## 2.2 Exercises

**1.** Explain in your own words what is meant by the equation

$$\lim_{x \to 2} f(x) = 5$$

Is it possible for this statement to be true and yet $f(2) = 3$? Explain.

**2.** Explain what it means to say that

$$\lim_{x \to 1^-} f(x) = 3 \quad \text{and} \quad \lim_{x \to 1^+} f(x) = 7$$

In this situation is it possible that $\lim_{x \to 1} f(x)$ exists? Explain.

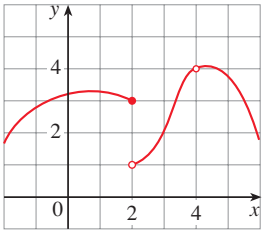
**3.** Explain the meaning of each of the following.

(a) $\lim_{x \to -3} f(x) = \infty$ (b) $\lim_{x \to 4^+} f(x) = -\infty$

**4.** Use the given graph of $f$ to state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 2^-} f(x)$ (b) $\lim_{x \to 2^+} f(x)$ (c) $\lim_{x \to 2} f(x)$

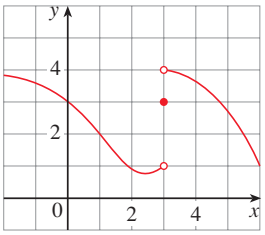
(d) $f(2)$ (e) $\lim_{x \to 4} f(x)$ (f) $f(4)$




**5.** For the function $f$ whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 1} f(x)$ (b) $\lim_{x \to 3^-} f(x)$ (c) $\lim_{x \to 3^+} f(x)$

(d) $\lim_{x \to 3} f(x)$ (e) $f(3)$



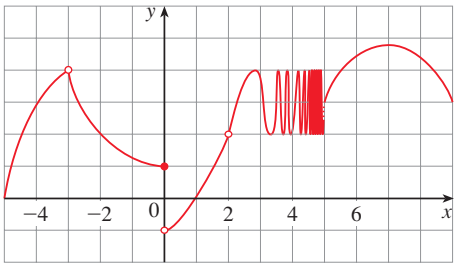

**6.** For the function $h$ whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to -3^-} h(x)$ (b) $\lim_{x \to -3^+} h(x)$ (c) $\lim_{x \to -3} h(x)$

(d) $h(-3)$ (e) $\lim_{x \to 0^-} h(x)$ (f) $\lim_{x \to 0^+} h(x)$

(g) $\lim_{x \to 0} h(x)$ (h) $h(0)$ (i) $\lim_{x \to 2} h(x)$

(j) $h(2)$ (k) $\lim_{x \to 5^+} h(x)$ (l) $\lim_{x \to 5^-} h(x)$



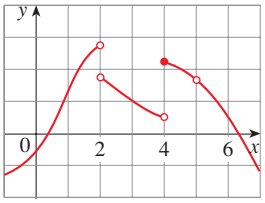

**7.** For the function $g$ whose graph is shown, find a number $a$ that satisfies the given description.

(a) $\lim_{x \to a} g(x)$ does not exist but $g(a)$ is defined.

(b) $\lim_{x \to a} g(x)$ exists but $g(a)$ is not defined.

(c) $\lim_{x \to a^-} g(x)$ and $\lim_{x \to a^+} g(x)$ both exist but $\lim_{x \to a} g(x)$ does not exist.

(d) $\lim_{x \to a^+} g(x) = g(a)$ but $\lim_{x \to a^-} g(x) \neq g(a)$.

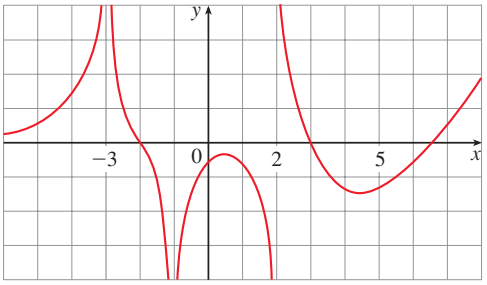



**8.** For the function $A$ whose graph is shown, state the following.

(a) $\lim_{x \to -3} A(x)$ (b) $\lim_{x \to 2^-} A(x)$

(c) $\lim_{x \to 2^+} A(x)$ (d) $\lim_{x \to -1} A(x)$

(e) The equations of the vertical asymptotes

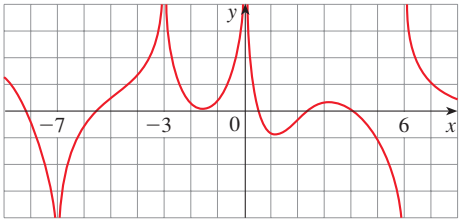



**9.** For the function $f$ whose graph is shown, state the following.

(a) $\lim_{x \to -7} f(x)$ (b) $\lim_{x \to -3} f(x)$ (c) $\lim_{x \to 0} f(x)$

(d) $\lim_{x \to 6^-} f(x)$ (e) $\lim_{x \to 6^+} f(x)$

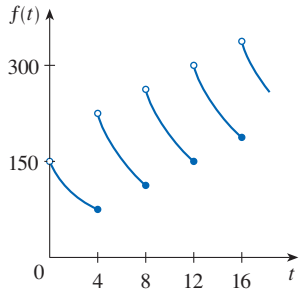
(f) The equations of the vertical asymptotes




