#1) (a) $\int_{-2}^{10} 4\cdot g(x)\,dx + \int_{1}^{1} -2 f(x)\,dx$

$= 4\int_{-2}^{10} g(x) - 2\cdot\int_{1}^{1} f(x)\,dx$

$= 4(15) - 2\cdot 0$

$= 60$

(b) $\int_{-2}^{10} (3\cdot f(x) - 5\,g(x))\,dx$

$= 3\cdot\int_{-2}^{10} f(x)\,dx - 5\cdot\int_{-2}^{10} g(x)\,dx$

$= 3(-9) - 5(15)$

$= -102$

(c) $\int_{10}^{-2} \frac{10}{3}\cdot f(x)\,dx$

$= -\frac{10}{3}\cdot\int_{-2}^{10} f(x)\,dx$

$= -\frac{10}{3}\cdot(-9)$

$= 30$

#2

(a)

$$f'(x) = \frac{(x^2+2)^{\frac{1}{2}}\cdot 1 - (x+2)\cdot \frac{1}{\cancel{2}}\cdot (x^2+2)^{-\frac{1}{2}}\cdot \cancel{2}x}{x^2+2}$$

$$= \frac{(x^2+2)^{-\frac{1}{2}}\left((x^2+2)^{1} - x^2 - 2x\right)}{x^2+2}$$

$$= \frac{-2x+2}{(x^2+2)^{3/2}}$$

$$f'(x) = \frac{-2(x-1)}{(x^2+2)^{3/2}}$$

(b) Critical numbers:

let $f'(x) = 0$, So $x = 1$

(c)

$f'(x)$: $+ + +$ 0 $- - -$; x

$\nearrow$ 1 $\searrow$

By I.D.T.:

<u>Increasing</u>: $x \in (-\infty, 1)$

<u>Decreasing</u>: $x \in (1, +\infty)$

By F.D.T. $(1, f(1)) = (1, \frac{3}{\sqrt{3}})$ is a local maximum point. But, by F.D.T. A.E.V, $(1, \frac{3}{\sqrt{3}})$ is an absolute maximum point.

#2

(d) $$f''(x) = \frac{(x^2+2)^{\frac{3}{2}}(-2) - (-2x+2)\cdot\frac{3}{\cancel{2}}\cdot(x^2+2)^{\frac{1}{2}}\cdot\cancel{2}x}{(x^2+2)^3}$$

$$= \frac{(x^2+2)^{\frac{1}{2}}\left((x^2+2)'(-2) - 3x(-2x+2)\right)}{(x^2+2)^3}$$

$$= \frac{-2x^2-4+6x^2-6x}{(x^2+2)^{\frac{5}{2}}}$$

$$= \frac{4x^2-6x-4}{(x^2+2)^{\frac{5}{2}}}$$

$$= \frac{2(2x^2-3x-2)}{(x^2+2)^{5/2}}$$

$$= \frac{2(2x+1)(x-2)}{(x^2+2)^{\frac{5}{2}}}$$

(e) Possible x-values for inflection points are solutions to $f''(x) = 0$, So

$$x = -\frac{1}{2} \text{ or } x = 2$$

#2 (e) continued

$$\begin{array}{ccccccccc} + & + & + & 0 & - - - & 0 & + & + & + \end{array} \quad f''(x)$$

$$\xrightarrow{\qquad\qquad\qquad\qquad} x$$

$$\cup \quad -\tfrac{1}{2} \quad \cap \quad 2 \quad \cup$$

By C.T.

