# Math 1B

## Final Review

A calculator may NOT be used to solve problems 1 - 5.

1. Match the equations below on the left with the graph of the surface on the right by writing the letter that corresponds to the surface in front of the equation. Note that one surface does not have a corresponding equation.

______ $\dfrac{x^2}{9} + \dfrac{y^2}{16} + \dfrac{z^2}{9} = 1$

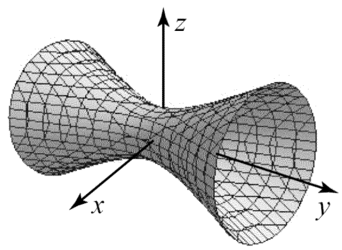
______ $15x^2 - 4y^2 + 15z^2 = -4$

______ $4x^2 - y^2 + 4z^2 = 4$

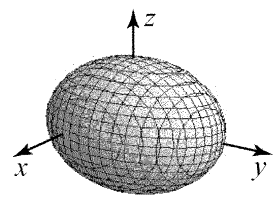
______ $y^2 = 4x^2 + 9z^2$

______ $4x^2 - 4y + z^2 = 0$

I




II

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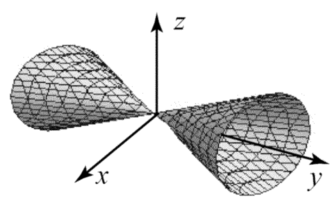


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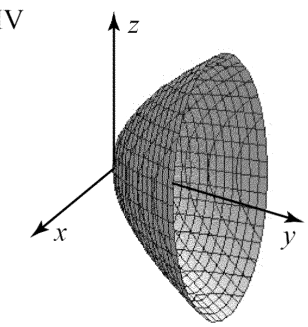


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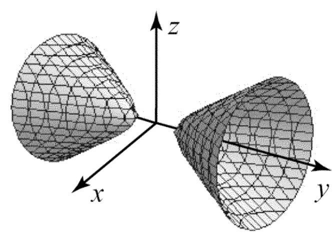
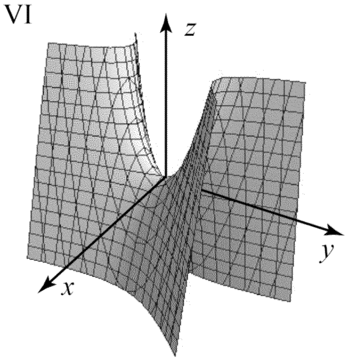



VI



2. Determine the interval of convergence for the following power series.

a) $\displaystyle\sum_{n=0}^{\infty} \left( \frac{e^n}{n+1} \right) x^n.$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n (x-4)^n}{n}.$

(*Note*: Be sure to check the endpoints.)

3. Consider the function $f(x) = \pi^x$. (*Hint*: Recall that $\dfrac{d}{dx}[a^x] = a^x \cdot \ln a$.)

a) Derive the formula for the coefficients $\dfrac{f^{(n)}(0)}{n!}$ for a Maclaurin series.

b) Find a Maclaurin series for $f(x)$, writing your answer using summation notation.

4. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{n^{2n}}$ converges or diverges. State any tests that you use and show all steps.

5. Consider the two planes $2x - y + 3z = 4$ and $x + 5y - 2z = 1$.

a) Find the angle between the two planes.

b) Determine the symmetric equations for the line of intersection between the two planes.

A calculator may be used to help solve problems 6 - 10.

6. Consider the points in space $A(-1, 2, 0)$, $B(2, 0, 1)$, and $C(-5, 3, 1)$.

a) Solve the triangle. (Find the lengths of all the sides and the measures of all the angles to the nearest degree.)

b) Find the area of the triangle $\triangle ABC$.

c) Find the equation of the plane passing through the points.

7. The Maclaurin series for $\sin x$ is given by $\sin x = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$.

a) Use the Maclaurin series for $\sin x$ to find the power series for $f(x) = \dfrac{\sin(x^2)}{x}$. Write your answer using summation notation.

b) Use the results of part (a) to obtain $\displaystyle\int_0^1 \frac{\sin(x^2)}{x}\,dx$ to four decimal place accuracy.

8. A bee is flying through space along a curve given by the position vector $\mathbf{r}(t) = \langle \cos t, \sin t, t \rangle$.

a) Find $\mathbf{T}(t)$

b) When $t = \dfrac{\pi}{2}$ the bee flies off along a tangent line to the curve. Find the parametric equations of its line of flight.

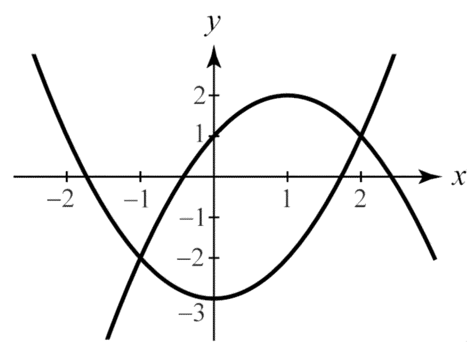
c) Show that the curvature of $\mathbf{r}(t)$ is constant for all $t$.

9. Consider the lamina determined by the region bounded by the graphs of $y = x^2 - 3$ and $y = -x^2 + 2x + 1$.

a) Find the area of the lamina.

b) Find the centroid of the lamina.

c) Using the Theorem of Pappus, determine the volume of the solid when the lamina is rotated about the line $x = 2$.



10. Consider the polar curves, $r = 2 - 2\cos\theta$ and $r = -6\cos\theta$.

a) Find the length of the arc from $\theta = 0$ to $\theta = 2\pi$ for $r = 2 - 2\cos\theta$

b) Find the points where the two curves intersect. Show all your work.

c) Write and evaluate an integral expression that will calculate the area of the region common to the two regions bounded by the given polar curves.