Test 1 Review Solutions

#1 (a) $\int (\tan x)\cdot[\ln(\cos x)]\,dx$

Let $u = \ln(\cos x)$

$du = \frac{1}{\cos x}(-\sin x)dx$

$-du = \tan x\,dx$

$= -\int u\,du$

$= -\frac{1}{2}u^2 + C$

$= -\frac{1}{2}\cdot\ln^2(\cos x) + C$

(b) $\int \cos x \cdot e^{5x}\,dx$

Let $u = e^{5x}$ $dv = \cos x\,dx$

$du = 5e^{5x}dx$ $v = \sin x$

$= e^{5x}\cdot\sin x - 5\int \sin x\cdot e^{5x}\,dx$

Let $u = e^{5x}$ $dv = \sin x\,dx$

$du = 5e^{5x}dx$ $v = -\cos x$

$= e^{5x}\cdot\sin x - 5\left[-e^{5x}\cdot\cos x + 5\int \cos x\cdot e^{5x}\,dx\right]$

$= e^{5x}\cdot\sin x + 5e^{5x}\cdot\cos x - 25\int \cos x\cdot e^{5x}\,dx$

Wrap around integral!

$26\int \cos x\cdot e^{5x}\,dx = e^{5x}\cdot\sin x + 5e^{5x}\cdot\cos x$

$= \frac{1}{26}\left[e^{5x}\cdot\sin x + 5e^{5x}\cdot\cos x\right] + C$

$= \frac{1}{26}\cdot e^{5x}\cdot\left[\sin x + 5\cdot\cos x\right] + C$

#1 © $\int \frac{9x-12}{x^3-x^2-6x}\,dx$

$$\frac{9x-12}{x(x-3)(x+2)} = \frac{A}{x} + \frac{B}{x-3} + \frac{c}{x+2}$$

$$9x-12 = A(x-3)(x+2) + Bx(x+2) + Cx(x-3)$$

Let $x=0$	$x=-2$	$x=3$
$-12=-6A$	$-30=10C$	$15=15B$
$A=2$	$c=-3$	$B=1$

$$2\cdot\int \frac{1}{x}dx + \int \frac{1}{x-3}dx - 3\int \frac{1}{x+2}\,dx$$

$$= 2\cdot\ln|x| + \ln|x-3| - 3\cdot\ln|x+2| + C$$

$$= \ln\left[\frac{x^2\cdot|x-3|}{|(x+2)^3|}\right] + C$$

#1 d

$$\int \sqrt{36 - x^2}\, dx \qquad \text{Let } x = 6\sin\theta \qquad \theta = \sin^{-1}\left(\frac{x}{6}\right)$$

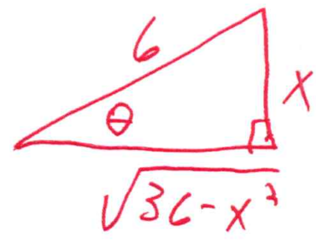
$$dx = 6\cos\theta\, d\theta$$

$$= \int \sqrt{36\cos^2\theta} \cdot 6\cos\theta\, d\theta \qquad 36 - x^2 = 36 - 36\sin^2\theta$$

$$= 36(1 - \sin^2\theta)$$

$$= 36 \cdot \int \cos^2\theta\, d\theta \qquad = 36\cos^2\theta$$

$$= 36 \cdot \frac{1}{2} \int (1 + \cos 2\theta)\, d\theta$$



$$= 18 \cdot \left(\theta + \frac{1}{2}\sin(2\theta)\right) + C$$

$$= 18 \cdot \left(\theta + \frac{1}{2} \cdot 2\sin\theta \cdot \cos\theta\right) + C$$

$$= 18 \cdot \left(\sin^{-1}\left(\frac{x}{6}\right) + \frac{x}{6} \cdot \frac{\sqrt{36 - x^2}}{6}\right) + C$$

$$= 18 \cdot \sin^{-1}\left(\frac{x}{6}\right) + \frac{x\sqrt{36 - x^2}}{2} + C$$

