above by inserting the first one for $k$ into the second one.

$$f = \frac{1}{2\pi} \sqrt{\frac{mg}{m \Delta L}} = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta L}} = \frac{1}{2\pi} \sqrt{\frac{9.8 \text{ m/s}^2}{3.0 \times 10^{-3} \text{ m}}} = 9.1 \text{ Hz}$$

The correct answer is C.

**Assess:** This is not an extremely high frequency, but within the range of spider detection, and one you could see.

P14.75. **Prepare:** Changing the orientation of the web (but otherwise leaving the web alone) changes nothing in the equations.

**Solve:** The frequency remains the same. The answer is C.
Assess: We’ve seen that a mass on a vertical spring is analyzed the same way as a mass and spring on a horizontal frictionless surface.

P14.76. Prepare: Equation 14.18 gives the maximum acceleration as $a_{\text{max}} = (2\pi f)^2 A$. We could separately compute $a_{\text{max}}$ for each case, but since we don’t care what the actual values are it might be better to try the ratio approach.

Let the subscript 1 refer to the low frequency case and 2 to the high frequency case; $f_1 = 1 \text{ Hz}$, $f_2 = 1 \text{ kHz}$, $A_1 = 0.1 \text{ mm}$, and $A_2 = 0.1 \mu\text{m}$.

Solve:

$$\frac{(a_{\text{max}})_1}{(a_{\text{max}})_2} = \frac{(2\pi f_1)^2 A_1}{(2\pi f_2)^2 A_2} = \frac{f_1^2 A_1}{f_2^2 A_2} = \frac{(1\text{ Hz})^2(0.1 \times 10^{-3} \text{ m})}{(1\times 10^3\text{ Hz})^2(0.1 \times 10^{-6} \text{ m})} = 10^{-3}$$

The $a_{\text{max}}$ for the low-frequency oscillation is only one-thousandth as much as $a_{\text{max}}$ for the high-frequency oscillation.

The correct answer is B.

Assess: The answer makes sense: The spider can feel the high-frequency oscillations better because the maximum acceleration is greater.