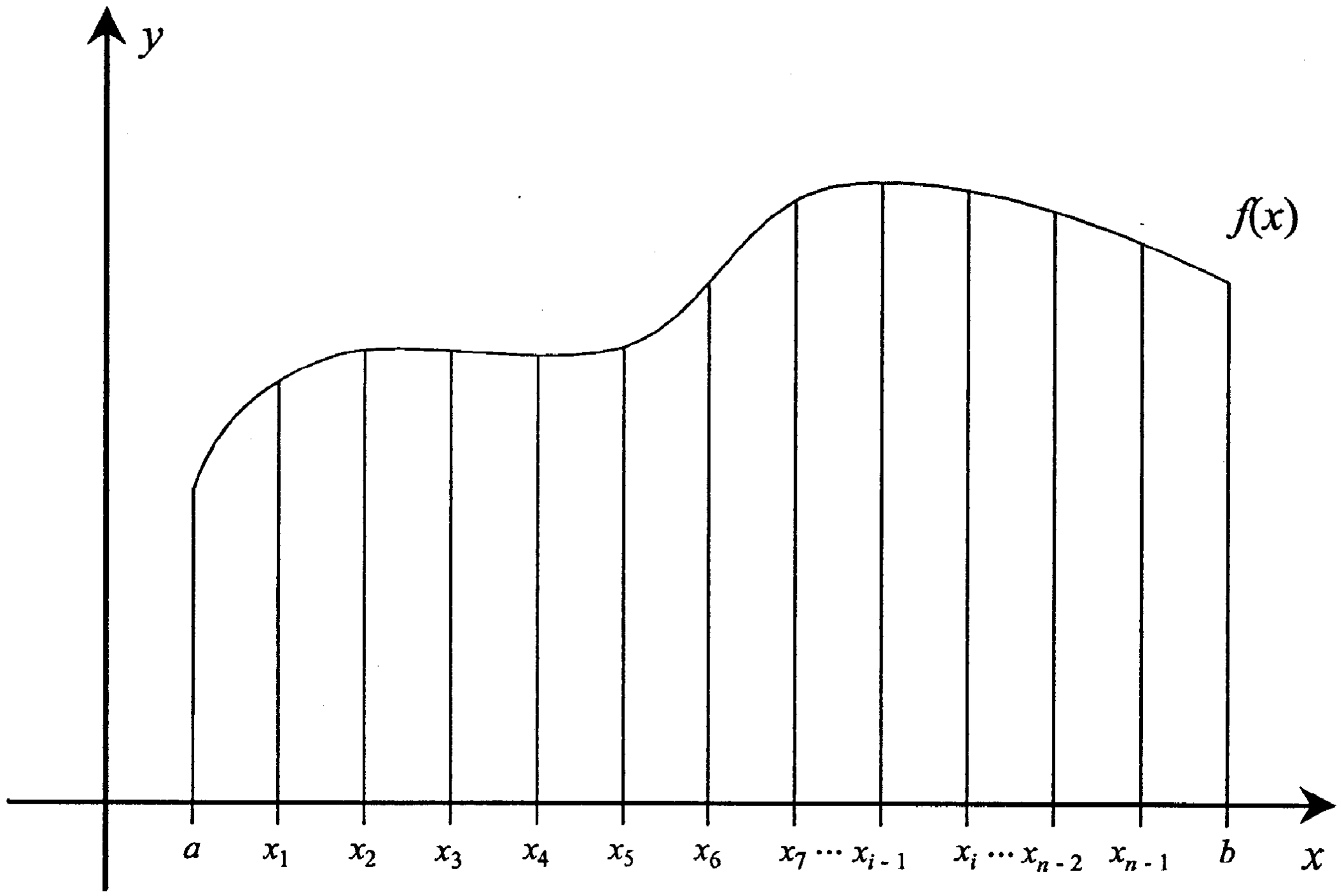
Definition of a Definite Integral: If f is a continuous function defined on a closed interval $[a,\ b]$, we divide the interval $[a,\ b]$ into n subintervals of equal width $\Delta x = \dfrac{b-a}{n}$. We let $x_0 = a,\ x_1,\ x_2,\ x_3, \ldots, x_{i-1},\ x_i, \ldots,\ x_{n-1},\ x_n = b$ be the endpoints of these subintervals and we choose sample points $x_1^*,\ x_2^*, \cdots, x_{i-1}^*,\ x_i^*, \cdots,\ x_{n-1}^*,\ x_n^*$ in these subintervals, so x_i^* lies in the ith subinterval $[x_{i-1},\ x_i]$. Then the **definite integral of f from a to b** is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$



$$\sum_{k=1}^{n} 1 = n$$

$$\sum_{k=1}^{n} k = \frac{n}{2} + \frac{n^2}{2} = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n}{6} + \frac{n^2}{2} + \frac{n^3}{3} = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2}{4} + \frac{n^3}{2} + \frac{n^4}{4} = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{k=1}^{n} k^4 = \frac{-n}{30} + \frac{n^3}{3} + \frac{n^4}{2} + \frac{n^5}{5} = \frac{n(n+1)(2n+1)\left(3n^2+3n-1\right)}{30}$$

$$\sum_{k=1}^{n} k^5 = \frac{-n^2}{12} + \frac{5n^4}{12} + \frac{n^5}{2} + \frac{n^6}{6} = \frac{n^2(n+1)^2\left(2n^2+2n-1\right)}{12}$$

$$\sum_{k=1}^{n} k^6 = \frac{n}{42} - \frac{n^3}{6} + \frac{n^5}{2} + \frac{n^6}{2} + \frac{n^7}{7} = \frac{n(n+1)(2n+1)\left(3n^4+6n^3-3n+1\right)}{42}$$

$$\sum_{k=1}^{n} k^7 = \frac{n^2}{12} - \frac{7n^4}{24} + \frac{7n^6}{12} + \frac{n^7}{2} + \frac{n^8}{8} = \frac{n^2(n+1)^2\left(3n^4+6n^3-n^2-4n+2\right)}{24}$$