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| Math 1A | | Name | Key |
| Test 1 - Summer 2026 | | Roll # ______ | Score ______ |

Part 1: This part has 5 questions. A calculator may not be used on this part of the exam. Show all your work on the space below each problem. Do not write on the back. Appropriate evidence will receive partial credit. If you need additional room for your solution please use "Spillage" on the last page. Good Luck!!

1. Using limit laws, evaluate the following limits. Show all your work. (10 points)

a) $\lim_{x \to 0} \frac{x}{\sqrt{1+3x}-1}$. What does this limit tell you about functional behavior?

$$\lim_{x \to 0} \frac{x}{\sqrt{1+3x}-1} \cdot \frac{\sqrt{1+3x}+1}{\sqrt{1+3x}+1}$$

$$= \lim_{x \to 0} \frac{x\cdot(\sqrt{1+3x}+1)}{1+3x-1}$$

$$= \lim_{x \to 0} \frac{\cancel{x}(\sqrt{1+3x}+1)}{3\cancel{x}}$$

$$= \frac{1}{3}\cdot \lim_{x \to 0} (\sqrt{1+3x}+1)$$

$$= \frac{1}{3}\cdot\left[\sqrt{\lim_{x\to 0}(1+3x)} + \lim_{x \to 0} 1\right]$$

$$= \frac{1}{3}\cdot\left[\sqrt{1+0}+1\right]$$

$$= \frac{1}{3}\cdot 2$$

$$= \frac{2}{3}$$

There is a hole in the graph at $(0, \frac{2}{3})$

b) $\lim_{x \to -\infty} \frac{\sqrt{1+4x^6}}{2-x^3}$. What does this limit tell you about functional behavior?

$$\lim_{x \to -\infty} \frac{\sqrt{1+4x^6}}{2-x^3}$$

$$\overset{DTE}{=} \lim_{x \to -\infty} \frac{\sqrt{4x^6}}{-x^3}$$

$$= \lim_{x \to -\infty} \frac{2|x^3|}{-x^3}$$

Note: $x^3 < 0$ $x \to -\infty$ $|x^3| = -x^3$

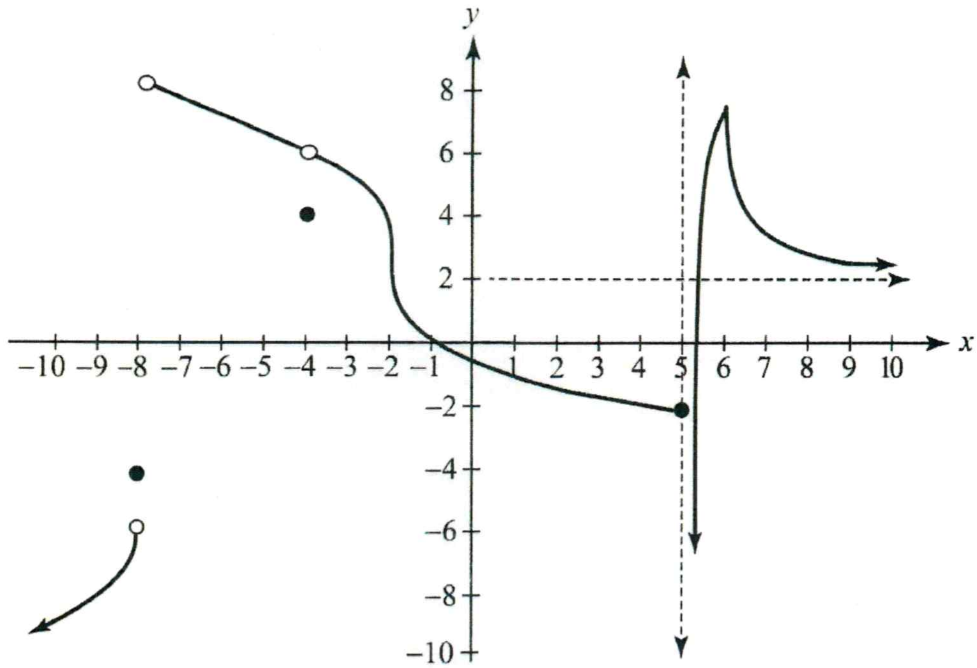
$$= \lim_{x \to -\infty} \frac{-2x^3}{-x^3}$$

$$= \lim_{x \to -\infty} 2$$

$$= 2$$

The limit represents a horizontal asymptote at $y=2$ as $x \to -\infty$.

