Math 1A

Test 2 Review

For problems 1 through 4 a calculator may NOT be used.

1. Use implicit differentiation to find $\frac{dy}{dx}$ for the implicit equation $y^3 + x^2y + x^2 - 3y^2 = 0$.

2. Consider the function $x^2 - 4xy + y^2 = 1$.

   a) Find $\frac{dy}{dx}$.

   b) Find $\frac{d^2y}{dx^2}$.

   c) Find the slope of the tangent line at (1, 0).

3. Consider the function $f(x) = x^n \sin x$.

   a) Find the derivative of the function $f(x)$ for $n = 1,\ 2,\ 3$, and 4.

   b) Use the results from part (a) to write a general rule for $f'(x)$ in terms of n.

4. Find $\dfrac{dy}{dx}$ for each function below. (Show all symbolic work.)

a) $y = \dfrac{1}{(x^2 + 5x - 7)^3}$

b) $y = x^3 \cdot \sin\left(\sqrt{x+1}\right)$

c) $y = \sqrt[4]{5 - 2x^3}$

d) $y = \tan(\sin\ x)$

e) $y = \sin^{-1} x + x\sqrt{1 - x^2}$

f) $y = \ln \dfrac{x(x^2+1)^2}{\sqrt{2x^3 - 1}}$

A calculator may be used to help solve problems 5 through 7.

5. Water runs into a conical tank at the rate of 9 ft^3/min. The tank stands point down and has a height of 10 feet and a base radius of 5 feet. How fast is the water level rising when the water is 6 feet deep? (Recall: $V = \dfrac{1}{3}\pi r^2 h$)

6. A rocket that is launched vertically is tracked by a radar station located on the ground 3 miles from the launch site. What is the vertical speed of the rocket at the instant that its distance from the radar station is 5 miles and this distance is increasing at the rate of 5000 mph?

7. Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.

8. The combined electrical resistance R of R_1 and R_2, connected in parallel, is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

where R, R_1, and R_2 are measured in ohms. R_1 and R_2 are increasing at rates of 1 and 1.5 ohms per second, respectively. At what rate is R changing when $R_1 = 50$ ohms and $R_2 = 75$ ohms?