#1

$$y^3 + x^2y + x^2 - 3y^2 = 0$$

$$3y^2\frac{dy}{dx} + x^2\frac{dy}{dx} + 2xy + 2x - 6y\frac{dy}{dx} = 0$$

$$\left(3y^2 + x^2 - 6y\right)\frac{dy}{dx} = -2xy - 2x$$

$$\frac{dy}{dx} = \frac{-2xy - 2x}{3y^2 + x^2 - 6y}$$

#2 $x^2 - 4xy + y^2 = 1$

a) $$2x - 4x\frac{dy}{dx} - 4y + 2y\frac{dy}{dx} = 0$$

$$(-4x + 2y)\frac{dy}{dx} = -2x + 4y$$

$$\frac{dy}{dx} = \frac{-2x + 4y}{-4x + 2y} = \frac{x - 2y}{2x - y}$$

b) $$\frac{d^2y}{dx^2} = \frac{(2x-y)\left(1 - 2\frac{dy}{dx}\right) - (x - 2y)\left(2 - \frac{dy}{dx}\right)}{(2x-y)^2}$$

b] continued

$$\frac{d^2y}{dx^2} = \frac{(2x-y)\left(1-2\cdot\frac{x-2y}{2x-y}\right)-(x-2y)\left(2-\frac{x-2y}{2x-y}\right)}{(2x-y)^2}$$

$$= \frac{2x-y-2x+4y-2x+4y+\frac{(x-2y)^2}{2x-y}}{(2x-y)^2}$$

$$= \frac{-2x+7y+\frac{(x-2y)^2}{2x-y}}{(2x-y)^2}\cdot\frac{2x-y}{2x-y}$$

$$= \frac{-4x^2+14xy+2xy-7y^2+x^2-4xy+4y^2}{(2x-y)^3}$$

$$= \frac{-3x^2+12xy-3y^2}{(2x-y)^3}$$

$$= \frac{-3(x^2-4xy+y^2)}{(2x-y)^3}$$

Note:
$x^2-4xy+y^2=1$

$$\frac{d^2y}{dx^2} = \frac{-3}{(2x-y)^3}$$

c) 2 continued

$$\left.\frac{dy}{dx}\right|_{(1,0)} = \frac{1-2(0)}{2(1)-0}$$

$$= \frac{1}{2}$$

#3)

a)

$n=1$

$f(x) = x \sin x$

$f'(x) = x\cos x + \sin x$

$n=2$

$f(x) = x^2 \sin x$

$f'(x) = x^2 \cos x + 2x \sin x$

$n=3$

$f(x) = x^3 \sin x$

$f'(x) = x^3 \cos x + 3x^2 \sin x$

$n=4$

$f(x) = x^4 \sin x$

$f'(x) = x^4 \cos x + 4x^3 \sin x$

b) $f'(x) = x^n \cos x + n x^{n-1} \sin x$

#4

a) $$y = \frac{1}{(x^2+5x-7)^3}$$

$$y = (x^2+5x-7)^{-3}$$

$$y' = -3(x^2+5x-7)^{-4}(2x+5)$$

$$y' = \frac{-6x-15}{(x^2+5x-7)^4}$$

b) $$y = x^3 \cdot \sin(\sqrt{x+1})$$

$$y' = x^3 \cdot \cos(\sqrt{x+1}) \cdot \tfrac{1}{2}(x+1)^{-\frac{1}{2}} \cdot 1 + 3x^2 \sin(\sqrt{x+1})$$

$$y' = \frac{x^3 \cos\sqrt{x+1}}{2\sqrt{x+1}} + 3x^2 \sin(\sqrt{x+1})$$

c) $$y = \sqrt[4]{5-2x^3}$$

$$y' = \tfrac{1}{4}(5-2x^3)^{-\frac{3}{4}} \cdot (-6x^2)$$

$$= \frac{-3x^2}{2\sqrt[4]{(5-2x^3)^3}}$$

#4) continued

d) $y = \tan(\sin x)$

$$\frac{dy}{dx} = \sec^2(\sin x) \cdot \cos x$$

e) $y = \sin^{-1} x + x\sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} + x \cdot \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) + \sqrt{1-x^2}$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{-x^2}{\sqrt{1-x^2}} + \sqrt{1-x^2}$$

$$\frac{dy}{dx} = \frac{2-2x^2}{\sqrt{1-x^2}}$$

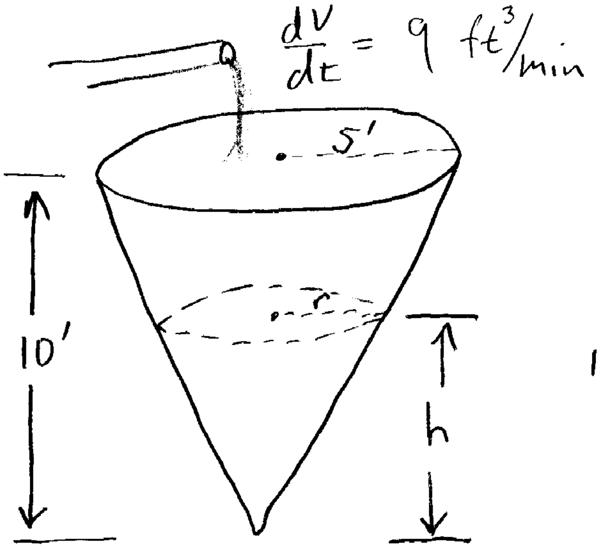
f) $y = \ln \dfrac{x(x^2+1)^2}{\sqrt{2x^3-1}}$