Concave Downward: $x \in \left(-\frac{1}{2}, 2\right)$

Concave Upward: $x \in \left(-\infty, -\frac{1}{2}\right) \cup (2, +\infty)$

By definition the points of inflection are:

$$\left(-\frac{1}{2}, f\left(-\frac{1}{2}\right)\right) = \left(-\frac{1}{2}, \frac{\frac{3}{2}}{\frac{3}{2}}\right) = \left(-\frac{1}{2}, 1\right)$$

$$\left(2, f(2)\right) = \left(2, \frac{4}{\sqrt{6}}\right) = \left(2, \frac{2\sqrt{6}}{3}\right)$$

#3 (a) $$\int \frac{5x^3+4}{x}\,dx = \int\left(5x^2+\frac{4}{x}\right)dx$$

$$= \frac{5}{3}x^3 + 4\cdot\ln|x| + C$$

(b) $$\int \sin^3(3t)\cos(3t)\,dt$$

let: $u=\sin(3t)$

$du = 3\cos(3t)\,dt$

$\frac{1}{3}du = \cos(3t)\,dt$

By substitution we get,

$$\frac{1}{3}\int u^3\,du$$

$$= \frac{1}{12}u^4 + C$$

$$= \frac{1}{12}\sin^4(3t) + C$$

#3 (c) $\int_0^{\frac{\pi}{4}} \sqrt{\tan x} \cdot \sec^2 x\, dx$

Let
$u = \tan x$
$du = \sec^2 x\, dx$

<u>New bounds</u>

<u>Upper</u>: $x = \frac{\pi}{4}$
$u = \tan \frac{\pi}{4}$
$u = 1$

<u>Lower</u> $x = 0$
$u = \tan 0$
$u = 0$

By sabstitution we get

$$\int_0^1 u^{\frac{1}{2}} du$$

$$= \frac{2}{3} u^{\frac{3}{2}} \Big|_0^1$$

$$= \frac{2}{3}(1)^{\frac{3}{2}} - \frac{2}{3}(0)^{\frac{3}{2}}$$

$$= \frac{2}{3}$$

(d) $\int_0^{\ln 3} e^x (1+e^x)^{1/2} dx$

Let $u = 1+e^x$
$du = e^x dx$

<u>New bounds</u>

<u>upper</u>: $x = \ln 3$
$u = 1 + e^{\ln 3}$
$u = 4$

<u>Lower</u>: $x = 0$
$u = 1 + e^0$
$u = 2$

By substitute we get,

$$\int_2^4 u^{\frac{1}{2}} du$$

$$= \frac{2}{3} u^{\frac{3}{2}} \Big|_2^4$$

#3) (d) continued

$$= \frac{2}{3}(4)^{\frac{3}{2}} - \frac{2}{3}(2)^{\frac{3}{2}}$$

$$= \frac{2}{3}(8) - \frac{2}{3}\cdot\sqrt{8}$$

$$= \frac{16 - 4\sqrt{2}}{3}$$

#4) (a) Let $f(x) = x(x^2+1)^3$

$$\Delta x = \frac{2-0}{20} = \frac{1}{10}$$

$$\bar{x}_i = a + (i - \tfrac{1}{2})\cdot\Delta x$$

$$= 0 + (i - \tfrac{1}{2})\tfrac{1}{10}$$

$$= (i - \tfrac{1}{2})\tfrac{1}{10}$$

$$A_m \approx \sum_{i=1}^{20} f(\bar{x}_i)\cdot\Delta x$$

(b) Let $\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$

$\bar{x}_i = (i - \tfrac{1}{2})\cdot\frac{2}{n}$ and $f(x) = x(x^2+1)^3$

#4 (b) continued

$$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(\bar{x}_i)\cdot \Delta x$$

---

c)