#1(e)

$$\int_0^{\frac{\pi}{2}} \cos^3 x \cdot \sin^8 x \, dx$$

$$= \int_0^{\frac{\pi}{2}} \cos x \cdot \cos^2 x \cdot \sin^8 x \, dx$$

$$= \int_0^{\frac{\pi}{2}} \cos x \cdot (1 - \sin^2 x) \cdot \sin^8 x \, dx$$

Let $u = \sin x$

$du = \cos x \, dx$

<u>New Bounds</u>

$u(0) = 0$

$u(\frac{\pi}{2}) = 1$

$$= \int_0^1 (1 - u^2) \cdot u^8 \, du$$

$$= \int_0^1 (u^8 - u^{10}) \, du$$

$$= \left[\frac{1}{9} u^9 - \frac{1}{11} u^{11} \right]_0^1$$

$$= \left(\frac{1}{9} - \frac{1}{11} \right) - (0 - 0)$$

$$= \frac{2}{99}$$

#1) (f)

$$\int_4^6 \frac{1}{\sqrt{x-4}}\,dx$$

$$= \lim_{x \to 4^+} \int_t^6 \frac{1}{\sqrt{x-4}}\,dx$$

$$= \lim_{x \to 4^+} \int_t^6 (x-4)^{-\frac{1}{2}}\,dx$$

$$= \lim_{x \to 4^+} \left[2(x-4)^{\frac{1}{2}}\right]_t^6$$

$$= \lim_{x \to 4^+} \left[2\sqrt{2} - \overbrace{2\cdot(t-4)^{\frac{1}{2}}}^{0}\right]$$

$$= 2\sqrt{2}$$ Converges!

#1) g

Tabular Integration

Diff	Int
$+\ x$	e^{-x}
$-\ 1$	$-e^{-x}$
0	e^{-x}

$$\int_0^{\infty} x \cdot e^{-x}\,dx$$

$$= \lim_{t \to \infty} \int_0^t x \cdot e^{-x}\,dx$$

$$= \lim_{t \to \infty} \left[-xe^{-x} - e^{-x}\right]_0^t$$

$$= \lim_{t \to \infty} \{(-te^{-t} - e^{-t}) - (0 - 1)$$

$$= \boxed{-\lim_{t \to \infty} \frac{t}{e^t}} - \overbrace{\lim_{x \to \infty} \frac{1}{e^t}}^{0} + 1$$

L'Hospital Rule

$$= -\overbrace{\lim_{t \to \infty} \frac{1}{e^t}}^{0} + 1$$

$$= 1$$ Converges

#2) $n = 4$ we get

$$\Gamma(4) = \int_0^{+\infty} (x^3 e^{-x})\,dx$$

Tabular Integration

Diff	Int
$+\ x^3$	e^{-x}
$-\ 3x^2$	$-e^{-x}$
$+\ 6x$	e^{-x}
$-\ 6$	$-e^{-x}$
0	e^{-x}

$$= \lim_{t \to +\infty} \left[\int_0^t (x^3 e^{-x})\,dx \right]$$

$$= \lim_{t \to +\infty} \left[-x^3 \cdot e^{-x} - 3x^2 \cdot e^{-x} - 6x \cdot e^{-x} - 6e^{-x} \right]_0^t$$

$$= \lim_{t \to +\infty} \left[\left(-\frac{t^3}{e^t} - \frac{3t^2}{e^t} - \frac{6t}{e^t} - \frac{6}{e^t} \right) - (0 + 0 + 0 - 6) \right]$$

L'Hospital Rule multiple times.

$$= \lim_{t \to +\infty} \left[-\cancelto{0}{\frac{6}{e^t}} - \cancelto{0}{\frac{6}{e^t}} - \cancelto{0}{\frac{6}{e^t}} - \cancelto{0}{\frac{6}{e^t}} + 6 \right]$$

$$= 6$$

#3) (a) $\frac{dy}{dx} = 4 \cdot \frac{x^3}{8} - 2 \cdot \frac{1}{4x^3}$

$= \frac{x^3}{2} - \frac{1}{2x^3}$

$= \frac{x^6 - 1}{2x^3}$

(b)

$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_1^2 \sqrt{1 + \left(\frac{x^6-1}{2x^3}\right)^2}\, dx$

