Math 1B		Name	Key
Test 3 - Spring 2026		Roll # ______	Score ______

Part 1: This part has 7 question. No calculator can be used on this part. Each numbered problem is worth 10 points. Show all your work on the test. Appropriate evidence will receive partial credit. Do not write on the back of the page. If you need more room, please use the "Spillage" page that is attached to the back of the exam. Good Luck!!

1. Consider the infinite series $\sum_{n=0}^{\infty} \frac{(-4)^{n+1}}{7^n}$.

a) List out the first three term. Analyze the $\lim_{n\to\infty} a_n$.

$S = -4 + \frac{16}{7} - \frac{64}{49} + \ldots$

$\lim_{n\to\infty} a_n \stackrel{?}{=} 0$ yes

b) Choose a test and determine if the infinite series converges or diverges. If it converges, find it sum. Show all work!

By "GST" $|r| = \left|\frac{\frac{16}{7}}{-4}\right| = \frac{4}{7} < 1$ with $a = -4$, $r = -\frac{4}{7}$

converges

$S = \frac{a}{1-r}$

$S = \frac{-4}{1+\frac{4}{7}}$

$S = -\frac{28}{11}$

c) Write a conclusion stating the test you used to determine convergence/divergence.

Thus, the infinite series converges by "GST", since $|r| < 1$ and it's sum is $-\frac{28}{11}$.

2. Consider the telescoping series $\sum_{n=1}^{\infty} \frac{3}{n^2+3n}$.

a) Find a formula a_i for the decomposition of the general term a_n of the given infinite series.

$a_i = \frac{3}{i(i+3)}$

$\frac{3}{i(i+3)} = \frac{A}{i} + \frac{B}{i+3}$

$3 = A(i+3) + Bi$

Let $i = -3$ and $i = 0$

$B = -1$ $A = 1$

$a_i = \frac{1}{i} - \frac{1}{i+3}$

b) Express the series as a partial sum s_n in sigma notation, $s_n = \sum_{i=1}^{n} a_i$.

$S_n = \sum_{i=1}^{n} \left(\frac{1}{i} - \frac{1}{i+3}\right)$

c) Expand the partial sum s_n and simplify.

$S_n = \left(1 - \cancel{\frac{1}{4}}\right) + \left(\frac{1}{2} - \cancel{\frac{1}{5}}\right) + \left(\frac{1}{3} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{4}} - \cancel{\frac{1}{7}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{8}}\right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{9}}\right) + \ldots$

$+ \left(\cancel{\frac{1}{n-3}} - \cancel{\frac{1}{n}}\right) + \left(\cancel{\frac{1}{n-2}} - \frac{1}{n+1}\right) + \left(\cancel{\frac{1}{n-1}} - \frac{1}{n+2}\right) + \left(\cancel{\frac{1}{n}} - \frac{1}{n+3}\right)$

$S_n = 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}$

d) Find the sum of the infinite series, if it exists.

$\lim_{n \to \infty} S_n = 1 + \frac{1}{2} + \frac{1}{3} - 0 - 0 - 0$

$= \frac{11}{6}$ thus $\sum_{n=1}^{\infty} \frac{3}{n^2+3n} = \frac{11}{6}$

by the definition of a convergent infinite series

3. Consider the infinite series $\sum_{k=1}^{\infty} k \cdot e^{-k^2}$.

a) List out the first three term. Analyze the $\lim_{n\to\infty} a_n$.

$S = \frac{1}{e} + \frac{2}{e^4} + \frac{3}{e^9} + \cdots$

$\lim_{k\to\infty} \frac{k}{e^{k^2}} \overset{L'H}{=} \lim \frac{1}{2k \cdot e^{k^2}} = 0$, may converge

b) Choose a test and determine if the infinite series converges or diverges. Show all work!

