#1

(a) $x=-5$ Infinity

$x=-3$ Removable

$x=-2$ Infinity

$x=1$ Undefined

$x=3$ Jump

$x=5$ Jump

(b) $x=-5$ From the left

$x=-3$ Neither

$x=-2$ From the left

$x=1$ Neither

$x=3$ From the right

$x=5$ From the left

(c) $x \in [-7,-5] \cup (-5,-3) \cup (-3,-2] \cup (-2,1) \cup (1,3) \cup [3,5] \cup (5,7]$

#2)

(a) Domain: $x \in (-\infty, -2) \cup (-2, +\infty]$

(b) Horizontal Asymptotes:

$$\lim_{x \to +\infty} \frac{\sqrt{4x^2+5}}{5x+10} \overset{DTE}{=} \lim_{x \to +\infty} \frac{\sqrt{4x^2}}{5x}$$

$$= \lim_{x \to +\infty} \frac{2|x|}{5x}$$

Note: $x > 0$, so $|x| = x$

$$= \lim_{x \to +\infty} \frac{2x}{5x}$$

$$= \frac{2}{5}$$

$$\lim_{x \to -\infty} \frac{\sqrt{4x^2+5}}{5x+10} \overset{DTE}{=} \lim_{x \to -\infty} \frac{\sqrt{4x^2}}{5x}$$

$$= \lim_{x \to -\infty} \frac{2|x|}{5x}$$

Note: $x < 0$, so $|x| = -x$

$$= \lim_{x \to -\infty} \frac{-2x}{5x}$$

$$= -\frac{2}{5}$$

#2b) Continued

Hence $y = \frac{2}{5}$ and $y = -\frac{2}{5}$ are horizontal asymptotes.

Vertical Asymptote: We need to examine $x = -2$ from the left or right.

$$\lim_{x \to -2^-} \frac{\sqrt{4x^2+5}}{5x+10}$$

Sign Test let $x = -2.1$

We get: $\frac{(+)}{(-)} = -$

Hence, $\lim_{x \to -2^-} \frac{\sqrt{4x^2+5}}{5x+10} = -\infty$

Thus $x = -2$ is a vertical asymptote.

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(c) Since: (1) $f(2) = \frac{\sqrt{4 \cdot 2^2 + 5}}{5(2)+10} = \frac{\sqrt{21}}{20}$

(2) $\lim_{x \to 2} \frac{\sqrt{4x^2+5}}{5x+10} = \frac{\sqrt{21}}{20}$

(3) $\lim_{x \to 2} \frac{\sqrt{4x^2+5}}{5x+10} = f(2)$

#2c) continued

Therefore, by definition $f(x)$ is continuous at $x=2$.

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#3)
$$\frac{d}{dx}[u \cdot v] = u \cdot \frac{dv}{dx} + \frac{du}{dx} \cdot v = u \cdot v' + u' \cdot v$$

$$\left.\frac{d}{dx}(u \cdot v)\right|_{x=-2} = u(-2) \cdot v'(-2) + u'(-2) \cdot v(-2)$$

$$= 5 \cdot 2 + (-3)(-1)$$

$$= 10 + 3$$

$$= 13$$

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#4) (a) Given a function $f(x)$ the derivative $f'(x)$ is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

#4b) continued

$$f(x) = -2x^2+3x-5$$

$$f'(x) = \lim_{h\to 0} \frac{-2(x+h)^2+3(x+h)-5-(-2x^2+3x-5)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-2x^2}-4xh-2h^2+\cancel{3x}+3h-\cancel{5}+\cancel{2x^2}-\cancel{3x}+\cancel{5}}{h}$$

$$= \lim_{h\to 0} \frac{-4xh-2h^2+3h}{h}$$

$$= \lim_{h\to 0} -4x-2h+3$$

$$= -4x+3$$

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(c) $f'(-2) = -4(-2)+3 = 11$

