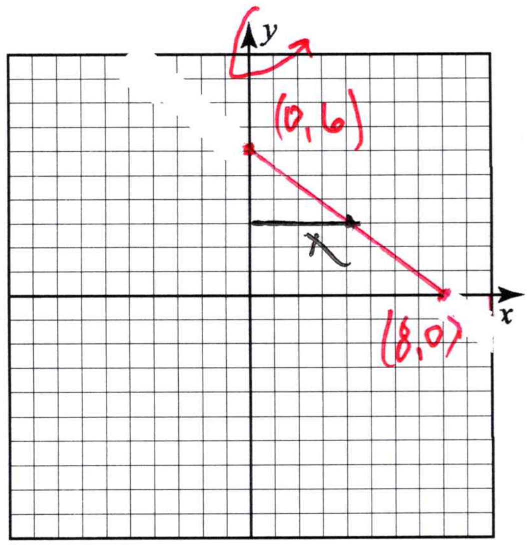
Math 1B	Name Key	
Test 2 - Spring 2026	Roll # ______	Score ______

Part 1: This part has 4 questions with subparts. Each subpart is worth 5 points. A calculator may NOT be used on this part of the exam. Show all your work on your exam. Appropriate evidence will receive partial credit. Do not write on the back of the page. If you need additional room for your solutions please use "Spillage" on the last page. Good Luck!!

$x = -\frac{4}{3}y + 8$

1. Consider the equation $y = -\frac{3}{4}x + 6$.

a) Sketch the graph of the equation given.



b) $A = \int_a^b 2\pi x \, ds$

$A = \int_0^8 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

$\frac{dy}{dx} = -\frac{3}{4}$

$A = 2\pi \int_0^8 x\sqrt{1 + \frac{9}{16}} \, dx$

$A = 2\pi \int_0^8 x \sqrt{\frac{25}{16}} \, dx$

b) Write the integral that will determine the area of the surface when the curve is revolved about the y-axis.

c) Calculate the integral you found in part (b).

c)

$A = \frac{5\pi}{2} \int_0^8 x \, dx$

$= \frac{5\pi}{2} \left[\frac{x^2}{2}\right]_0^8$

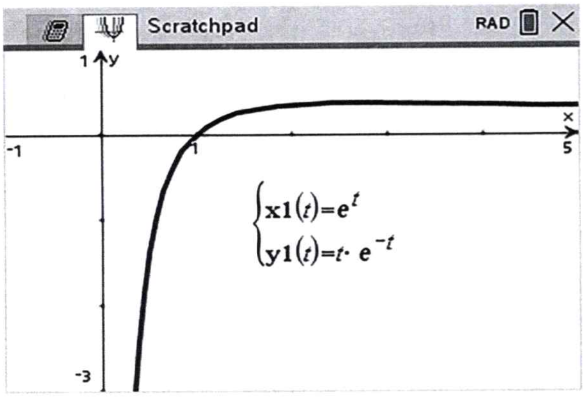
$= \frac{5\pi}{2} \cdot \frac{64}{2} = 80\pi \text{ units}^2$

2. Consider the parametric equations $x = e^t$ and $y = t \cdot e^{-t}$. The graph of the curve is below.

a) Find $\dfrac{dy}{dx}$.

b) Find $\dfrac{d^2y}{dx^2}$.

c) For which values of t is the curve concave downward and concave upward?



a) $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$= \dfrac{1 \cdot e^{-t} + t(-e^{-t})}{e^t}$

$= \dfrac{e^{-t}(1-t)}{e^t}$

$\dfrac{dy}{dx} = e^{-2t}(1-t)$

b) $\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)$

$= -2e^{-2t}(1-t) + e^{-2t} \cdot (-1)$

$= e^{-2t}[-2(1-t) - 1]$

b) continued

$= e^{-2t}(-2+2t-1)$

$= e^{-2t}(2t-3)$

$\dfrac{d^2y}{dx^2} = \dfrac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