Let's try the IT. Let $f(x) = x/e^{x^2}$. Clearly $f(x)$ is continuous, positive, and we will show it's decreasing by finding $f'(x)$

$$f'(x) = \frac{e^{x^2} \cdot 1 - x \cdot e^{x^2} \cdot 2x}{e^{2x^2}} = \frac{e^{x^2}(1-2x^2)}{e^{2x^2}} = \frac{1-2x^2}{e^{x^2}}$$

$f'(x) < 0$ for all $x \geq 1$. Hence $f(x)$ is decreasing.

$$\int_1^{\infty} x \cdot e^{-x^2} dx = \lim_{t\to\infty} \int_1^t x \cdot e^{-x^2} dx$$ Let $u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2} du = x\,dx$

Limits

$u \to -t^2$

$u = -1$

$$= \lim_{t\to\infty} \int_{-1}^{-t^2} -\frac{1}{2} e^u \, du$$

$$= \frac{1}{2} \lim_{t\to\infty} \left[e^{-1} - e^{-t^2}\right]$$

$$= \frac{1}{2e}$$ convergent!

c) Write a conclusion stating the test you used to determine convergence/divergence.

Thus, by "IT" the series $\sum_{k=1}^{\infty} k \cdot e^{-k^2}$ is convergent.

4. Consider the infinite series $\sum_{n=1}^{\infty}\left[\frac{2^n \cdot n!}{5\cdot 8\cdot 11\cdot \ldots \cdot (3n+2)}\right]$.

a) List out the first three term. Analyze the $\lim_{n\to\infty} a_n$.

$S = \frac{2}{5} + \frac{1}{5} + \frac{6}{55} + \cdots$

$\lim_{n\to\infty} a_n = 0$, probably.

b) Choose a test and determine if the infinite series converges or diverges. Show all work!

Let use the ratio test

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{2^{n+1}\cdot(n+1)!}{5\cdot 8\cdot 11\cdot\ldots\cdot(3n+2)(3n+5)} \cdot \frac{5\cdot 8\cdot 11\cdot\ldots\cdot(3n+2)}{2^n\cdot n!}$$

$$= \frac{2(n+1)}{(3n+5)}$$

$$\lim_{n\to\infty} \frac{2n+2}{3n+5} = \frac{2}{3} < 1 \quad \text{convergent!}$$

c) Write a conclusion stating the test you used to determine convergence/divergence.

Thus, since $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \frac{2}{3} < 1$, the series converges by Ratio Test.

5. Consider the infinite series $\sum_{n=1}^{\infty} \sin\left(\frac{n^2+4n+3}{5n^2+1}\right)$

a) List out the first three term. Analyze the $\lim_{n\to\infty} a_n$.

$S = \sin\left(\frac{4}{3}\right) + \sin\left(\frac{5}{7}\right) + \sin\left(\frac{12}{23}\right) + \dots$

$\lim_{n\to\infty} a_n \stackrel{?}{=} 0$, I don't think so.

b) Choose a test and determine if the infinite series converges or diverges. Show all work!

$$\lim_{n\to\infty}\left[\sin\left(\frac{n^2+4n+3}{5n^2+1}\right)\right]$$

$$= \sin\left[\lim_{n\to\infty}\left(\frac{n^2+4n+3}{5n^2+1}\right)\right]$$

$$\stackrel{\text{DTE}}{=} \sin\left(\frac{1}{5}\right) \neq 0$$

c) Write a conclusion stating the test you used to determine convergence/divergence.

Therefore the infinite series is divergent by "TD".

6. Consider the infinite series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$.

a) List out the first three term. Analyze the $\lim_{n\to\infty} a_n$.

$S = \frac{1}{2} + \left(\frac{2}{3}\right)^4 + \left(\frac{3}{4}\right)^9 + \cdots$

$\lim_{n\to\infty} a_n = 0$, maybe.

b) Choose a test and determine if the infinite series converges or diverges. Show all work!

Let's use the n-Root Test

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{n}{n+1}\right)^{n^2}}$$

$$= \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{n}$$

$$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^{-n}$$

$$= \lim_{n\to\infty} \left[\left(1+\frac{1}{n}\right)^{n}\right]^{-1}$$

$$= e^{-1}$$

$$= \frac{1}{e} < 1$$

c) Write a conclusion stating the test you used to determine convergence/divergence.

Hence, the series converges by the n-Root Test since $\lim_{n\to\infty} |\sqrt[n]{a_n}| < 1$.

