Chapter 6

Momentum and Collisions
Momentum

Momentum

- a property of moving things
- means inertia in motion
- more specifically, mass of an object multiplied by its velocity
- in equation form: mass $\times$ velocity

$(\text{momentum} = mv)$
The linear momentum $\mathbf{p}$ of an object of mass $m$ moving with a velocity $\mathbf{v}$ is defined as the product of the mass and the velocity

- $\mathbf{p} = m\mathbf{v}$

SI Units are kg m / s

Vector quantity, the direction of the momentum is the same as the velocity’s
Momentum components

- $p_x = mv_x$ and $p_y = mv_y$
- Applies to two-dimensional motion
Problem 6.4

- A 0.10-kg ball is thrown straight up into the air with an initial speed of 15 m/s. Find the momentum of the ball (a) at its maximum height and (b) halfway to its maximum height.
Change in Momentum

If the momentum of an object changes, then either the mass or the velocity or both change.
If the mass remains unchanged, then the velocity changes and acceleration, and therefore force, occurs.

\[ \Delta P = \Delta (mv) = m(\Delta v) = m(a\Delta t) = (ma)\Delta t = F\Delta t \]
Impulse

When a single, constant force acts on the object, there is an **impulse** delivered to the object

\[ \vec{I} = \vec{F} \Delta t \]

- \( \vec{I} \) is defined as the *impulse*
- Vector quantity, the direction is the same as the direction of the force
Impulse-Momentum Theorem

- The theorem states that the impulse acting on the object is equal to the change in momentum of the object.

\[ \vec{F} \Delta t = \Delta \vec{p} = m \vec{v}_f - m \vec{v}_i \]

- If the force is not constant, use the average force applied.
Problem 6.1

A ball of mass 0.150 kg is dropped from rest from a height of 1.25 m. It rebounds from the floor to reach a height of 0.960 m. What impulse was given to the ball by the floor?
Problem 6.10

A 0.500-kg football is thrown toward the east with a speed of 15.0 m/s. A stationary receiver catches the ball and brings it to rest in 0.0200 s. (a) What is the impulse delivered to the ball as it’s caught? (b) What is the average force exerted on the receiver?
Problem 6.17

A car of mass $1.6 \times 10^3$ kg is traveling east at a speed of 25 m/s along a horizontal roadway. When its brakes are applied, the car stops in 6.0 s. What is the average horizontal force exerted on the car while it is braking?
Problem 6.15

The front 1.20 m of a 1 400-kg car is designed as a “crumple zone” that collapses to absorb the shock of a collision. If a car traveling 25.0 m/s stops uniformly in 1.20 m, (a) how long does the collision last, (b) what is the magnitude of the average force on the car, and (c) what is the acceleration of the car? Express the acceleration as a multiple of the acceleration of gravity.
Impulse Changes
Momentum

The greater the impulse exerted on something, the greater the change in momentum.

- in equation form: $Ft = \Delta(mv)$
Newtons 2\textsuperscript{nd} Law - Restated

- In order to \textit{change} the momentum of an object, a force must be applied
- The time rate of change of momentum of an object is equal to the net force acting on it

\[
\frac{\Delta \vec{p}}{\Delta t} = \frac{m(v_f - v_i)}{\Delta t} = F_{\text{net}}
\]
- Gives an alternative statement of Newton’s second law
Case 1: increasing momentum

- apply the greatest force for as long as possible and you extend the time of contact
- force can vary throughout the duration of contact

examples:
- golfer swings a club and follows through
- baseball player hits a ball and follows through
Impulse Changes
Momentum

examples:

when a car is out of control, it is better to hit a haystack than a concrete wall

physics reason: same impulse either way, but extension of hitting time reduces the force
Impulse Applied to Auto Collisions

- The most important factor is the collision time or the time it takes the person to come to a rest
  - This will reduce the chance of dying in a car crash

- Ways to increase the time
  - Seat belts
  - Air bags
Air Bags

- The air bag increases the time of the collision
- It will also absorb some of the energy from the body
- It will spread out the area of contact
  - decreases the pressure
  - helps prevent penetration wounds
Impulse Changes
Momentum

example (continued):
in jumping, bend your knees when your feet make contact with the ground because the extension of time during your momentum decrease reduces the force on you.

\[ F \int t = \text{change in momentum} \]

\[ \int t = \text{change in momentum} \]
Case 3: decreasing momentum over a short time

- short time interval produces large force

example: Karate expert splits a stack of bricks by bringing her arm and hand swiftly against the bricks with considerable momentum. Time of contact is brief and force of impact is huge.
Bouncing

