Math 1B

Final Exam Review

1. The fraction $\dfrac{2x+3}{(x+2)^2}$ can be written as a sum of partial fractions where one of these fractions is $\dfrac{B}{(x+2)^2}$. Find B.

A) -4 B) -2 C) -1 D) 2 E) 7

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2. Which of the following is equal to the area of the region inside the polar curve $r = 3\sin\theta$ and outside the polar curve $r = \sin\theta$?

A) $8\displaystyle\int_0^{\pi} \sin^2\theta\, d\theta$ B) $\dfrac{9}{2}\displaystyle\int_0^{\frac{\pi}{2}} \sin^2\theta\, d\theta$ C) $8\displaystyle\int_0^{\frac{\pi}{2}} \sin^2\theta\, d\theta$

D) $8\displaystyle\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta$ E) $8\displaystyle\int_0^{\pi} \sin\theta\, d\theta$

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3. A direction vector of a line parallel to the intersection of the planes $x - 3y - z = 8$ and $2x - 2y - z = -7$ is

A) $\langle -1, -1, -4\rangle$ B) $\langle -1, -1, 4\rangle$ C) $\langle 1, -1, -4\rangle$

D) $\langle 1, -1, 4\rangle$ E) $\langle 1, 1, 4\rangle$

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4. The distance between the two foci of the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$ is

A) 8 B) 10 C) $8\sqrt{2}$ D) 20 E) 24

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5. If $a_n = \dfrac{(-1)^n n!}{n^n}$, then $\left|\dfrac{a_{n+1}}{a_n}\right| = ?$

A) $1 + \dfrac{1}{n}$ B) $1 + \left(\dfrac{1}{n}\right)^n$ C) $\dfrac{n+1}{n}$ D) $\left(\dfrac{n+1}{n}\right)^n$ E) $\left(\dfrac{n}{n+1}\right)^n$

6. Find the sum of the infinite series $1 - x^3 + x^6 - x^9 + \cdots$ for $|x| < 1$.

A) $\dfrac{-x}{1-x^3}$ B) $\dfrac{-x}{1+x^3}$ C) $\dfrac{x}{1-x^3}$ D) $\dfrac{-1}{1+x^3}$ E) $\dfrac{1}{1+x^3}$

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7. For what integer k, $k > 1$, will both $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\displaystyle\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge?

A) 6 B) 5 C) 4 D) 3 E) 2

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8. $\displaystyle\int xe^{2x}\,dx =$

A) $\dfrac{xe^{2x}}{2} - \dfrac{e^{2x}}{4} + C$ B) $\dfrac{xe^{2x}}{2} - \dfrac{e^{2x}}{2} + C$ C) $\dfrac{xe^{2x}}{2} + \dfrac{e^{2x}}{4} + C$

D) $\dfrac{xe^{2x}}{2} + \dfrac{e^{2x}}{2} + C$ E) $\dfrac{x^2e^{2x}}{4} + C$

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9. The curve $x = t$, $y = t^3$ for $0 \le t \le 1$ is rotated about the x-axis. Which integral gives the surface area of the resulting solid?

A) $\displaystyle\int_0^1 2\pi t^3\sqrt{1+9t^4}\,dt$ B) $\displaystyle\int_0^1 2\pi t\sqrt{1+3t^2}\,dt$ C) $\displaystyle\int_0^1 \pi t^6\,dt$

D) $\displaystyle\int_0^1 2\pi t^3\sqrt{1+3t^2}\,dt$ E) $\displaystyle\int_0^1 2\pi t\sqrt{1+9t^4}\,dt$

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10. Find the area of the region bounded by the curve $x = 1 + t^2$, $y = t - t^3$ and the x-axis for $0 \le t \le 1$.