7. Consider the infinite series $\sum_{n=0}^{\infty}(-1)^n \frac{2n}{4n^2+1}$.

a) Is the series absolutely convergent? Justify your answer.

Take $\sum_{n=1}^{\infty} \left| (-1)^n \frac{2n}{4n^2+1} \right| = \sum_{n=1}^{\infty} \frac{2n}{4n^2+1}$ is a series of positive terms. Let's use "LCT". Compare the series with $b_n = \frac{1}{n}$ where $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test $n=1$ (Harmonic)

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2n^2}{4n^2+1} \overset{\text{DTE}}{=} \frac{1}{2} > 0$, positive finite constant. So the series is divergent and is not absolute convergent.

b) Is the series conditionally convergent? Justify your answer.

The series is alternating so let $b_n = \frac{2n}{4n^2+1}$

1) $\lim_{n\to\infty} \frac{2n}{4n^2+1}$

$\overset{\text{DTE}}{=} \lim_{n\to\infty} \frac{2n}{4n^2}$

$= \lim_{n\to\infty} \frac{1}{2n}$

$= 0$

2) $b_{n+1} \leq b_n$

$\frac{2(n+1)}{4(n+1)^2+1} \overset{?}{\leq} \frac{2n}{4n^2+1}$

Let $f(x) = \frac{2x}{4x^2+1}$

$f'(x) = \frac{(4x^2+1)\cdot 2 - 2x\cdot(8x)}{(4x^2+1)^2}$

$f'(x) = \frac{8x^2+2-16x^2}{(4x^2+1)^2}$

See spillage

Math 1B

Spillage

7b) continued

$$f'(x) = \frac{2-8x^2}{(4x^2+1)^2} < 0 \quad \text{for } x \geq 1$$

Thus the series is decreasing

$b_{n+1} \leq b_n$ for all $n \geq 1$

Thus the orginal series is convergent and therefore conditionally convergent.

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Part 2: This part has 3 questions. Each numbered problem is worth 10 points. A calculator may be used on this part of the exam. Show all your work on your test. Appropriate evidence will receive partial credit. Do not write on the back of the page. If you need more room, please use the "Spillage" page that is attached to the back of the exam. Good Luck!!

1. Consider the ellipse with foci (2, −2), (4, −2), and vertices (1, −2), (5, −2).

   a) Find the coordinates of the center of the ellipse.

   Center: $\left(\frac{1+5}{2}, \frac{-2-2}{2}\right) = (3, -2)$

   b) Find the equation of the ellipse in the form $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

   $a = 2$ and $c = 1$

   $c^2 = a^2 - b^2$

   $1 = 4 - b^2$

   $b^2 = 3$

   $b = \pm\sqrt{3}$

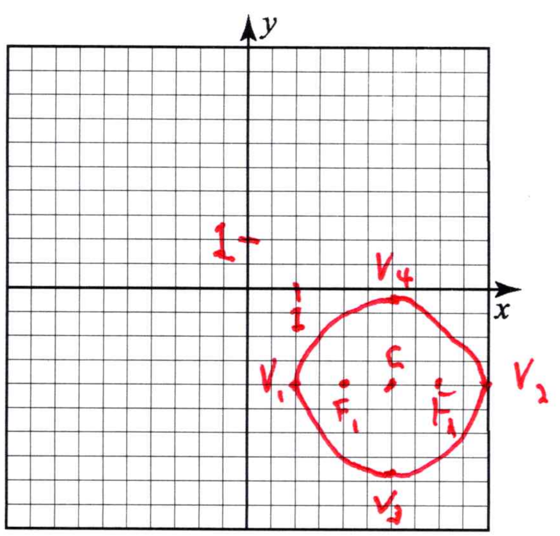
   Equation:

   $\frac{(x-3)^2}{4} + \frac{(y+2)^2}{3} = 1$

   Co-vertices:

   $(3, -2 \pm \sqrt{3})$

   c) Graph the ellipse you found in subpart (b).



