Math 1A

Test 3 Review

For problems 1 through 5 a calculator may NOT be used.

1. Given that $\int_{-2}^{10} f(x)\,dx = -9$ and $\int_{-2}^{10} g(x)\,dx = 15$, find the value of each expression below.

a) $\int_{-2}^{10} 4g(x)\,dx + \int_{1}^{1} -2f(x)\,dx$

b) $\int_{-2}^{10} (3f(x) - 5g(x))\,dx$

c) $\int_{10}^{-2} \frac{10}{3} f(x)\,dx$

2. Consider the function $f(x) = \dfrac{x+2}{\sqrt{x^2+2}}$. Determine the following.

a) Find $f'(x)$.

b) Find the critical numbers.

c) Find the intervals of increasing and decreasing, and the coordinates of all extrema, if any. Identify them as local or absolute.

d) Find $f''(x)$.

e) Find the intervals of concavity, and the coordinates of the inflection points, if any.

3. Evaluate each integral, show all steps.

a) $\int \frac{5x^3+4}{x}\,dx$

b) $\int \sin^3 3t \cos 3t\,dt$

c) $\int_{0}^{\pi/4} \sqrt{\tan x}\, \sec^2 x\,dx$

d) $\int_{0}^{\ln 3} e^x (1+e^x)^{1/2}\,dx$

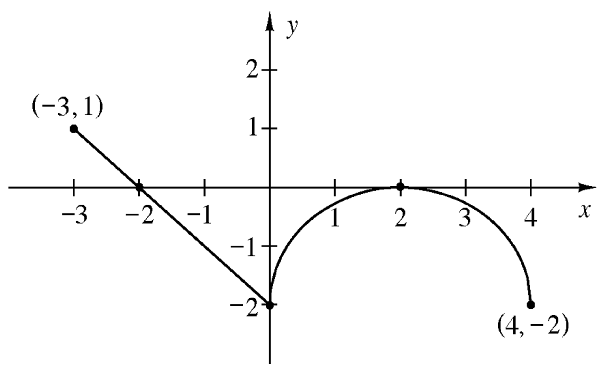
4. Consider the function $f(x) = x(x^2+1)^3$.

a) Write a Riemann Sum, but do not evaluate, that would approximate the area under the curve of $f(x)$ using 20 equal subintervals over the interval $[0, 2]$ using the midpoints of each subinterval.

b) Write the limit of the generalized Riemann Sum that would equal the exact area under the curve for the given interval.

c) Write the definite integral equal to the express in part (b), and evaluate the integral using The Fundamental Theorem of Calculus Part 2.

5. The graph of a differentiable function g on the closed interval $[-3, 4]$ with $g(0) = 3$ is shown in the figure below. The graph of f, the derivative of g, consists of one line segment and a semicircle, as show below.



Graph of f

a) Using the Fundamental Theorem of Calculus Part 1, express $g(x)$ as an integral.

b) On what intervals, if any, is $g(x)$ increasing? Justify your answer.

c) Find the x-coordinate of each point of inflection of the graph of g on the open interval $(-3, 4)$. Justify your answer.

d) Find the equation for the line tangent to the graph of $g(x)$ when $x = 4$.

e) Find $g(-3)$. Show the work that leads to your answer.

A calculator may be used to help solve problems 6 through 10.

6. A particle moves along the x-axis so that it's velocity at any time $t \geq 0$ seconds is given by $v(t) = t \sin t$. At time $t = 0$ seconds, the position of the particle is $x = 6$ feet to the right of the origin.

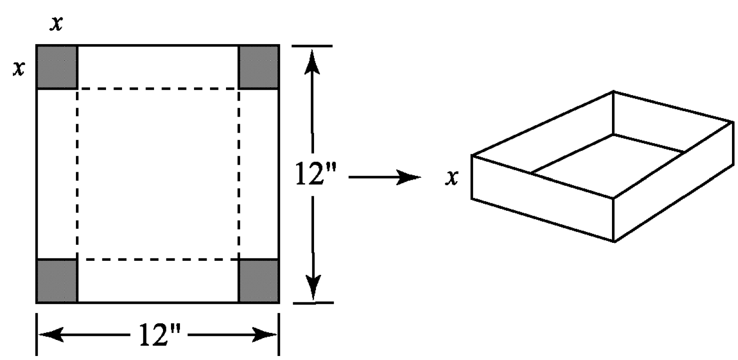
   a) For what values of t, $0 \leq t \leq 5$, is the particle moving right?

   b) Write an expression for the acceleration of the particle in terms of t.

   c) Write an expression for the position $x(t)$ of the particle.

   d) Find the total distance traveled for the particle when $0 \leq t \leq 5$.

7. An open-top box is to be made by cutting small congruent squares from the corners of a 12-by-12-in. sheet of tin and bending up the sides. How large should the squares cut from the corners be to make the box hold as much as possible?

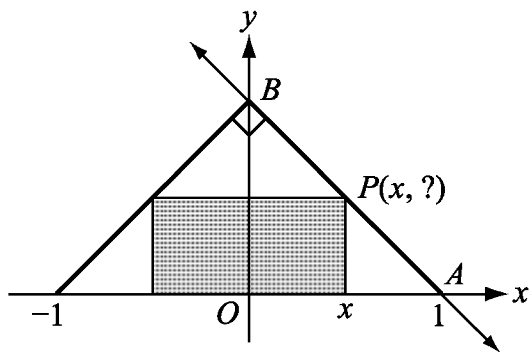


   a) Write a function of a single variable that describes the volume of the box.

   b) Indicate the domain for the function you found in part (a).

   c) Solve the problem.

8. The figure below shows a rectangle inscribed in an isosceles right triangle whose hypotenuse is 2 units long.



a) Express the y-coordinate of P in terms of x. (You might start by writing an equation for the line AB.)

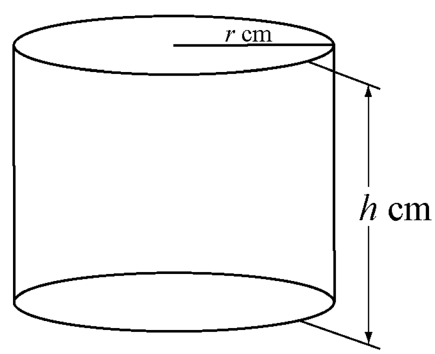
b) Express the area of the rectangle in terms of x.

c) What is the largest area the rectangle can have?

9. You have been asked to design a 1500 cm^3 container shaped like a right circular cylinder, see figure below. What dimensions will use least material?

$$V = \pi r^2 h$$

$$A = 2\pi r^2 + 2\pi rh$$



a) Write a function of a single variable that describes the surface area of the cylinder as a function of r.

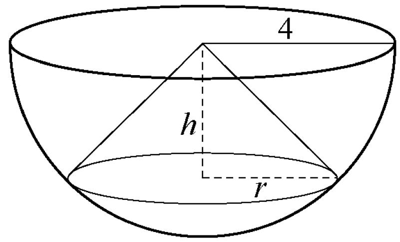
b) Find the derivative for the function you found in part (a).

c) Find and test the critical numbers for the derivative you found in part (b).

d) What are the dimensions of the cylinder with the least materials used.

10. Find the volume of a right circular cone with greatest volume that can be inscribed in a semisphere of radius 4 feet.

Volume of Cone: $V = \frac{1}{3}\pi r^2 h$



a) Express the radius of the cone r in terms of its height h.

b) Express the volume of the cone as a function of h, and state the domain.

c) What is the largest volume of the cone? Justify your answer.