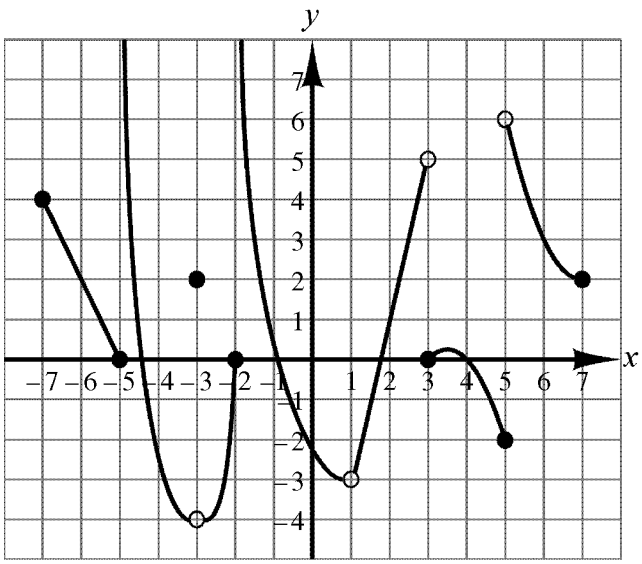
Math 1A

Test 1 Review

For problems 1 through 6 a calculator may NOT be used.

1. Consider the function $f(x)$ below over the interval $[-7,\ 7]$.



   a) From the graph of f, state the numbers at which f is discontinuous, and describe the type of discontinuity.

   b) For each of the numbers stated in part (a), determine whether f is continuous from the right, or from the left, or neither.

   c) State the intervals on which f is continuous.

2. Consider the function $f(x) = \dfrac{\sqrt{4x^2+5}}{5x+10}$.

   a) What is the domain of f?

   b) Using limits, determine the horizontal and vertical asymptotes for $f(x)$, if they exist.

   c) Is $f(x)$ continuous at $x = 2$? Explain your answer.

3. Suppose that u and v are functions of x that are differentiable at $x = -2$, and that $u(-2) = 5$, $u'(-2) = -3$, $v(-2) = -1$, and $v'(-2) = 2$ Find the value of $\dfrac{d}{dx}[u \cdot v]$ at $x = -2$.

4. Let $f(x) = -2x^2 + 3x - 5$. (Show all symbolic work, NO CAS!!)

   a) State a definition for the derivative of a function.

   b) Using the definition stated in part (a), find $f'(x)$.

   c) Find the slope of the tangent line to f at $x = -2$.

   d) What is the equation of the tangent line to f at $x = -2$.

5. Consider the functions below. (Show all symbolic work, NO CAS!!)

   a) Find $f'(x)$ for $f(x) = \dfrac{6\sqrt{x} - 2x^2}{x^3}$.

   b) Find $\dfrac{dy}{dx}\,|_{x=1}$, where $y = \sqrt[3]{x} \cdot e^x$.

   c) If $g(x) = \dfrac{f(x)}{2x^2 + x}$ where $f(3) = -2$ and $f'(3) = 4$, find $g'(3)$.

6. Evaluate the limits below by the indicated method. Answer with a number, $+\infty$, $-\infty$, or does not exist.

   a) $\lim\limits_{x \to 1} \dfrac{x - 1}{\sqrt{10 - x} - 3}$, Show all symbolic work, NO CAS!!

   b) $\lim\limits_{x \to \infty} \dfrac{\sin x^2}{x^2}$, Show all symbolic work, NO CAS!!

   c) $\lim\limits_{x \to -\infty} \dfrac{2x}{\sqrt{x^2 + 4}}$, Show all symbolic work, NO CAS!!

A calculator may be used to help solve problem 7 and 8.

7. If a tank holds 8000 gallons of gasoline, which drains from the bottom of the tank in 45 minutes, then Torricelli's law gives the volume V of gasoline remaining in the tank after t minutes as

$$V(t) = 8000\left(1 - \frac{t}{45}\right)^2 \qquad 0 \le t \le 45$$

a) Find the average rate of change of the gasoline emptying out of the tank over the time interval [10, 15], indicate your units.

b) Find the instantaneous rate of change of the gasoline emptying out of the tank at the time $t = 25$ minutes, indicate your units.

c) Find the time t where instantaneous rate of change is equal to the average rate of change found in part (a).

8. Consider the function $f(x) = \dfrac{-x^2 + 5x - 5}{e^x}$. (15 points)

a) Find $f'(x)$.

b) Find the equation of all horizontal tangent lines to the function $f(x)$.

c) Find the equation of the tangent line at $x = 3$.