$$A = \int_0^2 x(x^2+1)^3\,dx$$

Let

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

<u>New bounds</u>

<u>Upper</u>: $x=2$

$u = 2^2+1 = 5$

<u>Lower</u>: $x=0$

$u = 0^2+1 = 1$

$$A = \frac{1}{2}\int_1^5 u^3\,du$$

$$A = \frac{1}{2}\cdot\frac{1}{4}u^4 \Big|_1^5$$

$$A = \frac{1}{8}u^4 \Big|_1^5$$

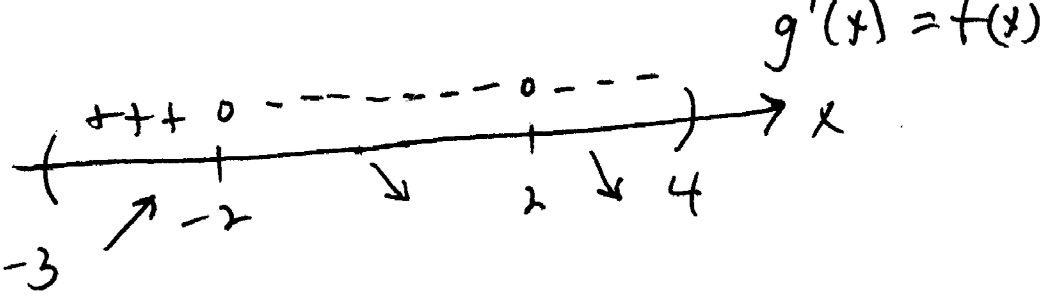
$$= \frac{1}{8}(5^4 - 1^4)$$

$$= \frac{624}{8}$$

$$= 78$$

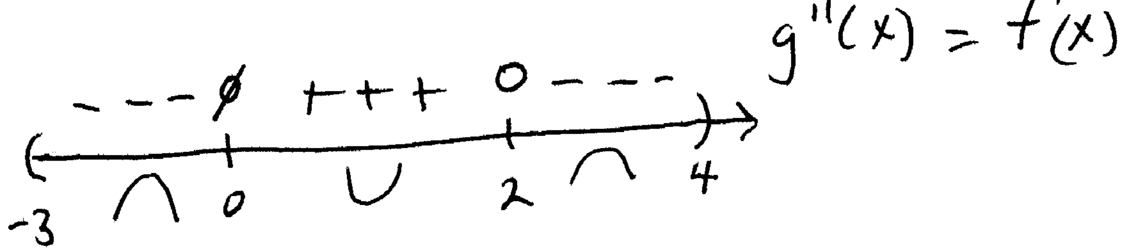
#5) (a) $g(x) = \int_0^x f(t)\,dt + 3$

(b) Since $f(x) = g'(x)$, the critical numbers for $g(x)$ are: $x = -2,\ x = 2$



From the sign number line above and by I.D.T., $g(x)$ is increasing on $x \in (-3, -2)$.

(c)

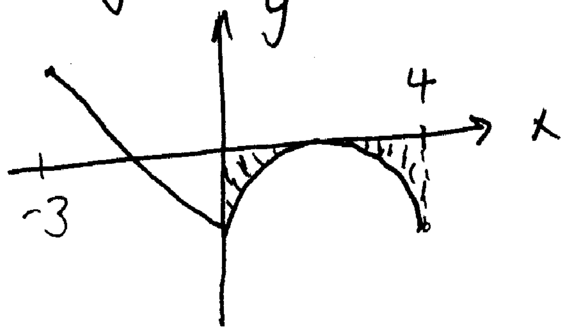


From the sign numberline above and by the definition of inflection points, concavity changes at $x = 0$ and $x = 2$. Hence, these are the x-coordinates of inflection points.

#5 continued

(d) $g(4) = \int_0^4 f(t)\,dt + 3$

From the graph of f we get



Area under the curve

$A = 4 \cdot 2 - \frac{\pi \cdot (2)^2}{2}$

$A = 8 - 2\pi$

But, A is below the x-axis, so

$A = -8 + 2\pi$, we then get $g(4) = \int_0^4 f(t)\,dt = -8 + 2\pi + 3$
$= -5 + 2\pi$