**10.** A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after $t$ hours. Find

$$\lim_{t\to 12^-} f(t) \qquad \text{and} \qquad \lim_{t\to 12^+} f(t)$$

and explain the significance of these one-sided limits.




**11–12** Sketch the graph of the function and use it to determine the values of $a$ for which $\lim_{x\to a} f(x)$ exists.

**11.** $f(x) = \begin{cases} e^x & \text{if } x \le 0 \\ x - 1 & \text{if } 0 < x < 1 \\ \ln x & \text{if } x \ge 1 \end{cases}$

**12.** $f(x) = \begin{cases} \sqrt[3]{x} & \text{if } x \le -1 \\ x & \text{if } -1 < x \le 2 \\ (x-1)^2 & \text{if } x > 2 \end{cases}$

**13–14** Use the graph of the function $f$ to state the value of each limit, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 0^-} f(x)$ (b) $\lim_{x\to 0^+} f(x)$ (c) $\lim_{x\to 0} f(x)$

**13.** $f(x) = x\sqrt{1 + x^{-2}}$

**14.** $f(x) = \dfrac{e^{1/x} - 2}{e^{1/x} + 1}$

**15–18** Sketch the graph of an example of a function $f$ that satisfies all of the given conditions.

**15.** $\lim_{x\to 1^-} f(x) = 3, \quad \lim_{x\to 1^+} f(x) = 0, \quad f(1) = 2$

**16.** $\lim_{x\to 0} f(x) = 4, \quad \lim_{x\to 8^-} f(x) = 1, \quad \lim_{x\to 8^+} f(x) = -3,$
$f(0) = 6, \quad f(8) = -1$

**17.** $\lim_{x\to -1^-} f(x) = 0, \quad \lim_{x\to -1^+} f(x) = 1, \quad \lim_{x\to 2} f(x) = 3,$
$f(-1) = 2, \quad f(2) = 1$

**18.** $\lim_{x\to -3^-} f(x) = 3, \quad \lim_{x\to -3^+} f(x) = 2, \quad \lim_{x\to 3^-} f(x) = -1,$
$\lim_{x\to 3^+} f(x) = 2, \quad f(-3) = 2, \quad f(3) = 0$

**19–22** Guess the value of the limit (if it exists) by evaluating the function at the given numbers (correct to six decimal places).

**19.** $\lim_{x\to 3} \dfrac{x^2 - 3x}{x^2 - 9}$,
$x = 3.1, 3.05, 3.01, 3.001, 3.0001,$
$2.9, 2.95, 2.99, 2.999, 2.9999$

**20.** $\lim_{x\to -3} \dfrac{x^2 - 3x}{x^2 - 9}$,
$x = -2.5, -2.9, -2.95, -2.99, -2.999, -2.9999,$
$-3.5, -3.1, -3.05, -3.01, -3.001, -3.0001$

**21.** $\lim_{t\to 0} \dfrac{e^{5t} - 1}{t}, \quad t = \pm 0.5, \pm 0.1, \pm 0.01, \pm 0.001, \pm 0.0001$

**22.** $\lim_{h\to 0} \dfrac{(2 + h)^5 - 32}{h}$,
$h = \pm 0.5, \pm 0.1, \pm 0.01, \pm 0.001, \pm 0.0001$

**23–28** Use a table of values to estimate the value of the limit. If you have a graphing device, use it to confirm your result graphically.