A) $\dfrac{4}{15}$ units^2 B) $\dfrac{8}{15}$ units^2 C) $\dfrac{2}{3}$ units^2 D) $\dfrac{1}{2}$ units^2 E) $\dfrac{7}{10}$ units^2

11. $\displaystyle\int_1^\infty \frac{x}{(1+x^2)^2}\,dx$ is

A) $-\frac{1}{2}$ B) $-\frac{1}{4}$ C) $\frac{1}{4}$ D) $\frac{3}{4}$ E) divergent

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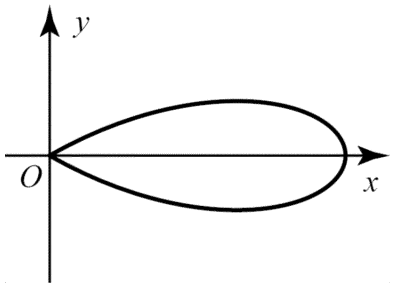
12.



Which of the following gives the area of the region enclosed by the loop of the polar curve $r(\theta) = 4\cos(3\theta)$ shown in the figure above?

A) $16\displaystyle\int_{-\pi/3}^{\pi/3} \cos(3\theta)\,d\theta$ B) $8\displaystyle\int_{-\pi/6}^{\pi/6} \cos(3\theta)\,d\theta$ C) $8\displaystyle\int_{-\pi/3}^{\pi/3} \cos^2(3\theta)\,d\theta$

D) $16\displaystyle\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\,d\theta$ E) $8\displaystyle\int_{-\pi/6}^{\pi/6} \cos^2(3\theta)\,d\theta$

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13. The length of the path described by the parametric equations $x = \frac{1}{3}t^3$ and $y = \frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

A) $\displaystyle\int_0^1 \sqrt{t^2+1}\,dt$ B) $\displaystyle\int_0^1 \sqrt{t^2+t}\,dt$ C) $\displaystyle\int_0^1 \sqrt{t^4+t^2}\,dt$

D) $\frac{1}{2}\displaystyle\int_0^1 \sqrt{t^4+4}\,dt$ E) $\frac{1}{6}\displaystyle\int_0^1 t^2\sqrt{4t^2+9}\,dt$

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14. What are all values of p for which the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^p+1}$ converges?

A) $p > 0$ B) $p \ge 1$ C) $p > 1$ D) $p \ge 2$ E) $p > 2$

15. What is the coefficient of the x^3 term in the Maclaurin series for $f(x) = \sin(2x)$?

A) $-\frac{4}{3}$ B) -1 C) $-\frac{3}{4}$ D) $-\frac{1}{6}$ E) 0

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16. Evaluate the indefinite integral $\int \frac{e^x - 1}{x}\,dx$ as a power series:

A) $C + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}$ B) $C + \sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!}$ C) $C + \sum_{n=1}^{\infty} \frac{x^n}{n!}$

D) $C + \sum_{n=1}^{\infty} \frac{x^{n-1}}{n!}$ E) $C + \sum_{n=0}^{\infty} \frac{x^n}{(n+1)!}$

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17. Find the area under one arch of the cycloid defined by $x = a(\theta - \sin\theta)$ and $y = a(1 - \cos\theta)$ for $0 \le \theta \le 2\pi$.

A) πa^2 units^2 B) $2\pi a^2$ units^2 C) $3\pi a^2$ units^2 D) $4\pi a^2$ units^2 E) $6\pi a^2$ units^2

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18. Find the radius and interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{1}{6^n}(x+5)^n$.

A) $R = 0$ and $x \in [-5, 5]$ B) $R = 6$ and $x \in (-11, 1)$ C) $R = 4$ and $x \in [-9, -1)$

D) $R = 3$ and $x \in (-8, -2]$ E) $R = 2$ and $x \in [-7, -3)$

19. Use the Ratio Test to determine the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{5^n}$. Which of the following is true?

A) The limit $L = 0$, so the series converges.

B) The limit $L = \frac{1}{5}$, so the series converges.

C) The limit $L = 1$, so the test is inconclusive.

D) The limit $L = \infty$, so the series diverges.

E) The limit $L = 5$, so the series diverges.

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20. Find the symmetric equations of the line that passes through the points $(-2, 1, -3)$ and $(3, 4, 1)$.