Impulses are generally greater when objects bounce.

example:

Catching a falling flower pot from a shelf with your hands. You provide the impulse to reduce its momentum to zero. If you throw the flower pot up again, you provide an additional impulse. This “double impulse” occurs when something bounces.
Bouncing

Pelton wheel designed to "bounce" water when it makes a U-turn against the curved paddle
Conservation of Momentum

Law of conservation of momentum:
In the absence of an external force, the momentum of a system remains unchanged.
Conservation of Momentum

- Momentum in an isolated system in which a collision occurs is conserved
  - A collision may be the result of physical contact between two objects
  - “Contact” may also arise from the electrostatic interactions of the electrons in the surface atoms of the bodies
  - An isolated system will have no external forces
The principle of conservation of momentum states when no external forces act on a system consisting of two objects that collide with each other, the total momentum of the system remains constant in time.

Specifically, the total momentum before the collision will equal the total momentum after the collision.
Conservation of Momentum, cont.

Mathematically:

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]

- Momentum is conserved for the system of objects
- The system includes all the objects interacting with each other
- Assumes only internal forces are acting during the collision
- Can be generalized to any number of objects
Momentum Video

Conservation of momentum in space
http://www.youtube.com/watch?v=4IYDb6K5UF8

Physics class room explanation
http://www.physicsclassroom.com/class/momentum/u4l2b.cfm
Notes About A System

- Remember conservation of momentum applies to the system
- You must define the isolated system
Problem 6.18

A 730-N man stands in the middle of a frozen pond of radius 5.0 m. He is unable to get to the other side because of a lack of friction between his shoes and the ice. To overcome this difficulty, he throws his 1.2-kg physics textbook horizontally toward the north shore at a speed of 5.0 m/s. How long does it take him to reach the south shore?
Types of Collisions

- Momentum is conserved in any collision
- Inelastic collisions
  - Kinetic energy is not conserved
    - Some of the kinetic energy is converted into other types of energy such as heat, sound, work to permanently deform an object
  - Perfectly inelastic collisions occur when the objects stick together
    - Not all of the KE is necessarily lost
More Types of Collisions

- Elastic collision
  - both momentum and kinetic energy are conserved

- Actual collisions
  - Most collisions fall between elastic and perfectly inelastic collisions
More About Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision.
- Conservation of momentum becomes
  \[ m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \]
Problem 6.25

A railroad car of mass $2.00 \times 10^4$ kg moving at 3.00 m/s collides and couples with two coupled railroad cars, each of the same mass as the single car and moving in the same direction at 1.20 m/s. (a) What is the speed of the three coupled cars after the collision? (b) How much kinetic energy is lost in the collision?
Some General Notes About Collisions

- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs
More About Elastic Collisions

- Both momentum and kinetic energy are conserved
- Typically have two unknowns
  \[
  m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}
  \]
  \[
  \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2
  \]
- Solve the equations simultaneously
Elastic Collisions, cont.

- A simpler equation can be used in place of the KE equation

\[ \mathbf{v}_{1i} - \mathbf{v}_{2i} = - (\mathbf{v}_{1f} - \mathbf{v}_{2f}) \]
Summary of Types of Collisions

- In an elastic collision, both momentum and kinetic energy are conserved.
- In an inelastic collision, momentum is conserved but kinetic energy is not.
- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same.
Problem Solving for One-Dimensional Collisions

- **Coordinates:** Set up a coordinate axis and define the velocities with respect to this axis
  - It is convenient to make your axis coincide with one of the initial velocities

- **Diagram:** In your sketch, draw all the velocity vectors and label the velocities and the masses
Conservation of Momentum:
Write a general expression for the total momentum of the system before and after the collision
- Equate the two total momentum expressions
- Fill in the known values
Problem Solving for One - Dimensional Collisions, 3

- **Conservation of Energy**: If the collision is elastic, write a second equation for conservation of KE, or the alternative equation
  - This only applies to perfectly elastic collisions

- **Solve**: the resulting equations simultaneously
Sketches for Collision Problems

- Draw “before” and “after” sketches
- Label each object
  - include the direction of velocity
  - keep track of subscripts
Sketches for Perfectly Inelastic Collisions

- The objects stick together
- Include all the velocity directions
- The “after” collision combines the masses
Problem 6.33

A 5.00-g object moving to the right at 20.0 cm/s makes an elastic head-on collision with a 10.0-g object that is initially at rest. Find (a) the velocity of each object after the collision and (b) the fraction of the initial kinetic energy transferred to the 10.0-g object.
Glancing Collisions