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(d) (1) $m = 11$

(2) Point of Tangency: $(-2, f(-2)) = (-2, -19)$

(3) $y - y_1 = m(x - x_1)$

$y + 19 = 11(x+2)$ or $y = 11x + 3$

#5) (a)

$$f(x) = \frac{6\sqrt{x} - 2x^2}{x^3} = \frac{6x^{\frac{1}{2}}}{x^3} - \frac{2x^2}{x^3} = 6x^{-\frac{5}{2}} - 2x^{-1}$$

$$f'(x) = -\frac{30}{2}x^{-\frac{7}{2}} + 2x^{-2}$$

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(b)

$$y = \sqrt[3]{x} \cdot e^x = x^{\frac{1}{3}} \cdot e^x$$

$$\frac{dy}{dx} = x^{\frac{1}{3}} \cdot e^x + \frac{1}{3}x^{-\frac{2}{3}} \cdot e^x$$

$$\left.\frac{dy}{dx}\right|_{x=1} = (1)^{\frac{1}{3}} \cdot e^1 + \frac{1}{3}(1)^{-\frac{2}{3}} \cdot e^1$$

$$= e + \frac{1}{3}e$$

$$= \frac{4}{3}e$$

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(c) $$g(x) = \frac{f(x)}{2x^2 + x}$$

$$g'(x) = \frac{(2x^2 + x) \cdot f'(x) - f(x)(4x+1)}{[2x^2 + x]^2}$$

#5c) continued

$$g'(3) = \frac{(2\cdot 3^2+3)\cdot f'(3) - f(3)(4\cdot 3+1)}{[2\cdot 3^2+3]^2}$$

$$= \frac{21\cdot 4 - (-2)(13)}{[21]^2}$$

$$= \frac{84+26}{}$$

$$= \frac{110}{441}$$

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#6)

$$\lim_{x\to 1} \frac{x-1}{\sqrt{10-x}-3} \cdot \frac{\sqrt{10-x}+3}{\sqrt{10-x}+3}$$

$$= \lim_{x\to 1} \frac{(x-1)\cdot(\sqrt{10-x}+3)}{10-x-9}$$

$$= \lim_{x\to 1} \frac{\overset{-1}{\cancel{(x-1)}}(\sqrt{10-x}+3)}{\cancel{1-x}}$$

$$= -1(\sqrt{9}+3)$$

$$= -6$$

#6 (b) $\lim_{x \to \infty} \frac{\sin(x^2)}{x^2}$

$$-1 \le \sin(x^2) \le 1$$

Since $x^2 \geq 0$, then $\frac{1}{x^2} \geq 0$. So we get

$$-\frac{1}{x^2} \le \frac{\sin(x^2)}{x^2} \le \frac{1}{x^2}$$

We know that $\lim_{x \to \infty} -\frac{1}{x^2} = 0$ and $\lim_{x \to \infty} \frac{1}{x^2} = 0$

Hence, by the "Squeeze Theorem" $\lim_{x \to \infty} \frac{\sin(x^2)}{x^2} = 0$

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(c) $\lim_{x \to -\infty} \frac{2x}{\sqrt{x^2+4}} \overset{DTE}{=} \lim_{x \to -\infty} \frac{2x}{\sqrt{x^2}}$

$$= \lim_{x \to -\infty} \frac{2x}{|x|}$$

Note:
$x < 0$, so
$|x| = -x$

$$= \lim_{x \to -\infty} \frac{2\cancel{x}}{-\cancel{x}}$$

$$= -2$$

#7

(a) $V(t) = 8000\left(1 - \frac{t}{45}\right)^2$

$V(10) \overset{CAS}{\approx} 4839.506$ gallons

$V(15) \overset{CAS}{\approx} 3555.556$ gallons

Average Rate at Change $= \dfrac{V(10) - V(15)}{10 - 15}$

$= -256.790$ gallons/minute

(b) $V'(t) \overset{CAS}{=} -\frac{3200}{9} + \frac{640}{81}t$

$V'(25) \overset{CAS}{\approx} -158.025$ gallons/minute

(c) $V'(t) = -256.790$

$-\frac{3200}{9} + \frac{640}{81}t = -256.790$

when $t \overset{CAS}{\approx} 12.5$ minutes

#8) $f(x) = \dfrac{-x^2+5x-5}{e^x}$

(a) $f'(x) \overset{CAS}{=} \dfrac{x^2-7x+10}{e^x}$

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(b) Let $f'(x)=0$. By CAS

$x=2$ or $x=5$

Horizontal Tangent Lines have slope zero, so points of tangency are

$(2, f(2)) \overset{CAS}{=} (2, e^{-2})$ & $(5, f(5)) \overset{CAS}{=} (5, -5e^{-5})$

Hence horizontal tangent lines are

$y = e^{-2}$ and $y = -5e^{-5}$

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(c) ① $m = f'(3) \overset{CAS}{=} -2e^{-3}$

② Point of Tangency $(3, f(3)) \overset{CAS}{=} (3, e^{-3})$

③ $y - y_1 = m(x - x_1)$

$y - e^{-3} = -2e^{-3}(x-3)$