$= \int_1^2 \sqrt{1 + \frac{x^{12} - 2x^6 + 1}{4x^6}}\, dx$

$= \int_1^2 \sqrt{\frac{4x^6}{4x^6} + \frac{x^{12} - 2x^6 + 1}{4x^6}}\, dx$

$= \int_1^2 \sqrt{\frac{x^{12} + 2x^6 + 1}{4x^6}}\, dx$

#4b) Continued

$$= \int_1^2 \sqrt{\frac{(x^6+1)^2}{4x^6}}\, dx$$

$$= \int_1^2 \frac{x^6+1}{2x^3}\, dx$$

$$= \int_1^2 \left(\frac{1}{2}x^3 + \frac{1}{2}x^{-3}\right) dx$$

$$= \frac{1}{2}\cdot\left[\frac{1}{4}x^4 - \frac{1}{2}x^{-2}\right]_1^2$$

$$= \frac{1}{2}\cdot\left[\left(4 - \frac{1}{8}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\cdot\left[\frac{31}{8} + \frac{1}{4}\right]$$

$$= \frac{1}{2}\cdot\left[\frac{33}{8}\right]$$

$$= \frac{33}{16} \text{ units}$$

4. Select the most appropriate technique for finding the antiderivatives from the given list. Write the letter corresponding to the method in the given space. A method may appear more than once in the list of answers, but only one answer per integral.

C a) $\int \cos^2 x \, dx$

E b) $\int \frac{3x^2 - 2}{x^2 - 2x - 8} \, dx$

A c) $\int \frac{e^{\tan^{-1} x}}{1 + x^2} \, dx$

B d) $\int x^4 \ln x \, dx$

D e) $\int \frac{1}{x\sqrt{4x^2 + 1}} \, dx$

TECHNIQUES

A. u-substitution

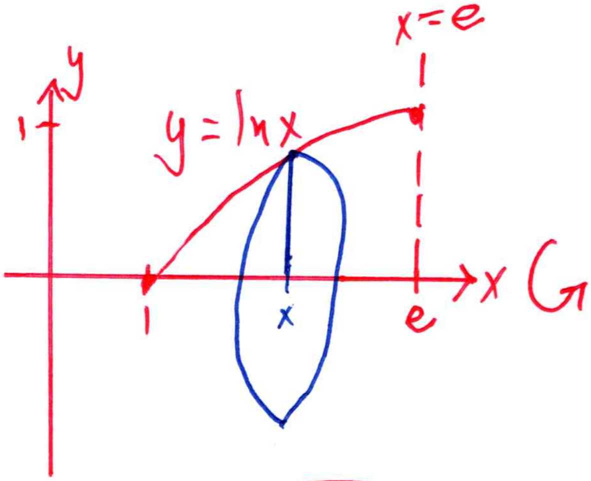
B. Parts

C. Trig Identity

D. Trig substitution

E. Partial Fractions

#5 a)



b) $V = \pi \int_1^e (\ln x)^2 \, dx$ "Disk Method"

c) $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

$S = \int ds = \int_1^e \sqrt{1 + \left(\frac{1}{x}\right)^2} \, dx$

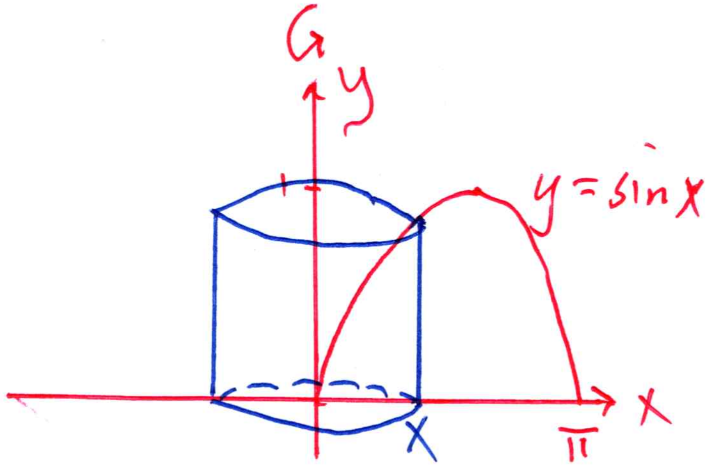
d) By CAS

Volume: $V = (e-2)\pi$ units^3 ≈ 2.257 units^3

Arc length: $S = \ln\left((\sqrt{2}+1)\cdot(\sqrt{e^2+1}-1)\right) + \sqrt{e^2+1} - \sqrt{2} - 1$ units

$= 2.003$ units

#6 a)



b) $V = 2\pi \int_0^{\pi} x \cdot \sin x \, dx$ "Shell Method"

c) $y = \sin x$

$\frac{dy}{dx} = \cos x$

$S = \int ds = \int_0^{\pi} \sqrt{1 + (\cos x)^2} \, dx$

d) By CAS

<u>Volume</u>: $V = 2\pi^2$ units^3 ≈ 19.739 units^3

<u>Arc length</u>: $S = 3.820$ units

<u>Note</u>: The antiderivative is not an elementary function, so no closed form for arc length.