2. Consider the graph of $y = f(x)$ below. All numerical answers will be integers. (10 points)




a) Determine all integer value(s) where $f(x)$ is NOT continuous. Explain.

x = -8 jump discontinuity
x = -4 removable discontinuity
x = 5 infinite discontinuity

b) Determine all integer value(s) where $f(x)$ is NOT differentible. Explain.

x = -8 discontinuity
x = -4 discontinuity
x = -2 Vertical Tangent line
x = 5 discontinuity
x = 6 cusp/corner.

c) Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to+\infty} f(x)$.

$\lim_{x\to-\infty} f(x) = -\infty$ | $\lim_{x\to+\infty} f(x) = 2$

d) Find $\lim_{x\to 5^+} f(x)$, $\lim_{x\to 5^-} f(x)$, and $\lim_{x\to 5} f(x)$.

$\lim_{x\to 5^+} f(x) = +\infty$ | $\lim_{x\to 5^-} f(x) = -2$ | $\lim_{x\to 5} f(x)$ DNE since $\lim_{x\to 5^-} f(x) \neq \lim_{x\to 5^+} f(x)$

e) Find the integer value $a$ where $\lim_{x\to a} f(x)$ exists and $f(a)$ exists, but $\lim_{x\to a} f(x) \neq f(a)$. Explain.

Let a = -4, $\lim_{x\to -4} f(x) = 6$ and $f(-4) = 4$

$\lim_{x\to -4} f(x) \neq f(-4)$

3. Consider the function $f(x) = -2x^2 + 3x - 5$. (20 points)

a) Complete the following definition.

The **derivative** of the function $f(x)$ is the function $f'(x)$ defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists !

b) Using the definition from part (a), find $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{-2(x+h)^2 + 3(x+h) - 5 - (-2x^2 + 3x - 5)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-2x^2} - 4hx \cancel{+3x} + 3h \cancel{-5} \cancel{+2x^2} \cancel{-3x} \cancel{+5}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-4x + 3)}{\cancel{h}}$$

$$= -4x + 3$$

c) Find the slope of the tangent line to $f(x)$ at $x = 2$.

$$f'(x) = -4x + 3$$

$$m = f'(2) = -8 + 3$$

$$m = -5$$

d) What is the equation of the tangent line to $f(x)$ at $x = 2$.

① <u>Slope</u>: $m = -5$

② <u>POT</u>: $(2, f(2)) = (2, -7)$

③ <u>Tangent Line</u>

$$y + 7 = -5(x - 2)$$

$$y = -5x + 3$$

4. Consider the functions below. Use your shortcut rules to differentiate. (15 points)

a) If $F(z) = \dfrac{A + Bz + Cz^2}{z^2}$, find $F'(z)$.

b) Find $\dfrac{dV}{dt}\Big|_{t=-2}$, where $V(t) = (3t^2 + 4) \cdot e^t$.

c) Let $g(x) = \dfrac{f(x)}{3x^2 + x - 1}$ where $f(-2) = 3$ and $f'(-2) = -4$, find $g'(-2)$.

a) $F(z) = Az^{-2} + Bz^{-1} + C$

$F'(z) = -2Az^{-3} - Bz^{-2}$

$= -z^{-3}(2A + Bz)$

$= -\dfrac{2A + Bz}{z^3}$

b) $\dfrac{dV}{dt} = (3t^2+4)\cdot e^t + e^t(6t)$

$= e^t \cdot (3t^2 + 6t + 4)$

$\dfrac{dV}{dt}\Big|_{t=-2} = e^{-2}(3(-2)^2 + 6(-2) + 4)$

$= e^{-2}(12 - 12 + 4)$

$= 4e^{-2}$

$= \dfrac{4}{e^2}$

c) $g'(x) = \dfrac{(3x^2+x-1)\cdot f'(x) - f(x)(6x+1)}{(3x^2+x-1)^2}$

$g'(-2) = \dfrac{(12-2-1)\cdot f'(-2) - f(-2)(-11)}{9^2}$

$= \dfrac{9(-4) - 3(-11)}{81}$

$= \dfrac{-36+33}{81}$

$= -\dfrac{3}{81}$

$g'(-2) = -\dfrac{1}{27}$

5. Consider the function $f(x) = \dfrac{3x}{1 + 5x^2}$. (15 points)

a) Find $\dfrac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{(1+5x^2)\cdot 3 - 3x\cdot(10x)}{(1+5x^2)^2}$$

$$= \frac{3+15x^2 - 30x^2}{(1+5x^2)^2}$$

$$= \frac{3-15x^2}{(1+5x^2)^2}$$

b) Find the slope of the tangent line and normal line at the point $\left(1, \frac{1}{2}\right)$

