Math 1A	Name Key	
Test 2 - Summer 2026	Roll # ______	Score ______

Part 1: This part has 4 problems with subparts. A calculator may not be used on this part of the exam. Show all your work on the test in the space provided after each question. Do not write on the back. Appropriate evidence will receive partial credit. If you need additional room for your solution please use "Spillage" on the last page. Good Luck!!

1. Consider the function $h(x) = \sqrt{4 + 3f(x)}$, where $f(1) = 7$ and $f'(1) = 4$. (20 points)

a) Find $h(1)$.

$$h(1) = \sqrt{4+3\cdot f(1)} = \sqrt{4+3\cdot 7} = \sqrt{25}$$

$h(1) = 5$

$(1, 5)$

b) Determine $h'(x)$.

$$h'(x) = \frac{1}{2}(4+3f(x))^{-\frac{1}{2}} \cdot 3f'(x) = \frac{3\cdot f'(x)}{2\sqrt{4+3\cdot f(x)}}$$

c) Find the slope of the tangent line to $h(x)$ at $x = 1$.

$$m = h'(1) = \frac{3\cdot f'(1)}{2\cdot\sqrt{4+3\cdot f(1)}} = \frac{3\cdot 4}{2\cdot 5} = \frac{6}{5}$$

d) Find the equations of the tangent line and normal line to $h(x)$ at $x = 1$.

Tangent Line

① Slope: $m = \frac{6}{5}$

② POT: $(1, 5)$

③ $y - 5 = \frac{6}{5}(x-1)$

$y = \frac{6}{5}x + \frac{19}{5}$

Normal Line

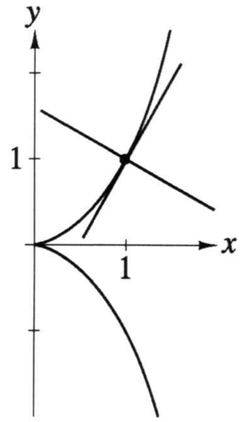
① Slope: $m = -\frac{5}{6}$

② POT: $(1, 5)$

③ $y - 5 = -\frac{5}{6}(x-1)$

$y = -\frac{5}{6}x + \frac{35}{6}$

2. Condsider the equation $2y^2 - xy^2 = x^3$, this curve is call the cissoid of Diocles (from about 200 B.C.). (20 points)



a) What type of equation is cissoid of Diocles.

Implicit Equation.

b) Find $\frac{dy}{dx}$.

$$4y \cdot y' - (x \cdot 2y \cdot y' + y^2 \cdot 1) = 3x^2$$

$$4yy' - 2xy \cdot y' - y^2 = 3x^2$$

$$(4y - 2xy)y' = 3x^2 + y^2$$

$$y' = \frac{3x^2 + y^2}{4y - 2xy}$$

c) Calculate the slope of the tangent line and normal line at the point (1, 1).

$$m = \frac{3+1}{4-2}$$

$$m = 2$$

Tangent Line

Normal Line:

$$m = -\frac{1}{2}$$

d) Find the equations for the tangent and normal lines at the point in part (c).

Tangent Line

$$y - 1 = 2(x - 1)$$

$$y = 2x - 1$$

Normal Line

$$y - 1 = -\frac{1}{2}(x - 1)$$

$$y = -\frac{1}{2}x + \frac{3}{2}$$

3. Find $\dfrac{dy}{dx}$ for each equation below. (15 points)

a) $y = \dfrac{1}{(x^5 + 3x^2 - x)^{-50}}$

$y = (x^5 + 3x^2 - x)^{50}$

$y' = 50(x^5 + 3x^2 - x)^{49} \cdot (5x^4 + 6x - 1)$

b) $y = x \cdot \sin^{-1} x + \sqrt{1 - x^2}$

$y' = x \cdot \dfrac{1}{\sqrt{1-x^2}} + \sin^{-1} x + \dfrac{1}{2} \cdot \dfrac{-2x}{\sqrt{1-x^2}}$

$= \dfrac{x}{\sqrt{1-x^2}} + \sin^{-1} x - \dfrac{x}{\sqrt{1-x^2}}$

$= \sin^{-1} x$

c) $y = \left(3^{\cos(x^2)} - 1\right)^4$

$y' = 4\left(3^{\cos(x^2)} - 1\right)^3 \cdot \dfrac{d}{dx}\left[3^{\cos(x^2)} - 1\right]$