- For a general collision of two objects in three-dimensional space, the conservation of momentum principle implies that the total momentum of the system in each direction is conserved.

\[ m_1 v_{1x} + m_2 v_{2x} = m_1 v_{1f_x} + m_2 v_{2f_x} \quad \text{and} \quad m_1 v_{1y} + m_2 v_{2y} = m_1 v_{1f_y} + m_2 v_{2f_y} \]

- Use subscripts for identifying the object, initial and final velocities, and components.
Glancing Collisions

- The “after” velocities have x and y components
- Momentum is conserved in the x direction and in the y direction
- Apply conservation of momentum separately to each direction
Problem Solving for Two-Dimensional Collisions

**Coordinates:** Set up coordinate axes and define your velocities with respect to these axes
- It is convenient to choose the x- or y-axis to coincide with one of the initial velocities

**Draw:** In your sketch, draw and label all the velocities and masses
Problem Solving for Two-Dimensional Collisions, 2

- **Conservation of Momentum:** Write expressions for the x and y components of the momentum of each object before and after the collision.

- Write expressions for the total momentum before and after the collision in the x-direction and in the y-direction.
Conservation of Energy: If the collision is elastic, write an expression for the total energy before and after the collision:
- Equate the two expressions
- Fill in the known values
- Solve the quadratic equations
  - Can’t be simplified
Problem Solving for Two-Dimensional Collisions, 4

- **Solve** for the unknown quantities
  - Solve the equations simultaneously
  - There will be two equations for inelastic collisions
  - There will be three equations for elastic collisions
An 8.00-kg object moving east at 15.0 m/s on a frictionless horizontal surface collides with a 10.0-kg object that is initially at rest. After the collision, the 8.00-kg object moves south at 4.00 m/s. (a) What is the velocity of the 10.0-kg object after the collision? (b) What percentage of the initial kinetic energy is lost in the collision?
Problem 6.41

A 2 000-kg car moving east at 10.0 m/s collides with a 3 000-kg car moving north. The cars stick together and move as a unit after the collision, at an angle of 40.0° north of east and a speed of 5.22 m/s. Find the speed of the 3 000-kg car before the collision.
Problem 6.42

Two automobiles of equal mass approach an intersection. One vehicle is traveling with velocity 13.0 m/s toward the east, and the other is traveling north with speed $v_{2i}$. Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 55.0° north of east. The speed limit for both roads is 35 mi/h, and the driver of the northward-moving vehicle claims he was within the limit when the collision occurred. Is he telling the truth?
Rocket Propulsion

- The operation of a rocket depends on the law of conservation of momentum as applied to a system, where the system is the rocket plus its ejected fuel.
  - This is different than propulsion on the earth where two objects exert forces on each other:
    - road on car
    - train on track
The rocket is accelerated as a result of the thrust of the exhaust gases.

This represents the inverse of an inelastic collision:
- Momentum is conserved
- Kinetic Energy is increased (at the expense of the stored energy of the rocket fuel)
The initial mass of the rocket is $M + \Delta m$
- $M$ is the mass of the rocket
- $m$ is the mass of the fuel
The initial velocity of the rocket is $\vec{v}$
Rocket Propulsion

- The rocket’s mass is $M$
- The mass of the fuel, $\Delta m$, has been ejected
- The rocket’s speed has increased to $\vec{v} + \Delta \vec{v}$
Rocket Propulsion, final

The basic equation for rocket propulsion is:

\[ v_f - v_i = v_e \ln \left( \frac{M_i}{M_f} \right) \]

- \( M_i \) is the initial mass of the rocket plus fuel
- \( M_f \) is the final mass of the rocket plus any remaining fuel
- The speed of the rocket is proportional to the exhaust speed
Thrust of a Rocket

- The thrust is the force exerted on the rocket by the ejected exhaust gases.
- The instantaneous thrust is given by

\[ Ma = M \frac{\Delta v}{\Delta t} = v_e \frac{\Delta M}{\Delta t} \]

- The thrust increases as the exhaust speed increases and as the burn rate \((\Delta M/\Delta t)\) increases.
Backup

20 minute video on crash physics
The average force can be thought of as the constant force that would give the same impulse to the object in the time interval as the actual time-varying force gives in the interval.
The impulse imparted by a force during the time interval $\Delta t$ is equal to the area under the force-time graph from the beginning to the end of the time interval.

Or, the impulse is equal to the average force multiplied by the time interval, $\vec{F}_{\text{av}} \Delta t = \Delta \vec{p}$.