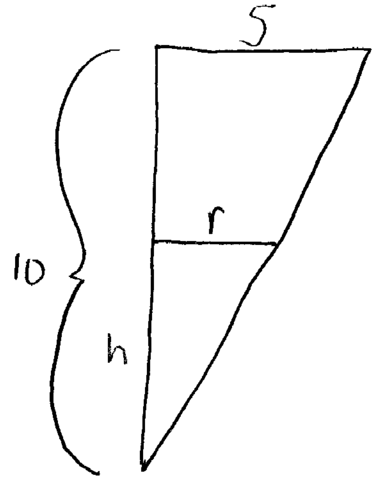
$$y = \ln x + 2\ln(x^2+1) - \frac{1}{2}\ln(2x^3-1)$$

$$\frac{dy}{dx} = \frac{1}{x} + 2 \cdot \frac{1}{x^2+1} \cdot 2x - \frac{1}{2} \cdot \frac{1}{2x^3-1} \cdot 6x^2$$

$$\frac{dy}{dx} = \frac{1}{x} + \frac{4x}{x^2+1} - \frac{3x^2}{2x^3-1}$$

#5]





$$\frac{r}{5} = \frac{h}{10}$$

$$r = \frac{1}{2}h$$

Given:

$$\frac{dV}{dt} = 9 \text{ ft}^3/\text{min}$$

Find:

$$\left.\frac{dh}{dt}\right|_{h=6}$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{1}{2}h\right)^2 h$$

$$V = \frac{\pi}{12} h^3$$

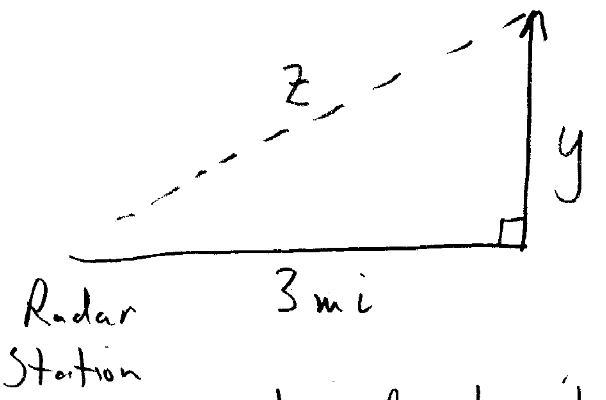
$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{\pi h^2} \cdot \frac{dV}{dt}$$

$$\left.\frac{dh}{dt}\right|_{h=6} = \frac{4}{\pi(6)^2} \cdot 9 = \frac{1}{\pi} \text{ ft}/\text{min}$$

The height is increasing at a rate of $\frac{1}{\pi}$ ft/min.

#6)



y = vertical height of rocket in miles
z = distance from station to rocket in miles

Given:

$$\left.\frac{dz}{dt}\right|_{z=5} = 5000 \text{ mph}$$

Find:

$$\left.\frac{dy}{dt}\right|_{z=5}$$

$$3^2 + y^2 = z^2$$

$$2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$\frac{dy}{dt} = \frac{z}{y}\cdot\frac{dz}{dt}$$

$z = 5$

$3^2 + y^2 = 5^2$

$y = 4$

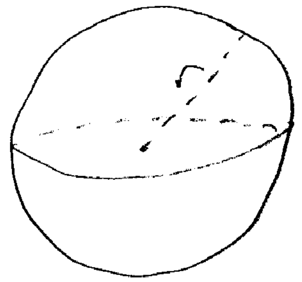
$$\left.\frac{dy}{dt}\right|_{\substack{y=4\\z=5}} = \frac{5}{4}(5000)$$

#6] Continued

$$\left.\frac{dy}{dt}\right|_{\substack{y=4\\ z=5}} = 6250 \text{ mph}$$

The rocket is traveling at a rate of 6250 mph when the rocket is 5 miles from the station.

#7]



r = radius of sphere in feet.

Given: $\frac{dV}{dt} = 4.5 \text{ ft}^3/\text{min}$

Find: $\left.\frac{dr}{dt}\right|_{r=2}$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4\pi r^2} \cdot \frac{dV}{dt}$$

$$\left.\frac{dr}{dt}\right|_{r=2} = \frac{1}{4\pi \cdot 2^2} \cdot \frac{9}{2}$$

$$\overset{\text{CAS}}{=} \frac{9}{32\pi} \text{ ft/min}$$

$$\approx 0.090 \text{ ft/min}$$

Radius is increasing at a rate of 0.090 ft/min.

#8

R, R_1, R_2 = resistance in ohms

Given: $\frac{dR_1}{dt} = 1$ ohms/sec

$\frac{dR_2}{dt} = 1.5$ ohms/sec

Find: $\left.\frac{dR}{dt}\right|_{\substack{R_1=50 \\ R_2=75}}$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R^{-1} = R_1^{-1} + R_2^{-1}$$

$$-R^{-2} \cdot \frac{dR}{dt} = -R_1^{-2} \cdot \frac{dR_1}{dt} - R_2^{-2} \cdot \frac{dR_2}{dt}$$

$$\frac{dR}{dt} = \frac{R_1^{-2} \cdot \frac{dR_1}{dt} + R_2^{-2} \frac{dR_2}{dt}}{R^{-2}}$$

$$\frac{1}{R} = \frac{1}{50} + \frac{1}{75}$$

$$R \overset{CAS}{=} 30$$

$$\left.\frac{dR}{dt}\right|_{\substack{R_1=50 \\ R_2=75}} = \frac{(50)^{-2} \cdot 1 + (75)^{-2} \cdot (1.5)}{(30)^{-2}}$$

$$\overset{CAS}{=} \frac{3}{5}$$

R is increasing at a rate of $\frac{3}{5}$ ohms/sec