$= 4\left(3^{\cos(x^2)} - 1\right)^3 \cdot 3^{\cos(x^2)} \cdot \ln 3 \cdot \dfrac{d}{dx}\left[\cos(x^2)\right]$

$= 4\left(3^{\cos(x^2)} - 1\right)^3 \cdot 3^{\cos(x^2)} \cdot \ln 3 \cdot (-\sin(x^2)) \cdot 2x$

$= -8x \cdot \ln 3 \cdot \sin(x^2) \cdot 3^{\cos(x^2)} \cdot \left(3^{\cos(x^2)} - 1\right)^3$

4. Find $\frac{dy}{dx}$ for each equation below. (15 points)

a) $y = x^2 \cdot \sinh(3x) + \tan^{-1}(\sec x)$

$$y' = x^2 \cdot \cosh(3x) \cdot 3 + 2x \cdot \sinh(3x) + \frac{1}{1+\sec^2 x} \cdot \sec x \cdot \tan x$$

$$= 3x^2 \cdot \cosh(3x) + 2x \cdot \sinh(3x) + \frac{\sec x \cdot \tan x}{1 + \sec^2 x}$$

b) $y = \sqrt{x} \cdot e^{x^2-x} \cdot (3x+1)^{2/3}$

$$\ln y = \frac{1}{2}\ln x + (x^2 - x) \cdot \ln e + \frac{2}{3} \cdot \ln(3x+1)$$

$$y' \cdot \frac{1}{y} = \frac{1}{2x} + 2x - 1 + \frac{2}{3} \cdot \frac{1}{3x+1} \cdot 3$$

$$y' = \sqrt{x} \cdot e^{x^2-x} \cdot (3x+1)^{2/3} \cdot \left[\frac{1}{2x} + 2x - 1 + \frac{2}{3x+1}\right]$$

c) $y = (\ln x)^{\tan x}$

$$\ln y = \tan x \cdot \ln(\ln x)$$

$$\frac{1}{y} \cdot y' = \tan x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} + \sec^2 x \cdot \ln(\ln x)$$

$$y' = (\ln x)^{\tan x} \cdot \left[\frac{\tan x}{x \cdot \ln x} + \sec^2 x \cdot \ln(\ln x)\right]$$

Math 1A Name Key

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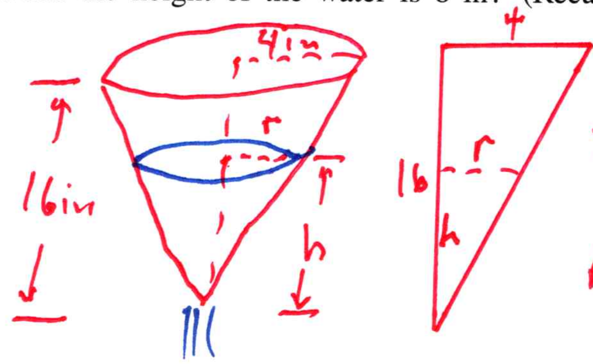
Part 2: This part has 3 problems with subparts. A calculator may be used on this part of the exam. Show all your work on your test. Do not write on the back. Appropriate evidence will receive partial credit. If you need additional room for your solution please use "Spillage" on the last page. Good Luck!!

5. Water is draining from the bottom of a cone-shaped funnel at the rate 2 in^3/min. The height of the funnel is 16 inches and the radius at the top of the funnel is 4 in. At what rate is the height of the water in the funnel changing when the height of the water is 8 in? (Recall: $V = \frac{1}{3}\pi r^2 h$) (10 points)



$\frac{r}{4} = \frac{h}{16}$

$r = \frac{h}{4}$

I. Introduction

Let V = Volume (in^3)
h = height (in)
r = radius (in)

II. Known Derivative

$\frac{dV}{dt} = -2$ in^3/min

III. Unknown Derivative

$\left.\frac{dh}{dt}\right|_{h=8\text{ in}}$

IV. Equation

$V = \frac{1}{3}\pi r^2 h \quad \Big| \quad V = \frac{1}{3}\pi\left(\frac{h}{4}\right)^2 h \quad \Big| \quad V = \frac{\pi}{48}h^3$

