In Problems 7–14, find i (the rate per period) and n (the number of periods) for each annuity.

7. Quarterly deposits of $500 are made for 20 years into an annuity that pays 8% compounded quarterly. \( i = 0.02; \) \( n = 80 \)

8. Monthly deposits of $350 are made for 6 years into an annuity that pays 6% compounded monthly. \( i = 0.005; \) \( n = 72 \)

9. Semiannual deposits of $900 are made for 12 years into an annuity that pays 7.5% compounded semiannually. \( i = 0.0375; \) \( n = 24 \)

10. Annual deposits of $2,500 are made for 15 years into an annuity that pays 6.25% compounded annually. \( i = 0.0625; \) \( n = 15 \)

11. Monthly deposits of $235 are made for 4 years into an annuity that pays 9% compounded monthly. \( i = 0.0075; \) \( n = 48 \)

12. Semiannual deposits of $1,900 are made for 7 years into an annuity that pays 8.5% compounded semiannually. \( i = 0.0425; \) \( n = 14 \)

13. Annual deposits of $3,100 are made for 12 years into an annuity that pays 5.95% compounded annually. \( i = 0.0595; \) \( n = 12 \)

14. Quarterly deposits of $1,200 are made for 18 years into an annuity that pays 7.6% compounded quarterly. \( i = 0.019; \) \( n = 72 \)

B In Problems 15–22, use the future value formula (6) to find each of the indicated values.

15. \( n = 20; \) \( i = 0.03; \) \( PMT = 500; \) \( FV = ? \) \( FV = 13,435.19 \)

16. \( n = 25; \) \( i = 0.04; \) \( PMT = 100; \) \( FV = ? \) \( FV = 4,164.59 \)

17. \( FV = 5,000; \) \( n = 15; \) \( i = 0.01; \) \( PMT = ? \) \( PMT = 310.62 \)

18. \( FV = 2,500; \) \( n = 10; \) \( i = 0.08; \) \( PMT = ? \) \( PMT = 172.57 \)

19. \( FV = 4,000; \) \( i = 0.02; \) \( PMT = 200; \) \( n = ? \) \( n = 17 \)

20. \( FV = 8,000; \) \( i = 0.04; \) \( PMT = 500; \) \( n = ? \) \( n = 13 \)

21. \( FV = 7,600; \) \( PMT = 500; \) \( n = 10; \) \( i = ? \) \( \text{Round answer to two decimal places.} \) \( i = 0.09 \)

22. \( FV = 4,100; \) \( PMT = 100; \) \( n = 20; \) \( i = ? \) \( \text{Round answer to two decimal places.} \) \( i = 0.07 \)

C 23. Explain what is meant by an ordinary annuity. \( \text{No answers required} \)

24. Explain why no interest is credited to an ordinary annuity at the end of the first period. \( \text{No answers required} \)

25. Solve the future value formula (6) for \( n \).

26. Solve the future value formula (6) for \( i \) if \( n = 2 \). \( i = \frac{FV}{PMT} - 2 \)

Applications

27. Guaranty Income Life offered an annuity that pays 6.65% compounded monthly. If $500 is deposited into this annuity every month, how much is in the account after 10 years? How much of this is interest? \( \text{Value: } \$84,895.40; \text{interest: } \$24,895.40 \)

28. USG Annuity and Life offered an annuity that pays 7.25% compounded monthly. If $1,000 is deposited into this annuity every month, how much is in the account after 15 years? How much of this is interest? \( \text{Value: } \$323,943.07; \text{interest: } \$143,943.07 \)

29. In order to accumulate enough money for a down payment on a house, a couple deposits $300 per month into an account paying 6% compounded monthly. If payments are made at the end of each period, how much will be in the account in 5 years? \( \$20,931.01 \)

30. A self-employed person has a Keogh retirement plan. (This type of plan is free of taxes until money is withdrawn.) If deposits of $7,500 are made each year into an account paying 8% compounded annually, how much will be in the account after 20 years? \( \$343,214.73 \)

31. Sun America offered an annuity that pays 6.35% compounded monthly. What equal monthly deposit should be made into this annuity in order to have $200,000 in 15 years? \( \$667.43 \)

32. The Hartford offered an annuity that pays 5.5% compounded monthly. What equal monthly deposit should be made into this annuity in order to have $100,000 in 10 years? \( \$626.93 \)

33. A company estimates that it will need $100,000 in 8 years to replace a computer. If it establishes a sinking fund by making fixed monthly payments into an account paying 7.5% compounded monthly, how much should each payment be? \( \$763.10 \)

34. Parents have set up a sinking fund in order to have $120,000 in 15 years for their children's college education. How much should be paid semiannually into an account paying 6.8% compounded semiannually? \( \$2,363.07 \)

