CONCEPTUAL INSIGHT

The two curves in Figure 7 intersect at \( t = 0 \) and again near \( t = 7 \). The \( t \) coordinate of each intersection point is a solution of the equation

\[
1,000(1 + 0.09r) = 1,000(1 + 0.07/12)^{12t}
\]

Don’t try to use algebra to solve this equation. It can’t be done. But the solutions are easily approximated on a graphing calculator (Fig. 8).

![Graph showing two curves intersecting](image)

Figure 8

Exercises 3.2

Find all dollar amounts to the nearest cent. When an interest rate is requested as an answer, express the rate as a percentage correct to two decimal places, unless directed otherwise. In all problems involving days, use a 365-day year.

1. \( P = 950 \)  
2. \( P = 2,500 \)  
3. \( x = 17 \)  
4. \( x = 3.5 \)

W 

Skills Warm-up Exercises

In Problems 1–8, solve the equation for the unknown quantity. (If necessary, review section A.7.)

1. \( 1.6416 = P(1.2)^3 \)  
2. \( 2,652.25 = P(1.03)^2 \)  
3. \( 12x^3 = 58,956 \)  
4. \( 100x^4 = 15,006.25 \)  
5. \( 6.75 = 3(1 + i)^2 \)  
6. \( 13.72 = 5(1 + i)^3 \)  
7. \( 14,641 = 10,000(1.1)^n \)  
8. \( 2,488.32 = 1,000(1.2)^n \)

5. \( i = 0.5 \)  
6. \( i = 0.4 \)  
7. \( n = 4 \)  
8. \( n = 5 \)

A

In Problems 9–12, use compound interest formula (1) to find each of the indicated values.

9. \( P = 5,000; i = 0.005; n = 36; A = ? \)  
10. \( P = 2,800; i = 0.003; n = 24; A = ? \)  
11. \( A = 8,000; i = 0.02; n = 32; P = ? \)  
12. \( A = 15,000; i = 0.01; n = 28; P = ? \)

In Problems 13–20, use the continuous compound interest formula (3) to find each of the indicated values.

13. \( P = 2,450; r = 8.12\%; t = 3 \) years; \( A = ? \)  
14. \( P = 995; r = 22\%; t = 2 \) years; \( A = ? \)  
15. \( A = 6,300; r = 9.45\%; t = 8 \) years; \( P = ? \)  
16. \( A = 19,000; r = 7.69\%; t = 5 \) years; \( P = ? \)  
17. \( A = 88,000; P = 71,153; r = 8.5\%; t = ? \)  
18. \( A = 32,982; P = 27,200; r = 5.93\%; t = ? \)

*Answer located in Additional Instructor’s Answers section.

\* 19. \( A = 15,875; P = 12,100; t = 48 \) months; \( r = ? \) 6.79%
\* 20. \( A = 23,600; P = 19,150; t = 60 \) months; \( r = ? \) 4.18%

In Problems 21–26, use the given annual interest rate \( r \) and the compounding period to find \( i \), the interest rate per compounding period.

21. 9% compounded monthly 0.75% per month
22. 6% compounded quarterly 1.5% per quarter
23. 14.6% compounded daily 0.04% per day
24. 15% compounded monthly 1.25% per month
25. 4.8% compounded quarterly 1.2% per quarter
26. 3.2% compounded semiannually 1.6% per half-year

In Problems 27–32, use the given interest rate \( i \) per compounding period to find \( r \), the annual rate.

27. 0.395% per month 4.74%
28. 0.012% per day 4.38%
29. 0.9% per quarter 3.6%
30. 0.175% per month 2.1%
31. 2.1% per half year 4.2%
32. 1.4% per quarter 5.6%

B

33. If \$100 is invested at 6\% compounded

- (A) annually (B) quarterly (C) monthly
  
  $126.25; 82.25  
  $126.90; 82.90  
  $127.05; 82.95

what is the amount after 4 years? How much interest is earned?

