Math 1B

Test 4 Review

1. Determine the interval of convergence for the following power series.

   a) $\displaystyle\sum_{n=0}^{\infty}\left(\frac{e^n}{n+1}\right)x^n.$

   b) $\displaystyle\sum_{n=1}^{\infty}\frac{2^n(x-4)^n}{n}.$

   (*Note*: Be sure to check the endpoints.)

2. Find a power series representation of $f(x) = \dfrac{x^2}{x+4}$, and the interval of convergence for the series.

3. Derive the Maclaurin series for $f(x) = \pi^x$ by working out the coefficients directly. Write your answer using sigma notation.

   (*Hint*: Recall that $\dfrac{d}{dx}[a^x] = a^x \cdot \ln a$.)

4. Consider the equation $x^2 + y^2 + z^2 + 4x + 6y - 10z + 2 = 0$.

   a) Find the center and the length of the radius.

   b) Find the greatest distance from the origin to a point on the sphere.

5. Match the equations below on the left with the graph of the surface on the right by writing the letter that corresponds to the surface in front of the equation. Note that one surface does not have a corresponding equation.

______ $\dfrac{x^2}{9} + \dfrac{y^2}{16} + \dfrac{z^2}{9} = 1$

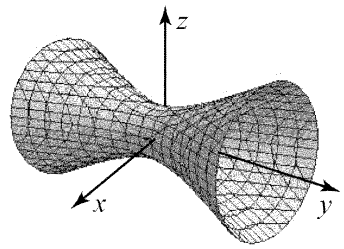
______ $15x^2 - 4y^2 + 15z^2 = -4$

______ $4x^2 - y^2 + 4z^2 = 4$

______ $y^2 = 4x^2 + 9z^2$

______ $4x^2 - 4y + z^2 = 0$

I



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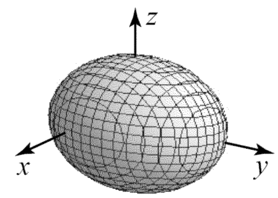

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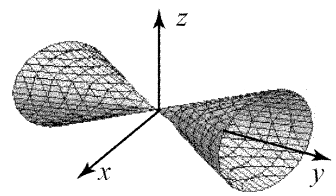

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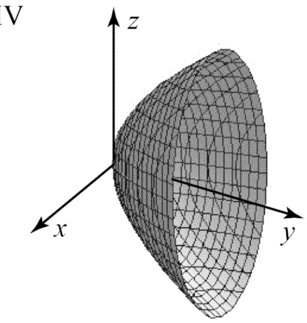

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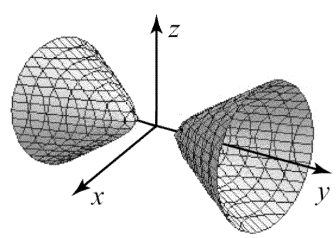

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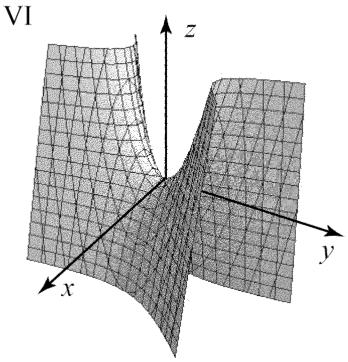

A calculator may be used to help solve problems 6 - 9.

6. Consider the points in space $A(-1, 2, 0)$, $B(2, 0, 1)$, and $C(-5, 3, 1)$.

a) Solve the triangle. (Find the lengths of all the sides and the measures of all the angles to the nearest degree.)

b) Find the area of the triangle $\triangle ABC$.

c) Find the equation of the plane passing through the points.

7. Consider the planes $x + y - z = 1$ and $2x - 3y + 4z = 5$.

a) Show that the planes are neither parallel nor perpendicular.

b) Find, correct to the nearest degree, the angle between these planes.

8. The Maclaurin series for $\sin x$ is given by $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$.

a) Use the Maclaurin series for $\sin x$ to find the power series for $f(x) = \frac{\sin(x^2)}{x}$. Write your answer using summation notation.

b) Use the results of part (a) to obtain $\int_0^1 \frac{\sin(x^2)}{x}\,dx$ to four decimal place accuracy.

9. Consider the function $f(x) = x\cos(x)$.

a) Find the Taylor Polynomial $T_5(x)$ centered at $a = 0$.

b) Find the maximum error in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

c) Find the values of x for which $T_5(x)$ has $|\text{error}| < 0.00001$.