Point of Tangency: $(4, -5 + 2\pi)$

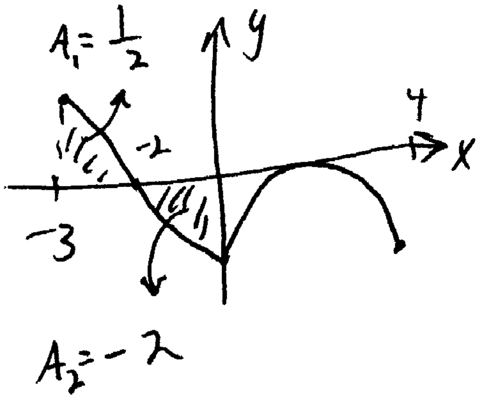
Slope of Tangent Line: $f(4) = -2$

Equation of Tangent Line:

$$y - (-5 + 2\pi) = -2(x - 4)$$

#5) Continued

(e) $g(-3) = \int_0^{-3} f(t)\,dt + 3$



$= -\int_{-3}^{0} f(t)\,dt + 3$

$= -\left(\frac{1}{2} + (-2)\right) + 3$

$= -\left(-\frac{3}{2}\right) + 3$

$= \frac{9}{2}$

#6) (a) $v(t) = t \sin t$

$v(t)$

$+ + + \; 0 \; - - -$

t: $0 \longrightarrow \pi \longleftarrow 5$

Right   Left

Moving right : $t \in (0, \pi)$ seconds

(b) $a(t) = v'(t)$

$= t \cdot \cos t + \sin t$

#6 continued

(c) $x(t) = \int v(t)\,dt$

$x(t) \overset{CAS}{=} \sin(t) - t\cdot\cos(t) + C$

Initial Conditions: $x(0) = 6$

$6 = \sin(0) - 0\cdot\cos(0) + C$

$C = 6$

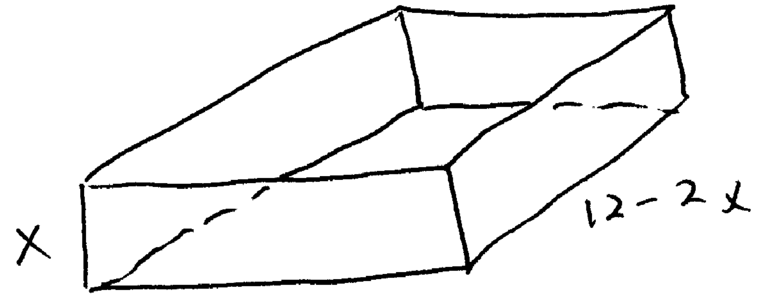
$x(t) = \sin(t) - t\cdot\cos(t) + 6$

(d) Total Distance $= \int_0^5 |v(t)|\,dt$

$= \int_0^5 |t\cdot\sin t|\,dt$

$\overset{CAS}{=} 8.660$ feet

#7



a) $V(x) = x(12-2x)^2$

b) Domain: $x \in [0, 6]$

c) $V'(x) \overset{CAS}{=} 12(x-6)(x-2)$

Let $V'(x) = 0$

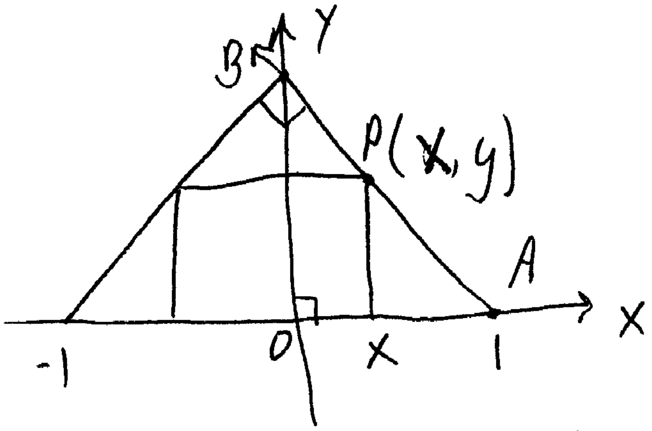
Critical Numbers: $x = 2$ or $x = 6$

By E.V.T

x	$V(x)$
0	0
2	128 $\Longrightarrow$ Absolute maximum point.
6	0

Hence, cut out a 2 inch square from each corner to maximize the volume at 128 in^3.