<u>Slope of tangent line:</u>

$$m = y'(1) = \frac{3-15(1)^2}{[1+5(1)^2]^2} = \frac{-12}{36} = -\frac{1}{3}$$

<u>Slope of normal line:</u>

$$m = 3$$

c) Find equations of the tangent line and normal line to the given curve at the point $\left(1, \frac{1}{2}\right)$.

<u>Tangent Line</u>

$$y - \frac{1}{2} = -\frac{1}{3}(x-1)$$

$$y = -\frac{1}{3}x + \frac{1}{3} + \frac{1}{2}$$

$$y = -\frac{1}{3}x + \frac{5}{6}$$

Normal Line

$$y - \frac{1}{2} = 3(x-1)$$

$$y = 3x - 3 + \frac{1}{2}$$

$$y = 3x - \frac{5}{2}$$

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Part 2: This part has 2 questions. A calculator may be used on this part of the exam. Show all your work on the space below each problem. Do not write on the back. Appropriate evidence will receive partial credit. If you need additional room for your solution please use "Spillage" on the last page. Good Luck!!

6. While visiting Mars, Thor tossed up his hammer (Mjölnir) vertically from the surface at a height of 9 feet. The hammer is tossed up with an initial velocity of 50 ft/sec. Determine the following if the position function of the hammer is given by $s(t) = 9 + 50t - 6.1t^2$, $t \geq 0$. (15 points)

   a) Determine the average velocity of the hammer over the time interval 2.108 seconds to 6.753 seconds.

   b) Determine the velocity function $v(t)$ for the hammer, and find the velocity and the direction of the hammer when $t = 2.108$ seconds.

   c) What is the instantaneous velocity of the hammer when it hits the ground?

a) $V_{avg} = \dfrac{s(2.108) - s(6.753)}{2.108 - 6.753} \overset{CAS}{=} -4.052 \text{ ft/sec}$

b) $v(t) = s'(t) = 50 - 12.2t$

$v(2.108) \overset{CAS}{=} 24.282 \frac{ft}{sec}$

The hammer is moving up.

c) Let $s(t) = 0$

By CAS we get $t = 8.373$ sec when the hammer hits the ground.

$v(8.373) \overset{CAS}{=} -52.151$ ft/sec is the velocity when the hammer hits the ground.

7. Consider the function $f(x) = -\dfrac{x^2 + 7x + 1}{e^x}$. (15 points)

a) Find $f'(x)$.

$$f'(x) \overset{CAS}{=} \frac{x^2+5x-6}{e^x} \quad \text{or} \quad f'(x) = (x^2+5x-6)\cdot e^{-x}$$

By hand we get

$$f'(x) = -\frac{e^x(2x+7) - (x^2+5x-6)\cdot e^x}{e^{2x}}$$

$$f'(x) = -\frac{e^x\cdot(-x^2-5x+6)}{e^{2x}} = \frac{x^2+5x-6}{e^x}$$

b) Find the $x$-coordinates of all horizontal tangent lines to the function $f(x)$.

Let $f'(x)=0$

$$\frac{x^2+5x-6}{e^x}=0$$

$$x^2+5x-6=0$$

$$(x+6)(x-1)=0$$

$x=-6$ and $x=1$ are the $x$-coordinates of all horizontal tangent lines to $f(x)$.

c) Find the equations of the tangent and normal lines at the $x$-values you found in part (b).

Points of Tangency

$$(-6, f(-6)) = (-6, 5e^6) = (-6, 2017.144)$$

$$(1, f(1)) = (1, -9e^{-1}) = (1, -3.311)$$

The horizontal tangents are $y=5e^6$ and $y=-9e^{-1}$!

The normal lines are $x=-6$ and $x=1$!