35. If $1,000 is deposited at the end of each year for 5 years into an ordinary annuity earning 8.32% compounded annually, construct a balance sheet showing the interest earned during each year and the balance at the end of each year. \( \text{No table provided} \)

36. If $2,000 is deposited at the end of each quarter for 2 years into an ordinary annuity earning 7.9% compounded quarterly, construct a balance sheet showing the interest earned during each quarter and the balance at the end of each quarter. \( \text{No table provided} \)

37. Beginning in January, a person plans to deposit $100 at the end of each month into an account earning 6% compounded monthly. Each year taxes must be paid on the interest earned during that year. Find the interest earned during each year for the first 3 years. \( \text{First year: } \$33.56; \text{second year: } \$109.64; \text{third year: } \$190.41 \)

38. If $500 is deposited each quarter into an account paying 8% compounded quarterly for 3 years, find the interest earned during each of the 3 years. \( \text{First year: } \$60.80; \text{second year: } \$250.68; \text{third year: } \$414.56 \)

39. Bob makes his first $1,000 deposit into an IRA earning 6.4% compounded annually on his 24th birthday and his last $1,000 deposit on his 35th birthday (12 equal deposits in all). With no additional deposits, the money in the IRA continues to earn 6.4% interest compounded annually until Bob retires on his 65th birthday. How much is in the IRA when Bob retires? \( \$1,110,050.27 \)

40. Refer to Problem 39, John procrastinates and does not make his first $1,000 deposit into an IRA until he is 36, but then he continues to deposit $1,000 each year until he is 65 (30 deposits in all). If John's IRA also earns 6.4% compounded annually, how much is in his IRA when he makes his last deposit on his 65th birthday? \( \$84,852.51 \)

41. Refer to Problems 39 and 40. How much would John have to deposit each year in order to have the same amount at retirement as Bob has? \( \$1,308.75 \)

\[ n = \frac{\ln(1 + i) \div FV}{\ln(1 + i)} \]
42. Refer to Problems 39 and 40. Suppose that Bob decides to continue to make $1,000 deposits into his IRA every year until his 65th birthday. If John still waits until he is 36 to start his IRA, how much must he deposit each year in order to have the same amount at age 65 as Bob has? $2,308.75

43. Compubank, an online banking service, offered a money market account with an APY of 1.551%.
   (A) If interest is compounded monthly, what is the equivalent annual nominal rate? 1.540%.
   (B) If you wish to have $10,000 in this account after 4 years, what equal deposit should you make each month? $202.12

44. American Express’s online banking division offered a money market account with an APY of 2.243%.
   (A) If interest is compounded monthly, what is the equivalent annual nominal rate? 2.220%.
   (B) If a company wishes to have $1,000,000 in this account after 8 years, what equal deposit should be made each month? $9,536.40

45. You can afford monthly deposits of $200 into an account that pays 5.7% compounded monthly. How long will it be until you have $7,000? (Round to the next-higher month if not exact.) 33 months

46. A company establishes a sinking fund for upgrading office equipment with monthly payments of $2,000 into an account paying 6.6% compounded monthly. How long will it be before the account has $100,000? (Round up to the next-higher month if not exact.) 45 months

47. A person makes annual payments of $1,000 into an ordinary annuity. At the end of 5 years, the amount in the annuity is $5,840. What annual nominal compounding rate has this annuity earned? 7.77%

48. A person invests $2,000 annually in an IRA. At the end of 6 years, the amount in the fund is $14,000. What annual nominal compounding rate has this fund earned? 6.14%

49. At the end of each month, an employee deposits $50 into a Christmas club fund. At the end of the year, the fund contains $620. What annual nominal rate compounded monthly has this fund earned? 7.13%

50. At the end of each month, an employee deposits $80 into a credit union account. At the end of 2 years, the account contains $2,100. What annual nominal rate compounded monthly has this account earned? 9.24%

In Problems 47–50, use graphical approximation techniques or an equation solver to approximate the desired interest rate. Express each answer as a percentage, correct to two decimal places.

3.4 Present Value of an Annuity; Amortization

- Present Value of an Annuity
- Amortization
- Amortization Schedules
- General Problem-Solving Strategy

Present Value of an Annuity

How much should you deposit in an account paying 6% compounded semiannually in order to be able to withdraw $1,000 every 6 months for the next 3 years? (After the last payment is made, no money is to be left in the account.)

Actually, we are interested in finding the present value of each $1,000 that is paid out during the 3 years. We can do this by solving for $P$ in the compound interest formula:

$$A = P(1 + i)^n$$

$$P = \frac{A}{(1 + i)^n} = A(1 + i)^{-n}$$

The rate per period is $i = \frac{0.06}{2} = 0.03$. The present value $P$ of the first payment is $1,000(1.03)^{-1}$, the present value of the second payment is $1,000(1.03)^{-2}$, and so on. Figure 1 shows this in terms of a time line.