A) $\frac{x+2}{5} = \frac{y-1}{3} = \frac{z+3}{4}$

B) $\frac{x-2}{5} = \frac{y+1}{3} = \frac{z+3}{4}$

C) $\frac{x+2}{5} = \frac{y+1}{-3} = \frac{z-3}{4}$

D) $\frac{x-5}{5} = \frac{y+1}{3} = \frac{z-3}{2}$

E) $\frac{x-2}{-5} = \frac{y-1}{-3} = \frac{z+3}{-4}$

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21. Evaluate the following indefinite integral using trigonometric substitution:

$$\int \frac{1}{(x^2+16)^{3/2}}\,dx$$

A) $\frac{1}{16\sqrt{x^2+16}} + C$

B) $\frac{1}{4}\arctan(x/4) + C$

C) $\frac{x}{\sqrt{x^2+16}} + C$

D) $\frac{x}{16\sqrt{x^2+16}} + C$

E) $\frac{x^2}{4(x^2+16)} + C$

22. What is the correct form of the partial fraction decomposition for the following rational function?

$$f(x) = \frac{x^2 + 1}{(x-5)^2(x^2+8)}$$

A) $\dfrac{A}{x-5} + \dfrac{B}{(x-5)^2} + \dfrac{C}{x^2+8}$

B) $\dfrac{A}{x-5} + \dfrac{B}{x-5} + \dfrac{Cx+D}{x^2+8}$

C) $\dfrac{A}{x-5} + \dfrac{B}{(x-5)^2} + \dfrac{Cx+D}{x^2+8}$

D) $\dfrac{A}{(x-5)^2} + \dfrac{Bx+C}{x^2+8}$

E) $\dfrac{Ax+B}{(x-5)^2} + \dfrac{Cx+D}{x^2+8}$

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23. Let $\mathbf{r}(t) = \langle t, 4\sin t, 2\cos t \rangle$, find $\mathbf{T}(t)$ at $t = \dfrac{\pi}{3}$.

A) $\left\langle -\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{4}, \dfrac{\sqrt{6}}{4} \right\rangle$

B) $\left\langle \dfrac{1}{4}, -\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{6}}{2} \right\rangle$

C) $\left\langle -\dfrac{1}{2}, -\dfrac{\sqrt{2}}{6}, -\dfrac{\sqrt{6}}{4} \right\rangle$

D) $\left\langle \dfrac{\sqrt{2}}{4}, \dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{6}}{4} \right\rangle$

E) $\left\langle -\dfrac{\sqrt{6}}{4}, \dfrac{\sqrt{2}}{4}, \dfrac{1}{4} \right\rangle$

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24. Find $\dfrac{d^2y}{dx^2}$ for the parametric curve $x = t \cdot e^{-t}$, $y = e^t$.

A) $-\dfrac{e^{-3t}(3-2t)}{(1-t)^2}$

B) $\dfrac{e^{-t}(2-3t)}{(1-t)^3}$

C) $e^{3t}(3-2t)$

D) $\dfrac{e^{-3t}(3-2t)}{(1-t)^2}$

E) $\dfrac{e^{3t}(3-2t)}{(1-t)^3}$

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25. The cosine of the angle between the planes $x + 2y + z = 0$ and $x + y + 3z = 0$ is

A) $\dfrac{4\sqrt{33}}{11}$

B) $\dfrac{\sqrt{66}}{11}$

C) $\dfrac{\sqrt{6}}{11}$

D) $\dfrac{6\sqrt{11}}{11}$

E) $\dfrac{6}{11}$

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1.
Answer: C

2.
Answer: C

3.
Answer: D

4.
Answer: B

5.
Answer: E

6.
Answer: E

7.
Answer: D

8.
Answer: A

9.
Answer: A

10.
Answer: A

11.
Answer: C

12.
Answer: E

13.
Answer: C

14.
Answer: E

15.
Answer: A

16.
Answer: A

17.
Answer: C

18.
Answer: B

19.
Answer: D

20.
Answer: A

21.
Answer: D

22.
Answer: C

23.
Answer: D

24.
Answer: E

25.
Answer: B