$V_1(1, -2)$

$V_2(5, -2)$

$V_3(3, -2-\sqrt{3})$

$V_4(3, -2+\sqrt{3})$

$C(3, -2)$

$F_1(2, -2)$

$F_2(4, -2)$

2. Consider the hyperbola $9x^2 - 4y^2 - 72x + 8y + 176 = 0$.

a) Write the hyperbola in the form $\dfrac{(y-k)^2}{a^2} + \dfrac{(x-h)^2}{b^2} = 1$.

$(9x^2 - 72x) - (4y^2 - 8y) = -176$

$9(x^2 - 8x + 16) - 4(y^2 - 2y + 1) = -176 + 144 - 4$

$9(x-4)^2 - 4(y-1)^2 = -36$

$\dfrac{(y-1)^2}{9} - \dfrac{(x-4)^2}{4} = 1$

North-South Hyperbola

b) Find the center, vertices, foci and asymptotes for the hyperbola.

Center: $(h,k) = (4,1)$

$a = 3,\ b = 2$

$c^2 = a^2 + b^2$

$c^2 = 9 + 4$

$c = \pm\sqrt{13}$

Vertices: $(4,4)$ & $(4,-2)$

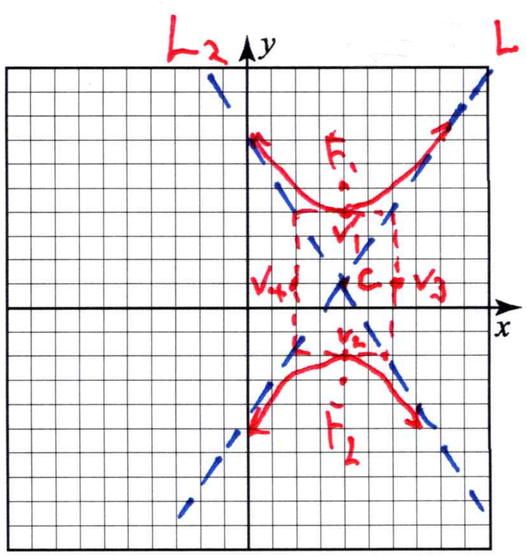
Co-Vertices: $(6,1)$ & $(2,1)$

Foci: $(4, 1 \pm \sqrt{13})$

Asymptotes:

$y - 1 = \pm \frac{3}{2}(x-4)$

c) Graph the hyperbola.



$V_1(4,4)$

$V_2(4,-2)$

$V_3(6,1)$

$V_4(2,1)$

$F_1(4, 1+\sqrt{13})$

$F_2(4, 1-\sqrt{13})$

$C(4,1)$

$L_1: y = \frac{3}{2}x - 5$

$L_2: y = -\frac{3}{2}x + 7$

3. Solve the problems below.

a) Consider the infinite series $\sum_{n=1}^{\infty} \frac{1}{n^6}$. Using the **Remainder Estimate for the Integral Test**, find the smallest n such that the approximation is accurate to five decimal places ($R_n \leq 0.000001 = 1 \times 10^{-6}$). Using the n you found, determine with your calculator an approximation for the given infinite series to five decimal places of accuracy.

Clearly the series converges by IT.

Let $\int_n^\infty \frac{1}{x^6}dx \leq 1 \times 10^{-6}$ by "REIT."

By CAS we get $\frac{1}{5n^5} \leq 1 \times 10^{-6}$

Solving for n, $n \geq 11.487$

Let $n = 12$, smallest n.

$$\sum_{n=1}^{12} \frac{1}{n^6} \approx 1.01734$$

b) Consider the infinite series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^6}$. Using the **Alternating Series Estimation Theorem**, find the smallest n such that the approximation is accurate to five decimal places ($R_n \leq 0.000001 = 1 \times 10^{-6}$). Using the n you found, determine with your calculator an approximation for the given infinite series to five decimal places of accuracy.

Clearly the series converges by AST.

Let $b^{n+1} \leq 1 \times 10^{-6}$ by "ASET."

$$\frac{1}{(n+1)^6} \leq 1 \times 10^{-6}$$

By CAS $n \geq 9$

Let $n = 9$, smallest n.

$$\sum_{n=1}^{9} (-1)^n \cdot \frac{1}{n^6} \approx -0.98555$$