34. If \$2,000 is invested at 7\% compounded

- (A) annually (B) quarterly (C) monthly
  
  $2,805.10; 805.10  
  $2,829.56; 829.56  
  $2,835.25; 835.25

what is the amount after 5 years? How much interest is earned?
35. If $5,000 is invested at 5% compounded monthly, what is the amount after
   (A) 2 years? $5,524.71  (B) 4 years? $6,104.48
36. If $20,000 is invested at 4% compounded monthly, what is the amount after
   (A) 5 years? $24,419.93  (B) 8 years? $27,527.90
37. If $8,000 is invested at 7% compounded continuously, what is the amount after 6 years? $12,175.69
38. If $23,000 is invested at 13.5% compounded continuously, what is the amount after 15 years? $174,250.55

39. Discuss the similarities and the differences in the graphs of future value A as a function of time t if $1,000 is invested for 8 years and interest is compounded monthly at annual rates of 4%, 8%, and 12%, respectively (see the figure).

40. Discuss the similarities and differences in the graphs of future value A as a function of time t for loans of $4,000, $8,000, and $12,000, respectively, each at 7.5% compounded monthly for 8 years (see the figure).

41. If $1,000 is invested in an account that earns 9.75% compounded annually for 6 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.
42. If $2,000 is invested in an account that earns 8.25% compounded annually for 5 years, find the interest earned during each year and the amount in the account at the end of each year. Organize your results in a table.
43. If an investment company pays 6% compounded semiannually, how much should you deposit now to have $10,000
   (A) 5 years from now? $7,440.94  (B) 10 years from now? $5,536.76
44. If an investment company pays 8% compounded quarterly, how much should you deposit now to have $6,000
   (A) 3 years from now? $4,730.96  (B) 6 years from now? $3,730.33
45. If an investment earns 9% compounded continuously, how much should you deposit now to have $25,000
   (A) 36 months from now? $19,084.49  (B) 9 years from now? $11,121.45
46. If an investment earns 12% compounded continuously, how much should you deposit now to have $4,800
   (A) 48 months from now? $2,970.16  (B) 7 years from now? $2,072.21
47. What is the annual percentage yield (APY) for money invested at an annual rate of
   (A) 3.9% compounded monthly? 3.97%
   (B) 2.3% compounded quarterly? 2.32%
48. What is the annual percentage yield (APY) for money invested at an annual rate of
   (A) 4.32% compounded monthly? 4.41%
   (B) 4.31% compounded daily? 4.40%
49. What is the annual percentage yield (APY) for money invested at an annual rate of
   (A) 5.15% compounded continuously? 5.28%
   (B) 5.20% compounded semiannually? 5.27%
50. What is the annual percentage yield (APY) for money invested at an annual rate of
   (A) 3.05% compounded quarterly? 3.09%
   (B) 2.95% compounded continuously? 2.99%
51. How long will it take $4,000 to grow to $9,000 if it is invested at 7% compounded monthly? 11 $\frac{2}{3}$ yr
52. How long will it take $5,000 to grow to $7,000 if it is invested at 6% compounded quarterly? 5 $\frac{3}{4}$ yr
53. How long will it take $6,000 to grow to $8,600 if it is invested at 9.6% compounded continuously? 3.75 yr
54. How long will it take $42,000 to grow to $60,276 if it is invested at 4.25% compounded continuously? 8.5 yr

C In Problems 55 and 56, use compound interest formula (1) to find n to the nearest larger integer value.

55. $A = 2P; i = 0.06; n = ? \quad n \approx 12$
56. $A = 2P; i = 0.05; n = ? \quad n \approx 15$
57. How long will it take money to double if it is invested at
   (A) 10% compounded quarterly? $7 \frac{1}{4}$ yr
   (B) 12% compounded quarterly? 6 yr
58. How long will it take money to double if it is invested at
   (A) 8% compounded semiannually? 9 yr
   (B) 7% compounded semiannually? 10 $\frac{1}{2}$ yr
59. How long will it take money to double if it is invested at
   (A) 9% compounded continuously? 7.7 yr
   (B) 11% compounded continuously? 6.3 yr
60. How long will it take money to double if it is invested at
(A) 21% compounded continuously? 3.3 yr
(B) 33% compounded continuously? 2.1 yr

Applications

61. A newborn child receives a $20,000 gift toward college from her grandparents. How much will the $20,000 be worth in 17 years if it is invested at 7% compounded quarterly? $65,068.44

62. A person with $14,000 is trying to decide whether to purchase a car now, or to invest the money at 6.5% compounded semiannually and then buy a more expensive car. How much will be available for the purchase of a car at the end of 3 years? $16,961.66

63. What will a $210,000 house cost 10 years from now if the inflation rate over that period averages 3% compounded annually? $282,222.44

