

Math 1B  
Test 3 Review

1. Determine whether  $\sum_{n=0}^{\infty} \left(\frac{3}{\pi}\right)^n$  converges or diverges. If it converges, find the sum.
  
2. Consider the telescoping series  $\sum_{n=1}^{\infty} \frac{4}{n^2 + 2n}$ .
  - a) Find a formula  $a_i$  for the decomposition of the general term of the given infinite series.
  - b) Express the series as a partial sum  $s_n$  in sigma notation,  $s_n = \sum_{i=1}^n a_i$ .
  - c) Expand the partial sum  $s_n$  and simplify.
  - d) Find the sum of the infinite series, if it exists. Show all your work. NO CAS!

For problems 3-6, determine whether the series converges or diverges. State any tests that you use and show all steps.

3.  $\sum_{n=1}^{\infty} \frac{1 + 3^n}{2^n}$

4.  $\sum_{k=1}^{\infty} \left(\frac{4k^2 - 3}{7k^2 + 6}\right)^k$

5.  $\sum_{n=0}^{\infty} \frac{1 + \sin^2 n}{5^n}$

6.  $\sum_{n=1}^{\infty} \frac{(2n)!}{n^{2n}}$

7. Classify the series below as absolutely convergent, conditionally convergent, or divergent. State any tests you use and show all steps.

a)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$

b)  $\sum_{k=1}^{\infty} (-1)^k \frac{k^2}{\sqrt{k^6+1}}$

c)  $\sum_{n=2}^{\infty} \frac{(-1)^n(2n-1)}{n^3-1}$

A calculator may be used to help solve problem 8.

8. Solve the problems below. CAS may be used to help solve these problem.

- a) Consider the infinite series  $\sum_{n=1}^{\infty} \frac{1}{n^6}$ . Using the **Remainder Estimate for the Integral Test**, find the smallest  $n$  such that the approximation is accurate to five decimal places ( $R_n \leq 0.000001 = 1 \times 10^{-6}$ ). Using the  $n$  you found, determine with your calculator an approximation for the given infinite series to four decimal places of accuracy.

- b) Consider the infinite series  $\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^6}$ . Using the **Alternating Series Estimation Theorem**, find the smallest  $n$  such that the approximation is accurate to five decimal places ( $R_n \leq 0.0000001 = 1 \times 10^{-6}$ ). Using the  $n$  you found, determine with your calculator an approximation for the given infinite series to four decimal places of accuracy.