$= \dfrac{e^{-2t}(2t-3)}{e^t}$

$= e^{-3t}(2t-3)$

c) Let $\dfrac{d^2y}{dx^2} = 0 \quad t = \dfrac{3}{2}$

y'': $-$ | 0 | $+$ on the t axis, with $\dfrac{3}{2}$ marked at the 0

concave down: $t \in (-\infty, \frac{3}{2})$

concave up: $t \in (\frac{3}{2}, +\infty)$

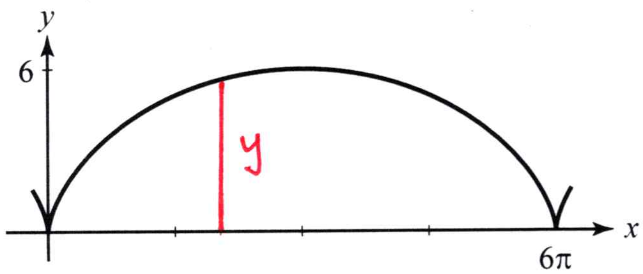
3. Consider the cycloid curve below, it is described parametrically by

$$\begin{cases} x = 3\theta - 3\sin\theta \\ y = 3 - 3\cos\theta\,. \end{cases}$$

a) Determine $\dfrac{dx}{d\theta}$ and $\dfrac{dy}{d\theta}$.

b) Write the integral that will compute area under one arch of the cycloid.

c) Find the value of your integral in part (b).



a) $\dfrac{dx}{d\theta} = 3 - 3\cos\theta$

$\dfrac{dy}{d\theta} = 3\sin\theta$

b) $A = \int_a^b y\,dx$

$dx = (3 - 3\cos\theta)\,d\theta$

$A = \int_0^{2\pi} (3 - 3\cos\theta)(3 - 3\cos\theta)\,d\theta$

$A = \int_0^{2\pi} (9 - 18\cos\theta + 9\cos^2\theta)\,d\theta$

$A = 9\int_0^{2\pi} (1 - 2\cos\theta + \cos^2\theta)\,d\theta$

c) $A = 9\int_0^{2\pi} \left[1 - 2\cos\theta + \dfrac{1+\cos(2\theta)}{2}\right]d\theta$

$A = 9\left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{4}\sin(2\theta)\right]_0^{2\pi}$

$A = 9\left[(3\pi - 0 + 0) - (0 - 0 + 0)\right]$

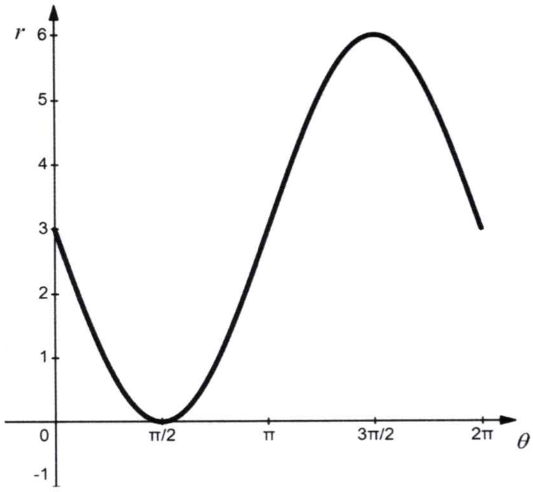
$A = 27\pi$ units^2

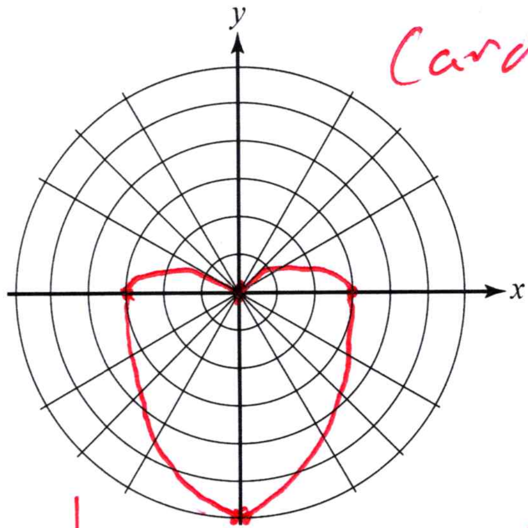
4. Consider the polar curve $r = 2 - 2\sin\theta$.

a) Given the graph of (r, θ) in a rectangular coordinate system below on the left, graph the given polar equation in the provided polar coordinate system on the right. Determine $\dfrac{dy}{dx}$ for the polar curve.

b) Find the slope of the tangent line to the polar curve at the point where $\theta = \dfrac{\pi}{6}$, and the polar and rectangular coordinates of the point of tangency.

c) Find the total area bounded by the polar curve.