**23.** $\lim_{x\to 4} \dfrac{\ln x - \ln 4}{x - 4}$

**24.** $\lim_{p\to -1} \dfrac{1 + p^9}{1 + p^{15}}$

**25.** $\lim_{\theta\to 0} \dfrac{\sin 3\theta}{\tan 2\theta}$

**26.** $\lim_{t\to 0} \dfrac{5^t - 1}{t}$

**27.** $\lim_{x\to 0^+} x^x$

**28.** $\lim_{x\to 0^+} x^2 \ln x$

**29–40** Determine the infinite limit.

**29.** $\lim_{x\to 5^+} \dfrac{x + 1}{x - 5}$

**30.** $\lim_{x\to 5^-} \dfrac{x + 1}{x - 5}$

**31.** $\lim_{x\to 2} \dfrac{x^2}{(x - 2)^2}$

**32.** $\lim_{x\to 3^-} \dfrac{\sqrt{x}}{(x - 3)^5}$

**33.** $\lim_{x\to 1^+} \ln\left(\sqrt{x} - 1\right)$

**34.** $\lim_{x\to 0^+} \ln(\sin x)$

**35.** $\lim_{x\to (\pi/2)^+} \dfrac{1}{x} \sec x$

**36.** $\lim_{x\to \pi^-} x \cot x$

**37.** $\lim_{x\to 1} \dfrac{x^2 + 2x}{x^2 - 2x + 1}$

**38.** $\lim_{x\to 3^-} \dfrac{x^2 + 4x}{x^2 - 2x - 3}$

**39.** $\lim_{x\to 0} \left(\ln x^2 - x^{-2}\right)$

**40.** $\lim_{x\to 0^+} \left(\dfrac{1}{x} - \ln x\right)$

**41.** Find the vertical asymptote of the function

$$f(x) = \frac{x - 1}{2x + 4}$$

**42.** (a) Find the vertical asymptotes of the function

$$y = \frac{x^2 + 1}{3x - 2x^2}$$

(b) Confirm your answer to part (a) by graphing the function.

**43.** Determine $\lim_{x \to 1^-} \dfrac{1}{x^3 - 1}$ and $\lim_{x \to 1^+} \dfrac{1}{x^3 - 1}$

(a) by evaluating $f(x) = 1/(x^3 - 1)$ for values of $x$ that approach 1 from the left and from the right,

(b) by reasoning as in Example 7, and

(c) from a graph of $f$.

**44.** (a) By graphing the function

$$f(x) = \frac{\cos 2x - \cos x}{x^2}$$

and zooming in toward the point where the graph crosses the $y$-axis, estimate the value of $\lim_{x \to 0} f(x)$.

(b) Check your answer in part (a) by evaluating $f(x)$ for values of $x$ that approach 0.

**45.** (a) Estimate the value of the limit $\lim_{x \to 0} (1 + x)^{1/x}$ to five decimal places. Does this number look familiar?

(b) Illustrate part (a) by graphing the function $y = (1 + x)^{1/x}$.

**46.** (a) Graph the function $f(x) = e^x + \ln|x - 4|$ for $0 \leqslant x \leqslant 5$. Do you think the graph is an accurate representation of $f$?

(b) How would you get a graph that represents $f$ better?

**47.** (a) Evaluate the function $f(x) = x^2 - (2^x/1000)$ for $x = 1, 0.8, 0.6, 0.4, 0.2, 0.1$, and 0.05, and guess the value of

$$\lim_{x \to 0} \left( x^2 - \frac{2^x}{1000} \right)$$

(b) Evaluate $f(x)$ for $x = 0.04, 0.02, 0.01, 0.005, 0.003$, and 0.001. Guess again.

**48.** (a) Evaluate the function

$$h(x) = \frac{\tan x - x}{x^3}$$

for $x = 1, 0.5, 0.1, 0.05, 0.01$, and 0.005.

(b) Guess the value of $\lim_{x \to 0} \dfrac{\tan x - x}{x^3}$.

(c) Evaluate $h(x)$ for successively smaller values of $x$ until you finally reach a value of 0 for $h(x)$. Are you still confident that your guess in part (b) is correct? Explain why you eventually obtained 0 values. (In Section 4.4 a method for evaluating this limit will be explained.)

(d) Graph the function $h$ in the viewing rectangle $[-1, 1]$ by $[0, 1]$. Then zoom in toward the point where the graph crosses the $y$-axis to estimate the limit of $h(x)$ as $x$ approaches 0. Continue to zoom in until you observe distortions in the graph of $h$. Compare with the results of part (c).

**49.** Use a graph to estimate the equations of all the vertical asymptotes of the curve

$$y = \tan(2 \sin x) \qquad -\pi \leqslant x \leqslant \pi$$

Then find the exact equations of these asymptotes.

**50.** Consider the function $f(x) = \tan \dfrac{1}{x}$.

(a) Show that $f(x) = 0$ for $x = \dfrac{1}{\pi}, \dfrac{1}{2\pi}, \dfrac{1}{3\pi}, \ldots$

(b) Show that $f(x) = 1$ for $x = \dfrac{4}{\pi}, \dfrac{4}{5\pi}, \dfrac{4}{9\pi}, \ldots$

(c) What can you conclude about $\lim_{x \to 0^+} \tan \dfrac{1}{x}$?

**51.** In the theory of relativity, the mass of a particle with velocity $v$ is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where $m_0$ is the mass of the particle at rest and $c$ is the speed of light. What happens as $v \to c^-$?