#8



First

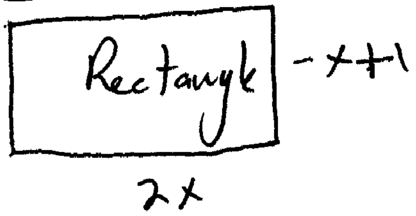
$OB = 1$

So point B is $(0,1)$

Slope of line AB is -1. So the equation is $y = -x + 1$.

a) P has a y-coordinate of $y = -x + 1$.

b)



$A(x) = 2x(-x+1)$ Domain: $x \in [0,1]$

$A'(x) \stackrel{CAS}{=} 2 - 4x$

Let $A'(x) = 0$

Critical Number: $x = \frac{1}{2}$

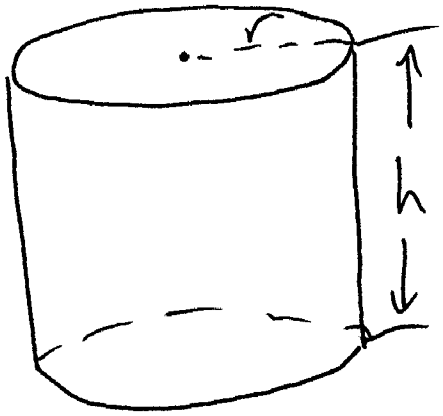
So by E.V.T.

x	$A(x)$
0	0
$\frac{1}{2}$	$\frac{1}{2}$
1	0

$\Longrightarrow$ Absolute maximum point.

Hence, the largest maximum area is $\frac{1}{2}$ units^2.

#9



Given:

$V = 1500$

$V = \pi r^2 h$

$1500 = \pi r^2 h$

$h = \dfrac{1500}{\pi r^2}$

a) $A = 2\pi r^2 + 2\pi r h$

$A = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left(\dfrac{1500}{\pi r^2}\right)$

$A(r) = 2\pi r^2 + \dfrac{3000}{r}$ Domain: $r \in (0, +\infty)$

b) $A'(r) \overset{CAS}{=} 4\pi r - \dfrac{3000}{r^2}$

c) let $A'(r) \overset{CAS}{=} 0$

<u>critical number</u>: $r = \dfrac{5 \cdot 6^{1/3}}{\pi^{1/3}}$

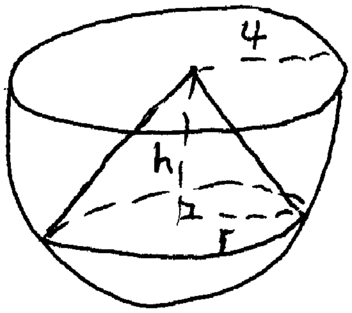
By FOTAEV,

$$\begin{array}{ccccc} & & & A'(r) & \\ - - - & 0 & + + + & \rightarrow & r \\ \searrow & \dfrac{5 \cdot 6^{1/3}}{\pi^{1/3}} & \nearrow & & \end{array}$$

Absolute minimizer

d) The dimension of the cylinder are radius is $\dfrac{5 \cdot 6^{1/3}}{\pi^{1/3}}$ cm and the height is $\dfrac{10 \cdot 6^{1/3}}{\pi^{1/3}}$ cm.
With a minimum amount of materials of $150\pi^{1/3} 6^{2/3}$ cm^2.

#10



$V = \frac{1}{3}\pi r^2 h$

a) $h^2 + r^2 = 4^2$

$r^2 = 16 - h^2$

b) $V(h) = \frac{1}{3}\pi(16-h^2)h \quad h \in [0,4]$

c) $V'(h) \overset{CAS}{=} \frac{-(3h^2-16)\pi}{3}$

Let $V'(h) \overset{CAS}{=} 0$

$h = \frac{4\sqrt{3}}{3}$ or $\boxed{h = \frac{-4\sqrt{3}}{3}}$ Not in domain

By EVT

h	$V(h)$
0	0
$\frac{4\sqrt{3}}{3}$	$\frac{128\pi\sqrt{3}}{27}$ $\Rightarrow$ Absolute maximum point.
4	0

Hence, the largest possible volume is $\frac{128\pi\sqrt{3}}{27}$ ft^3.