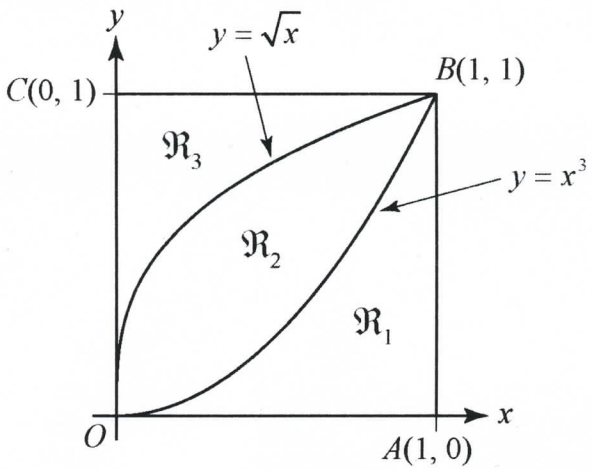
Math 1A

Test 4 - Review

1. The figure below shows the graphs of $y = x^3$ and $y = \sqrt{x}$. Three bounded regions are formed in a square of side length one $\mathfrak{R}_1$, $\mathfrak{R}_2$, and $\mathfrak{R}_3$. For parts (a) through (d), set up and EVALUATE an intergral for finding the given quantities. (25 points)



a) The area of region $\mathfrak{R}_2$.

b) The volume generated by revolving the region $\mathfrak{R}_1$ about the x-axis using the disk method.

c) The volume generated by revolving the region $\mathfrak{R}_3$ about the y-axis using the shell method.

d) The volume generated by revolving the region $\mathfrak{R}_2$ about the y-axis, you choose the method.

e) The volume generated by letting $\mathfrak{R}_2$ be the foot print of a solid such that when the solid is sliced by a plane perpendicular to the x-axis the intersection is a square.

a) $A = \int_0^1 (\sqrt{x} - x^3)\,dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{4}\cdot x^4\right]_0^1$

$= \frac{2}{3} - \frac{1}{4}$

$= \frac{5}{12}$ units^2

b) $V = \pi\int_0^1 [x^3]^2\,dx$

$V = \pi\cdot\int_0^1 x^6\,dx$

$= \pi\cdot\left[\frac{1}{7}x^7\right]_0^1$

$= \frac{\pi}{7}$ units^3

c) $V = 2\pi \int_0^1 x\,(1-\sqrt{x})\,dx$

$= 2\pi \int_0^1 \left(x - x^{\frac{3}{2}}\right) dx$

$= 2\pi \left[\frac{1}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}}\right]_0^1$

$= 2\pi \left[\frac{1}{2} - \frac{2}{5}\right]$

$= \frac{\pi}{5}$ units^3

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d) Shell Method

$V = 2\pi \int_0^1 x\left(x^{\frac{1}{2}} - x^3\right) dx$

$= 2\pi \cdot \int_0^1 \left(x^{\frac{3}{2}} - x^4\right) dx$

$= 2\pi \cdot \left[\frac{2}{5}x^{\frac{5}{2}} - \frac{1}{5}x^5\right]_0^1$

$= 2\pi \cdot \left[\frac{2}{5} - \frac{1}{5}\right]$

$= \frac{2\pi}{5}$ units^3

d)

$$V = \int_0^1 \left(x^{\frac{1}{2}} - x^3\right)^2 dx$$

$$= \int_0^1 \left(x - 2x^{\frac{7}{2}} + x^6\right) dx$$

$$= \left[\frac{1}{2}x^2 - \frac{4}{9}x^{\frac{9}{2}} + \frac{1}{7}x^7\right]_0^1$$

$$= \frac{1}{2} - \frac{4}{9} + \frac{1}{7}$$

$$= \frac{63 - 56 + 18}{126}$$

$$= \frac{25}{126} \text{ units}^3$$