V. Differential Equation

$\frac{dV}{dt} = \frac{\pi}{16}h^2 \cdot \frac{dh}{dt} \quad \Big| \quad \frac{dh}{dt} = \frac{16}{\pi h^2}\cdot\frac{dV}{dt}$

VI. Snap Shot $\frac{dV}{dt} = -2$, $h = 8$

$\frac{dh}{dt} = \frac{16}{\pi \cdot 8^2}\cdot(-2) \overset{CAS}{=} -\frac{1}{2\pi} \approx -0.159$ in/min

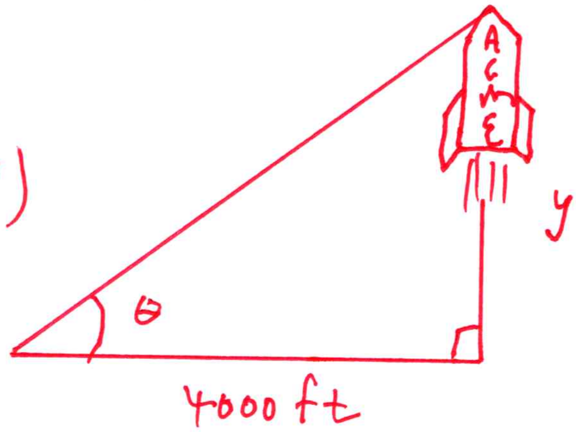
VII. Conclusion The height is decreasing at a rate of $\frac{1}{2\pi}$ in/min when the water height is 8 inches.

6. A television camera is positioned horizontally 4,000 feet from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Let's assume the rocket rises vertically and its speed is 600 ft/sec when it has risen 3,000 ft. If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that same moment? (10 points)

I. Introduction

y = heigth of rocket (ft)

θ = angle of camera (rad)



II. Known Derivative

$$\frac{dy}{dt} = 600 \text{ ft/sec at } 3000 \text{ ft.}$$

III. Unknown Derivative

$$\left.\frac{d\theta}{dt}\right|_{y=3000\text{ ft}}$$

IV. Equation

$$\tan\theta = \frac{y}{4000} \quad \Big| \quad \theta = \tan^{-1}\left(\frac{y}{4000}\right)$$

V. Differential Equation

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{y}{4000}\right)^2} \cdot \frac{1}{4000} \cdot \frac{dy}{dt}$$

VI. Snap Shot

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{3000}{4000}\right)^2} \cdot \frac{1}{4000} \cdot 600 \overset{CAS}{=} \frac{12}{125} \approx 0.096 \text{ rad/sec}$$

VII. Conclusion

The camera's angle is increasing at a rate of 0.096 rad/sec when the rocket is 3000 ft high.

7. A particle moves along a horizontal line so that its position at any time $t \geq 0$ (in seconds) is given by the function:

$$s(t) = t^3 - 6t^2 + 9t + 2$$

where s is measured in meters. (10 points)

a) Find the velocity $v(t)$ and the acceleration $a(t)$ of the particle at any time t.

$v(t) = 3t^2 - 12t + 9$

$a(t) = 6t - 12$

b) Find the position, velocity, and acceleration of the particle at $t = 3.671$ seconds.

$s(3.671) \overset{CAS}{=} 3.610$ meters

$v(3.671) \overset{CAS}{=} 5.319$ m/s

$a(3.671) \overset{CAS}{=} 10.026$ m/s^2

c) Is the particle speeding up or slowing down at this instant? Explain your reasoning.

At $t = 3.671$ sec, the particle is speeding up since velocity and acceleration have the same sign.

d) At what time(s) is the particle at rest?

Let $v(t) = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-1)(t-3) = 0$

The particle is stopped at $t = 1$ sec and $t = 3$ sec.

e) Find the total distance traveled by the particle during the first 5 seconds (from $t = 0$ to $t = 5$).

Total Distance $= |s(1) - s(0)| + |s(3) - s(1)| + |s(5) - s(3)|$

$\overset{CAS}{=} 28$ meters