Math 1A

Test 4 Review

For problems 1 through 2 a calculator may NOT be used.

1. Compute the following expressions.

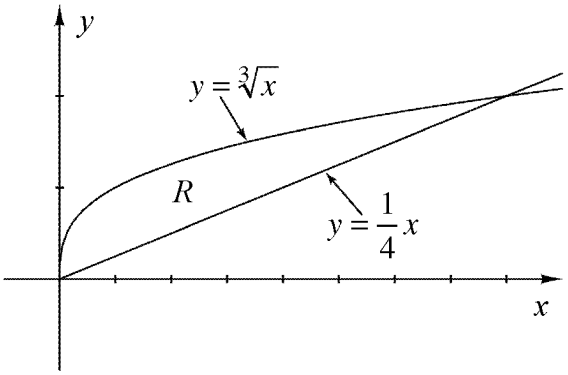
   a) $\lim_{x\to 0} \dfrac{1-\cos x}{x^2+x}$

   b) $\lim_{x\to 0^+} x^2 \ln x$

   c) $\dfrac{d}{dx}\left[\sqrt{x\ln(x^4)}\right]$

   d) $\displaystyle\int_0^4 \frac{x}{\sqrt{1+2x}}\,dx$

2. The figure below shows the graphs of $y = \sqrt[3]{x}$ and $y = \frac{1}{4}x$ enclose an area in the first quadrant. This area will be referred to as region R in parts (a) through (d). For parts (a) through (d), set up but DO NOT EVALUATE an intergral for finding the given quantity.



   a) The area of region R.

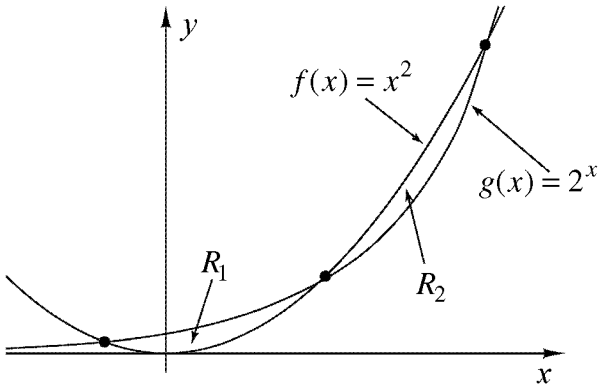
   b) The volume generated by revolving the region R about the x-axis using the washer method.

   c) The volume generated by revolving the region R about the y-axis using the shell method.

   d) The volume generated by revolving the region R about the line $x = 8$, you choose the method.

A calculator may be used to help solve problems 3 through 6.

3. The regions R_1 and R_2 shown in the figure below are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.



a) Find the $x-$ and $y-$coordinate of the three points of intersection of the graphs of f and g.

b) Find the total area of the regions $R_1 + R_2$.

c) Find the volume of the solid generated by revolving R_1 about the line $y = 5$.

4. Let R be the region in the first quadrant under the graph of $y = \ln x$ for $e \leq x \leq e^2$.

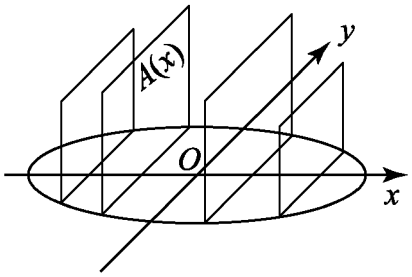
a) Find the length of the arc over the given interval.

b) Find the area of R.

c) If the line $x = k$ divides the region R into two regions of equal area, what is the value of k?

d) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are equilateral triangles.

5. For the solid below, the base is a circle of radius 3, and each cross section perpendicular to the x-axis is a square.



a) Find a function $A(x)$ that describes the area of the face of each cross-section.

b) Write an integral that will calculate the volume of the solid.

c) Compute the volume of the solid.

6. Consider the definite integral below.

$$\int_2^4 \frac{e^x}{x} dx$$

Find the value of the definite integral by the indicated methods using 10 subintervals and approximating your answers to six decimal places. (15 points)

a) Midpoint Rule

b) Trapezoidal Rule

c) Simpson's Rule