Review Exam 3

You have 1 hour and 15 minutes to take the exam. You may have a 3 by 5 in. note card, your graphing calculator, pen/pencil and eraser, but nothing else, not even extra paper. You will have to show all work, some calculator programs can find derivatives for you, but you will not be allowed to use that feature on the exam.

1. Use sigma notation to write the following Riemann sum. Then, evaluate the Riemann sum using formulas for the sums of powers of positive integers. The right Riemann sum for \( f(x) = x + 10 \) on \([0, 5]\) with \( n = 30 \).

2. Calculate the left and right Riemann sums to estimate the area under \( f(x) = \frac{3}{x} + 3 \) on \([1, 5]\) and \( n = 4 \). You may be asked to draw the curve and the rectangles that represent the Riemann sum for a problem like this.

3. Express \( \lim_{\Delta \to 0} \sum_{k=1}^{n} (x_{k}^*)^8 \Delta x_k \) on \([7, 11]\) as a definite integral.

4. Use \( n = 20 \) subintervals and midpoints to estimate \( \int_0^{10} (10x - x^2) \, dx \)

5. Find the area of the region bounded by the graph of \( f(x) = x^2 - 20 \) and the \( x \)-axis on the interval \([-1, 3]\).

6. Simplify: \( \frac{d}{dx} \int_9^x (20 \sin(t^2) + t^2 + 2) \, dt \).

7. Simplify: \( \frac{d}{dx} \int_2^{x^3} \left( \frac{dp}{p^2} \right) \).

8. Evaluate:
   a. \( \int_0^{\ln 7} e^x \, dx \).
   b. \( \int_{5\pi/4}^{7\pi/4} 7 \csc \theta \cot \theta \, d\theta \).
   c. \( \int_1^{64} \sqrt[3]{y} \, dy \).
   d. \( \int_{-\pi}^{\pi} 5 \sin(x) \, dx \)
   e. \( \int_{-1}^{1} 4 \cos(x) \, dx \)

9. Find the average value of \( f(x) = \frac{7}{x} \) on the interval \([3, 3e]\).

10. Find the point(s) at which the function \( f(x) = 6 - |x| \) equals its average value on the interval \([-6, 6]\).
11. Find the following antiderivatives using substitution:

   a. \( \int (x - 8)^{16} \, dx \).

   b. \( \int \sqrt{8x + 4} \, dx \).

   c. \( \int 2x(x^2 + 15)^{15} \, dx \).

   d. \( \int -6x \sin(3x^2 - 8) \, dx \).

   e. \( \int (5x^4 + 2)\sqrt{x^5 + 2x} \, dx \).

   f. \( \int \frac{(\sqrt{x} + 4)^3}{2\sqrt{x}} \, dx \).

12. Evaluate the following definite integrals using substitution:

   a. \( \int_{0}^{1} \frac{2x}{(x^2 + 2)^2} \, dx \)

   b. \( \int_{0}^{\pi/2} 2 \sin^2(x) \cos(x) \, dx \).

   c. \( \int_{1}^{2} x^2 e^{x^3 - 1} \, dx \).
1. \[ \sum_{k=1}^{30} \left[ \frac{1}{6}k + 10 \right] \frac{1}{6} \approx 62.92. \]

2. The left Riemann sum yields: 18.25. The right Riemann sum yields 15.85.

3. \[ \int_{7}^{11} x^8 \, dx \]

4. \[ \sum_{k=1}^{20} \left( 5 \left( \frac{1}{4} + \frac{k}{2} \right) - \frac{1}{2} \left( \frac{1}{4} + \frac{k}{2} \right)^2 \right) = 166.8750 \]

5. \[ \frac{212}{3} \]

6. \[ 20 \sin(x^2) + x^2 + 2 \]

7. \[ \frac{3}{x^4} \]

8. a. 6, b. 0, c. \( \frac{765}{4} \), d. 0 This is the integral of an odd function, e. \( 8 \sin(1) \) This is the integral of an even function.

9. \[ \bar{f} = \frac{7}{3e - 3} \]

10. The function equals its average value at \( x = 3, -3 \).

11. a. \( \frac{1}{17}(x - 8)^{17} + C \), b. \( \frac{1}{12}(8x + 4)^{3/2} + C \), c. \( \frac{1}{16}(x^2 + 15)^{16} + C \), d. \( \cos(3x^2 - 8) + C \), e. \( \frac{2}{3}(x^5 + 2x)^{3/2} + C \), f. \( \frac{1}{4}(\sqrt{x} + 4)^4 + C \)

12. a. \( \int_{2}^{3} \frac{1}{u^2} \, du = \frac{1}{6} \), b. \( \int_{0}^{1} 2u^2 \, du = \frac{2}{3} \), c. \( \int_{0}^{7} \frac{1}{3} e^u \, du = \frac{e^7 - 1}{3} \)