64. If the inflation rate averages 4% per year compounded annually for the next 5 years, what will a car that costs $17,000 now cost 5 years from now? $20,683.10

65. Rental costs for office space have been going up at 4.8% per year compounded annually for the past 5 years. If office space rent is now $25 per square foot per month, what were the rental rates 5 years ago? $19.78 per ft² per mo

66. In a suburb, housing costs have been increasing at 5.2% per year compounded annually for the past 8 years. A house worth $260,000 now would have had what value 8 years ago? $173,319.50

67. If the population in a particular country is growing at 1.7% compounded continuously, how long will it take the population to double? (Round up to the next-higher year if not exact.) 41 yr

68. If the world population is now about 7.5 billion people and is growing at 1.1% compounded continuously, how long will it take the population to grow to 10 billion people? (Round up to the next-higher year if not exact.) 27 yr

69. (A) If an investment of $100 were made in 1776, and if it earned 3% compounded quarterly, how much would it be worth in 2026? In 2026, 250 years after the signing, it would be worth $175,814.55.

(B) Discuss the effect of compounding interest monthly, daily, and continuously (rather than quarterly) on the $100 investment.

(C) Use a graphing calculator to graph the growth of the investment of part (A).

70. (A) Starting with formula (1), derive each of the following formulas:

\[ P = \frac{A}{(1 + i)^n}, \quad i = \left(\frac{A}{P}\right)^{1/n} - 1, \quad n = \frac{\ln A - \ln P}{\ln(1 + i)} \]

(B) Explain why it is unnecessary to memorize the formulas above for \( P, i, \) and \( n \) if you know formula (1).

No answer required

71. A promissory note will pay $50,000 at maturity 6 years from now. If you pay $28,000 for the note now, what rate compounded continuously would you earn? 9.66%

72. If you deposit $10,000 in a savings account now, what rate compounded continuously would be required for you to withdraw $12,500 at the end of 4 years? 5.58%

73. You have saved $7,000 toward the purchase of a car costing $9,000. How long will the $7,000 have to be invested at 9% compounded monthly to grow to $9,000? (Round up to the next-higher month if not exact.) 2 yr, 10 mo

74. A married couple has $15,000 toward the purchase of a house. For the house that the couple wants to buy, a down payment of $20,000 is required. How long will the money have to be invested at 7% compounded quarterly to grow to $20,000? (Round up to the next-higher quarter if not exact.) 17 quarters or 4 1/4 yr

75. An Individual Retirement Account (IRA) has $20,000 in it, and the owner decides not to add any more money to the account other than interest earned at 6% compounded daily. How much will be in the account 35 years from now when the owner reaches retirement age? $163,295.21

76. If $1 had been placed in a bank account in the year 1066 and forgotten until now, how much would be in the account at the end of 2026 if the money earned 2% interest compounded annually? 2% simple interest? (Now you can see the power of compounding and why inactive accounts are closed after a relatively short period of time.) $180,370,243.40; $20,20

77. How long will it take money to double if it is invested at 7% compounded daily? 8.2% compounded continuously? 3.615 days; 8.453 yr

78. How long will it take money to triple if it is invested at 5% compounded daily? 6% compounded continuously? 8.021 days; 18.310 yr

79. In a conversation with a friend, you note that you have two real estate investments, one that has doubled in value in the past 9 years and another that has doubled in value in the past 12 years. Your friend says that the first investment has been growing at approximately 8% compounded annually and the second at 6% compounded annually. How did your friend make these estimates? The rule of 72 states that the annual compound rate of growth \( r \) of an investment that doubles in \( n \) years can be approximated by \( r = \frac{72}{n} \). Construct a table comparing the exact rate of growth and the approximate rate provided by the rule of 72 for doubling times of \( n = 6, 7, \ldots, 12 \) years. Round both rates to one decimal place.

80. Refer to Problem 79. Show that the exact annual compound rate of growth of an investment that doubles in \( n \) years is given by \( r = 100(2^{1/n} - 1) \). Graph this equation and the rule of 72 on a graphing calculator for \( 5 \leq n \leq 20 \).

Solve Problems 81–84 using graphical approximation techniques on a graphing calculator.

81. How long does it take for a $2,400 investment at 13% compounded quarterly to be worth more than a $3,000 investment at 6% compounded quarterly? 14 quarters