Cardioid

a) $\dfrac{dy}{dx} = \dfrac{r'\cdot\sin\theta + r\cdot\cos\theta}{r'\cdot\cos\theta - r\cdot\sin\theta}$

$= \dfrac{-2\cos\theta\cdot\sin\theta + (2-2\sin\theta)\cdot\cos\theta}{-2\cos\theta\cdot\cos\theta - (2-2\sin\theta)\cdot\sin\theta}$

$= \dfrac{2\cos\theta - 4\sin\theta\cdot\cos\theta}{-2\sin\theta - 2\cos^2\theta + 2\sin^2\theta}$

$= \dfrac{\cos\theta - 2\sin\theta\cdot\cos\theta}{-\sin\theta - \cos^2\theta + \sin^2\theta}$

$= \dfrac{\cos\theta - \sin(2\theta)}{-\sin\theta - \cos(2\theta)}$

$= \dfrac{\sin(2\theta) - \cos\theta}{\cos(2\theta) + \sin\theta}$

b) $\left.\dfrac{dy}{dx}\right|_{\theta=\frac{\pi}{6}} = \dfrac{\sin(\frac{\pi}{3}) - \cos(\frac{\pi}{6})}{\cos(\frac{\pi}{3}) + \sin(\frac{\pi}{6})}$

$= \dfrac{\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}{\frac{1}{2} + \frac{1}{2}}$

$= \dfrac{0}{1} = 0$

$r = 2 - 2\sin(\frac{\pi}{6}) = 2 - 1 = 1$

Polar coordinates: $(1, \frac{\pi}{6})$

$x = 1\cdot\cos(\frac{\pi}{6}) = \frac{\sqrt{3}}{2}$

$y = 1 \sin(\frac{\pi}{6}) = \frac{1}{2}$

Rectangular coordinates: $(\frac{\sqrt{3}}{2}, \frac{1}{2})$

Math 1B

Spillage

4c) $A=\int_{\alpha}^{\beta}\frac{1}{2}r^2\,d\theta$

$A=\int_0^{2\pi}\frac{1}{2}(2-2\sin\theta)^2\,d\theta$

$A=\frac{1}{2}\int_0^{2\pi}(4-8\sin\theta+4\sin^2\theta)\,d\theta$

$A=2\int_0^{2\pi}\left[1-2\sin\theta+\frac{1}{2}(1-\cos 2\theta)\right]d\theta$

$A=2\int_0^{2\pi}\left[\frac{3}{2}-2\sin\theta-\frac{1}{2}\cos(2\theta)\right]d\theta$

$A=2\left[\frac{3}{2}\theta+2\cos\theta-\frac{1}{4}\sin(2\theta)\right]_0^{2\pi}$

$A=2\left[(3\pi+2-0)-(0+2-0)\right]$

$A=6\pi\ \text{units}^2$

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Part 2: This part has 3 questions with subparts. Each subpart is worth 5 points. A calculator may be used on this part of the exam. Show all your work on your test. Appropriate evidence will receive partial credit. Do not write on the back of the page. If you need more room, please use the "Spillage" page that is attached to the back of the exam. Good Luck!!

