1. Classify each of the following statements as true (T) or false (F).

   _____   a) There exists a function \( f(t) \) such that \( \mathcal{L}\{f(t)\} = \int_{0}^{\pi} e^{-st} f(t) \, dt \).

   _____   b) If \( F(s) = \mathcal{L}\{[f(t)]\} \), then \( \lim_{s \to 0} F(s) = 0 \).

   _____   c) The graph of \( u(t-a)f(t-a) \) is a translation of the graph of \( f(t) \) \( a \) units to the right.

   _____   d) One of the terms in the partial fraction expansion for \( \frac{s}{(s^2 + a^2)(s^2 - b^2)} \) is \( \frac{As^2 + B}{s^2 + a^2} \).

   _____   e) Any linear ODE of order 2 or higher can be transformed into a system of first-order differential equations.

2. Compute \( \mathcal{L}\{u(t-2)e^{3t}\} \) directly from the definition of the Laplace transform.
3. a) Solve the initial-value problem: \( y'' + \omega^2 y = 0; \quad y(0) = 0, \quad y'(0) = \omega \) without using the Laplace transform.

b) Apply the Laplace transform to both sides of the initial-value problem in part (a) and calculate \( \mathcal{L}\{y\} \).

c) What can you conclude from these two calculations?

4. Find the Laplace transforms of the following functions.

a) \( f(t) = \sin^2 3t \)  
\[ \text{Hint: } \sin^2 \theta = \frac{1 - \cos 2\theta}{2} \]

b) \( f(t) = \begin{cases} 
  t^2 & \text{if } t < 3 \\
  1 & \text{if } t \geq 3 
\end{cases} \)
5. Find \( \mathcal{L}^{-1}\{F(s)\} \) of each of the following. [Show work for any partial fraction expansion.]

a) \( F(s) = \frac{1}{s^2 - 6s + 8} \)

b) \( F(s) = \frac{6}{(s - 2)^6} \)

6. Solve the following initial-value problem using the method of Laplace transforms.
\( x'' + x = \sin 2t; \quad x(0) = 2, \quad x'(0) = 1 \). [Use your calculator for any partial fraction expansion.]
7. Find $\mathcal{L}^{-1}\{F(s)\}$ of each of the following.

   a) $F(s) = e^{-3s} \left( \frac{2}{s^2 - 1} \right)$

   b) $F(s) = \frac{2s}{s^2 - 6s + 11}$

8. Compute the Laplace transform of the periodic function $f(t)$ whose graph is shown below. [Hint: On the interval $0 \leq t < a$, $f(t) = t/a$] You may use your calculator to integrate.
9. Consider the linear system: \[
\begin{align*}
x'' + y' - x + y &= -1 \\
x' + y' - x &= t^2
\end{align*}
\]

a) Rewrite the system using operator notation.

b) Eliminate \( y \) from the system in part (a).

c) Find the operational determinate of the system and use the result to determine the number of arbitrary constants in the general solution.

d) Find the general solution to the homogeneous equation for \( x \) only.

10. FREE!