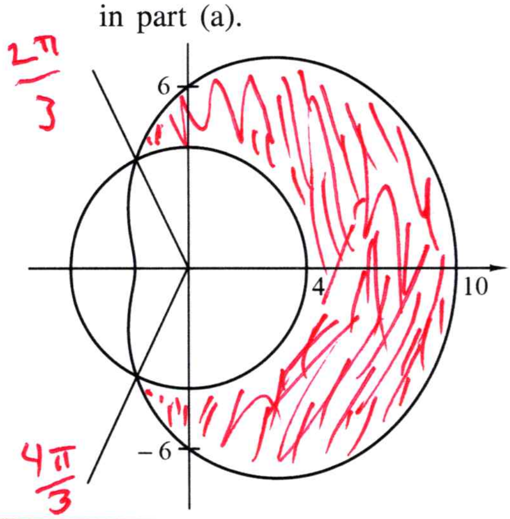
5. Consider the polar curves, graph below, $r = 6 + 4\cos\theta$ and $r = 4$.

   a) Shade the region inside of $r = 6 + 4\cos\theta$ and outside of $r = 4$.

   b) Find the points where the two curves intersect. Show all your work.

   c) Write and evaluate an integral expression that will calculate the area of the shaded region in part (a).

a)



b) $4 = 6 + 4\cos\theta$

$-2 = 4\cos\theta$

$\cos\theta = -\frac{1}{2}$

$\theta = \frac{2\pi}{3}$

$\theta = \frac{4\pi}{3}$

Points

$\left(4, \frac{2\pi}{3}\right)$

$\left(4, \frac{4\pi}{3}\right)$

c) $A = \frac{1}{2}\int_{\alpha}^{\beta} \left[(f(\theta))^2 - (g(\theta))^2\right] d\theta$

$A = 2 \cdot \frac{1}{2}\int_{0}^{\frac{2\pi}{3}} \left[(6 + 4\cos\theta)^2 - (4)^2\right] d\theta$

$A \overset{\text{calc}}{=} \frac{56\pi}{3} + 22\sqrt{3}$ units^2

$A \approx 96.748$ units^2

6. Consider the polar equation $r = 4 - 4\cos\theta$.

a) Find $\dfrac{dr}{d\theta}$.

b) Write and evaluate an integral expression that will calculate the total length of the curve.

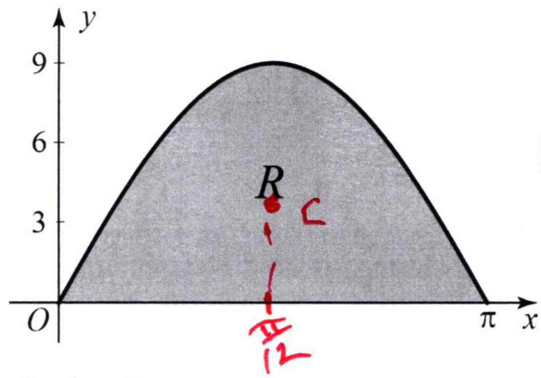
a) $\dfrac{dr}{d\theta} = 4\sin\theta$

b) $L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\dfrac{dr}{d\theta}\right)^2}\, d\theta$

$= \int_0^{2\pi} \sqrt{(4-4\cos\theta)^2 + (4\sin\theta)^2}\, d\theta$

$\overset{\text{CAS}}{=} 32$ units

7. Consider the lamina determined by $y = 9\sin x$ over the interval $[0, \pi]$. See the figure below.



$C = \left(\frac{\pi}{2}, \frac{9\pi}{8}\right)$

a) Find the area of the lamina R.

$y = -3$

b) Find the coordinates of the centroid for the lamina.

c) Using the Theorem of Pappus, determine the volume of the solid when region R is rotated about the line $y = -3$.

a) $A = \int_0^{\pi} 9\sin x \, dx$

CAS 18 units^2

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b) $\bar{x} = \frac{1}{A}\int_0^{\pi} x \cdot 9\sin x \, dx \overset{CAS}{=} \frac{\pi}{2} = 1.571$ units

$\bar{y} = \frac{1}{A}\int_0^{\pi} \frac{1}{2}\cdot\left[9\sin^2 x\right] dx \overset{CAS}{=} \frac{9\pi}{8} = 3.534$ units

Centroid: $\left(\frac{\pi}{2}, \frac{9\pi}{8}\right)$

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c) $V = A \cdot d \qquad d = 2\pi r$

$V = 18\left(\frac{9\pi}{8} + 3\right)\cdot 2\pi$

$\overset{CAS}{=} \frac{81\pi^2}{2} + 108\pi$ units^3 $= 739.011$ units^3